

$$\hat{\beta}_5 \equiv \hat{\beta}_{\text{Advert}}$$

$$a = (0, 0, 0, 0, 1, 0, 0, 1, 0)$$

$$\hat{\beta}_8 \equiv \hat{\beta}_{\text{region south: Advert}}$$

$$a^T \hat{\beta} = (0, \dots, 0 \overset{\text{rth pos}}{1}, 0 \overset{\text{8th pos}}{-1}) \hat{\beta} = \hat{\beta}_5 + \hat{\beta}_8$$

$$\begin{aligned} \text{var} [a^T \hat{\beta}] &= a^T \text{var} [\hat{\beta}] a \\ &= [0 \dots 0 \underset{\text{rth pos}}{1} \underset{\text{8th pos}}{-1} \dots 0] \text{var} [\hat{\beta}] \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ \vdots \end{bmatrix} \\ &= \underbrace{\text{var} [\hat{\beta}_5]}_{(5,5) \text{ entry}} + \underbrace{\text{var} [\hat{\beta}_8]}_{(8,8) \text{ entry}} \\ &\quad + \underbrace{\text{cov} [\hat{\beta}_5, \hat{\beta}_8]}_{(5,8) \text{ entry}} + \underbrace{\text{cov} [\hat{\beta}_8, \hat{\beta}_5]}_{(8,5) \text{ entry}} \end{aligned}$$

$$[x^T A]_z = \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_i y_j$$