Computational Stability Analysis of Chatter in Turning

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Machine tool chatter is one of the major constraints that limits productivity of the turning process. It is a self-excited vibration that is mainly caused by the interaction between the machine-tool/workpiece structure and the cutting process dynamics. This work introduces a general method which avoids lengthy algebraic (symbolic) manipulations in deriving, a characteristic equation. The solution scheme is simple and robust since the characteristic equation is numerically formulated as a single variable equation whose variable is well bounded rather than two nonlinear algebraic equations with unbounded variables. An asymptotic stability index is also introduced for a relative stability analysis. The method can be applied to other machining processes, as long as the system equations can be expressed as a set of linear time invariant difference-differential equations.

1 Introduction

Machine tool chatter is one of the major constraints that limits productivity of the turning process. It is a self-excited vibration that is mainly caused by the interaction between the machine-tool/workpiece structure and the cutting process dynamics. In addition to the steady cutting force in turning, there are also other small force disturbances in the cutting force due to the chip breakage, interference from continuous chips, or hard spots on the workpiece. Such disturbances make the machine-tool/workpiece structure respond with a change in the relative displacement between the cutting tool and the workpiece. This change leads to a variation in the cutting parameters (e.g., chip thickness) and, therefore, again affects the resultant cutting force. Consequently, the interaction between the machine-tool/workpiece structure and the cutting process dynamics can be described as a closed loop system. Chatter occurs when this closed loop system becomes unstable. From the energy perspective, chatter occurs when the energy from the machine spindle drive is not completely consumed by the structural damping and the cutting friction. Chatter leads to poor surface finish and dimensional accuracy in the machined part, fast wear and breakage of the cutting tool, and even severe damage to the machine tool.

From the process planning point of view, a stability chart (e.g., Fig. 1) is usually constructed by experiment or theoretical prediction in order to select appropriate cutting parameters without causing chatter problems. In the chart, the stability limit (e.g., limit of width of cut in orthogonal cutting) is generally a function of feed, cutting speed, and spindle speed [Minis, 1990]. When the feed and the cutting speed are given, the stability limit can be plotted as a function of spindle speed, i.e., the lobed borderline. Furthermore, the lowest stability limit for given feed and cutting speed is defined as the asymptotic borderline (Merrit, 1965), which is independent of the spindle speed. The asymptotic borderlines with respect to different cutting speeds then form a tangent plane. When the tangent plane is subject to a constant workpiece radius, there results a constraint that the cutting speed is proportional to the spindle speed. Eventually, the constraint on the tangent plane forms a tangent borderline (Wu, 1985).

Many methods have been developed to predict the onset of chatter. Based on a specific expression in the cutting process, Tobias et al. (1965) developed a graphical method and an algebraic method to determine the onset of instability of a system with multiple degrees of freedom. Merrit (1965) assumed no dynamics in the cutting process and developed a theory to calculate the stability boundary by plotting the harmonic solutions of the system characteristic equation on a special chart. He also proposed a simple asymptotic stability criterion to assure chatter-free performance at all spindle speeds. Opitz and Bernardi (1970) developed a general closed loop representation of the cutting system dynamics for both turning and milling processes. The machine structural dynamics was generally expressed in terms of transfer matrices, while the cutting process was limited by two assumptions: (1) the direction of the dynamic cutting force is fixed during dynamic cutting, and (2) the effect of feed and cutting speed are neglected. These assumptions were later removed by Minis et al. (1990). They described the system stability by a characteristic equation, and then applied the Nyquist stability criterion to determine the system stability boundaries.

This work introduces a general computational method which avoids lengthy algebraic (symbolic) manipulations in deriving a characteristic equation. Since the characteristic equation is numerically formulated as an equation in a single unknown variable which is well bounded (rather than two nonlinear algebraic equations with unbounded variables), the solution scheme is simple and robust. An asymptotic stability index is also introduced for the purpose of relative stability analysis. Two examples are provided to demonstrate the proposed method.

2 Mathematical Formulation

In order to predict the onset of chatter, the chatter model is usually linearized about an equilibrium point (i.e., a steady cutting condition). Then the linearized model can be expressed by a set of linear time invariant differential equations with time delays (i.e., time difference-differential equations) (Minis, 1990). Therefore, the mathematical formulation starts from a system of linear time difference-differential equations expressed in terms of the state space form, i.e.,

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}_1\mathbf{x}(t) + \mathbf{A}_2\mathbf{x}(t-T), \tag{1}$$

where T is the time needed for one revolution (i.e., the inverse of the spindle speed), $\mathbf{A}_1$ and $\mathbf{A}_2$ are the linearized equation matrices of the process model, and are functions of the machine-tool/workpiece structural parameters (e.g., natural frequency, damping, and stiffness), of the cutting parameters (feed, speed, and depth of cut), and of the cutting coefficients (e.g., cutting

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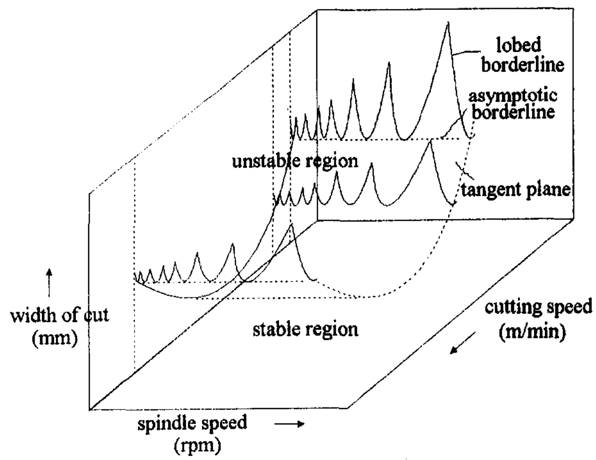

Fig. 1 Typical stability chart for 1-DOF models in orthogonal cutting

stiffness). Now, based on Eq. (1), new algorithms are proposed to calculate the stability boundary and also to provide an asymptotic stability index for relative stability analysis. Instead of solving two simultaneous nonlinear algebraic equations as in previous studies [Chiriacescu, 1990], the stability chart is constructed by solving one single-variable equation whose variable is well bounded. First assume a sinusoidal solution for Eq. (1). For a nontrivial solution, one obtains

$$\det(\omega j \mathbf{I} - \mathbf{A}_1 - e^{-T\omega j}\mathbf{A}_2) = 0 \tag{2}$$

Equation (2) is defined as the characteristic equation of Eq. (1) and is usually used for calculating the stability boundaries. In fact, Eq. (2) is given in an alternative form in many previous works. With further symbolic manipulations, the characteristic equation has been used for chatter prediction by applying the Nyquist stability criterion (Minis, 1990). However, in order to avoid unnecessary symbolic manipulations, Eq. (2) is utilized here without any further symbolic manipulation. Since the term $e^{-T\omega j}$ in Eq. (2) is of unit magnitude, the following definition is given:

Definition 1

$2\pi(m + v) = T\omega$, where m is a natural number, and the variable v is well bounded, i.e., $0 \le v < 1$.

After replacing the term $T\omega$ with $2\pi(m + v)$, the unbounded variable T in Eq. (2) can be replaced by a well bounded variable v. That is,

$$\det(\omega j \mathbf{I} - \mathbf{A}_e(v)) = 0, \quad 0 \le v < 1, \tag{3}$$

where $\mathbf{A}_e(v) = \mathbf{A}_1 + e^{-2\pi v j}\mathbf{A}_2$, and ω is the chatter frequency.

Traditionally Eq. (3) is solved by taking both the real and imaginary parts to be zero, i.e., by solving two nonlinear simultaneous equations with respect to two independent variables ω and v (Chiriacescu, 1990). However, since ω is not well bounded, care must be taken to define the appropriate domain of ω in searching for numerical solutions. The chatter frequency ω is closely related to the system's fundamental vibration modes. Therefore, the appropriate selection of the domain of ω is a case-by-case problem. In order to avoid such a drawback, a different approach is proposed. Consider the following equation:

$$\det(\lambda \mathbf{I} - \mathbf{A}_e(v)) = 0, \quad 0 \le v < 1, \tag{4}$$

where λ is a complex variable. Note that λ does not have to be a purely imaginary number. For a given parameter v, solving for the variable λ is equivalent to solving for the eigenvalues of the matrix $\mathbf{A}_e(v)$. When the solution λ is located on the positive imaginary axis, the magnitude of λ and its corresponding v become a solution of Eq. (3). This motivates the proposed new scheme for stability analysis. The main idea of the proposed algorithm is to look for all the v's in $0 \le v < 1$ that induce eigenvalue(s) on the positive imaginary axis for $\mathbf{A}_e(v)$. First, in order to detect if λ is an imaginary number, a function of v, gap(v), is defined to be the real part of the eigenvalue of $\mathbf{A}_e(v)$ that has the shortest distance to the imaginary axis, i.e.,

Definition 2

gap $(v) = \mathbf{min}_{\lambda_i}(|\mathrm{Re}(\lambda_i)|)$, where λ_i's are all the eigenvalues of $\mathbf{A}_e(v)$.

Therefore, if gap(v) is zero for a specified v, $\mathbf{A}_e(v)$ has imaginary eigenvalue(s), and vice versa. It is also claimed that all the v's in $0 \le v < 1$ leading to eigenvalue(s) on the positive imaginary axis in $\mathbf{A}_e(v)$ can be determined by all the v's in $0 \le v \le 0.5$ leading to imaginary eigenvalue(s) in $\mathbf{A}_e(v)$. Since the scalar $e^{-2\pi v j}$ is the complex conjugate of $e^{-2\pi(1-v)j}$, the matrix $\mathbf{A}_e(v)$ is, by definition, a complex conjugate of $\mathbf{A}_e(1 - v)$. For each v in $0.5 \le v < 1$ that leads to eigenvalue(s) on the positive imaginary axis in $\mathbf{A}_e(v)$, there is a corresponding $w\,(=1 - v)$ in $0 < w \le 0.5$ that leads to eigenvalue(s) on the negative imaginary axis in $\mathbf{A}_e(w)$. In this way, the problem of solving Eq. (3) can be converted into a problem of solving a single variable equation:

$$\text{gap}(v) = 0, \quad 0 \le v \le 0.5 \tag{5}$$

After solving Eq. (5), the solution v's obtained are used as parameters to determine the corresponding chatter frequencies and spindle speeds. The chatter frequency ω is equivalent to the magnitude of the imaginary solution in Eq. (4), i.e., Im(λ)'s. Also, by definition and from the above discussion, the spindle speed is expressed as

$$\begin{aligned} \frac{1}{T} &= \frac{\mathrm{Im}(\lambda)}{(m + v)2\pi}, \quad \mathrm{Im}(\lambda) \ge 0, \\ &= \frac{-\mathrm{Im}(\lambda)}{(m + 1 - v)2\pi}, \quad \mathrm{Im}(\lambda) < 0. \end{aligned} \tag{6}$$

where m is any natural number, and Im(λ) is also a function of v.

3 Computational Stability Analysis

The proposed mathematical formulation is rather convenient for computational chatter prediction. Once the system matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ are given, a computer program can take care of the rest of the tasks. Lengthy algebraic (symbolic) manipulations in deriving a characteristic equation are also avoided. In order to demonstrate the use of Eq. (5), (6), two simple examples from the work of Merrit [1965] are given. The first example is a one degree-of-freedom dynamic system, and the second example is a three degree-of-freedom dynamic system. Figure 2 shows the block diagram of the chatter loop. When the dynamics of the cutting process is neglected, the structural dynamics, for one or multiple degree(s) of freedom, can be expressed in terms of the structure in Fig. 2.

In the first example,

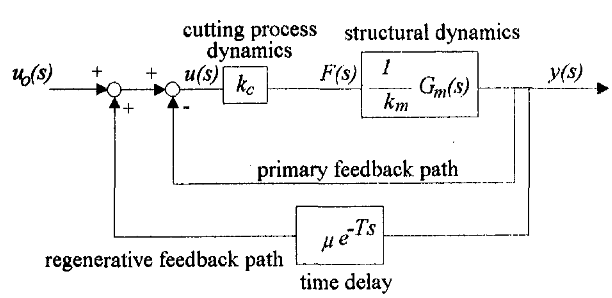

Fig. 2 Block diagram of chatter loop [Merrit, 1965]

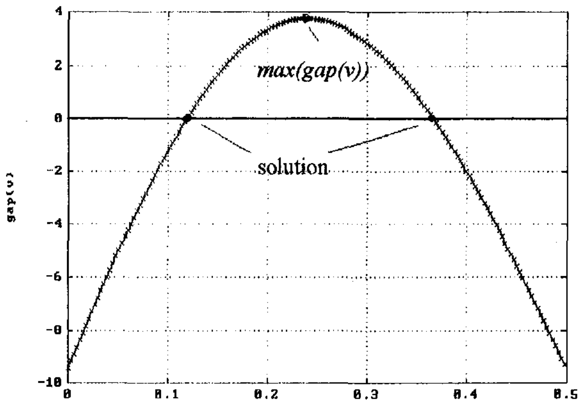

Fig. 3 Typical sketch of the function *gap*(*v*) for one degree-of-freedom models

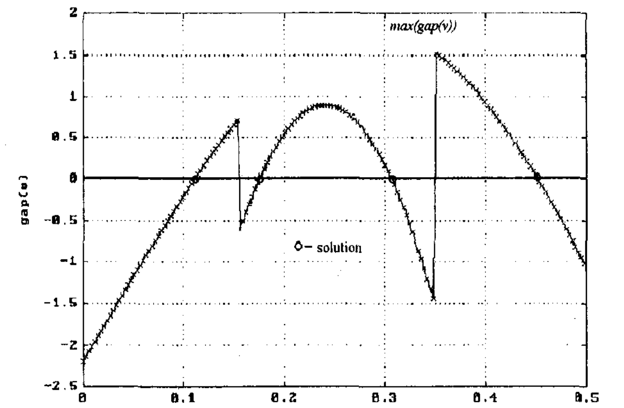

Fig. 5 Typical sketch of the function *gap*(*v*) for multiple degree-of-freedom models

$$G_m(s) = \frac{1}{\frac{1}{\omega_n^2} s^2 + \frac{2\zeta}{\omega_n} s + 1},$$

where

ω_n: natural frequency of structure (rad/sec),
ζ: damping ratio of structure, = 0.05,
k_m: structural stiffness (N/m),
k_c: cutting stiffness (N/m),
T: time for one revolution (sec),
μ: overlapping factor, =1.

The corresponding time difference-differential equation is expressed as

$$\frac{1}{\omega_n^2}\ddot{x} + \frac{2\zeta}{\omega_n}\dot{x} + x = -\frac{k_c}{k_m}(x(t) - \mu x(t-T)).$$

If one defines the state vector $\mathbf{x} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$, the system matrices in Eq. (1) can be expressed as

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ -\left(1 + \frac{k_c}{k_m}\right)\omega_n^2 & -2\zeta\omega_n \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 0 & 0 \\ \mu \frac{k_c}{k_m}\omega_n^2 & 0 \end{bmatrix}$$

When the stiffness ratio, (k_c/k_m), is specified, the corresponding solution of Eq. (6) can be evaluated. Figure 3 shows the typical shape of the gap(v) function for one degree-of-freedom models. There are two solutions to gap(v) = 0 for a given stiffness ratio in this sketch. In the first example, the value of stiffness ratio is gradually increased from 0 to 0.6 with a step change 0.01. Then the stability chart, i.e., the stability boundary with respect to the spindle speed (rps, revolutions per second), is depicted in Fig. 4, where m is chosen to be 0, 1, and 2. When the given stiffness ratio is smaller than 0.1, there is no solution to gap(v) = 0; when the stiffness ratio is 0.1, there is one solution; when the stiffness ratio is greater than 0.1, there are two solutions, which is the case in Fig. 3.

In the second example, a three degree-of-freedom model is utilized. In this case,

$$G_m(s) = \frac{\left(\frac{1}{\omega_b^2}s^2 + \frac{2\zeta_b}{\omega_b}s + 1\right)}{\left(\frac{1}{\omega_1^2}s^2 + \frac{2\zeta_1}{\omega_1}s + 1\right)\left(\frac{1}{\omega_2^2}s^2 + \frac{2\zeta_2}{\omega_2}s + 1\right)\left(\frac{1}{\omega_3^2}s^2 + \frac{2\zeta_3}{\omega_3}s + 1\right)}$$

where ω_b, ζ_b, ω_1, ζ_1, ω_2, ζ_2, ω_3, and ζ_3 are assigned the values 100 Hz, 0.05, 30 Hz, 0.05, 70 Hz, 0.005, 150 Hz, and 0.01 respectively. Figure 5 shows the typical shape of the gap(v) function for multiple degree-of-freedom models. There are four solutions for gap(v) = 0 in this sketch. Similarly, after obtaining all the solutions of Eq. (5) for given stiffness ratios from 0 to 0.8, the stability chart is plotted in Fig. 6, where m is chosen to be 0, 1, and 2. The stability boundary, i.e., the maximal stiffness ratio for chatter-free cutting, is sketched with respect to spindle speed (rps).

Based on the definition of the function gap(v), an asymptotic stability index, Max (gap (v)), which is the largest value of gap(v) over v in $0 \le v \le 0.5$, is now defined to indicate the

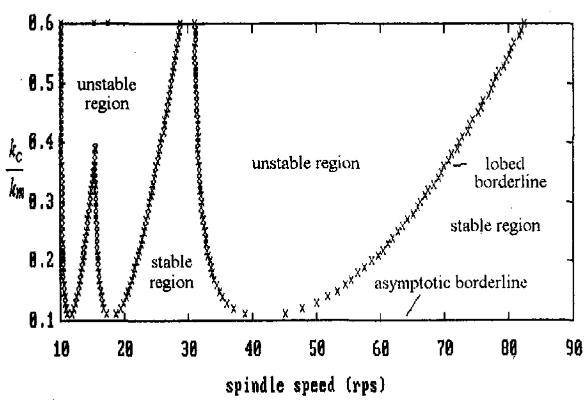

Fig. 4 Typical stability chart for one degree-of-freedom models

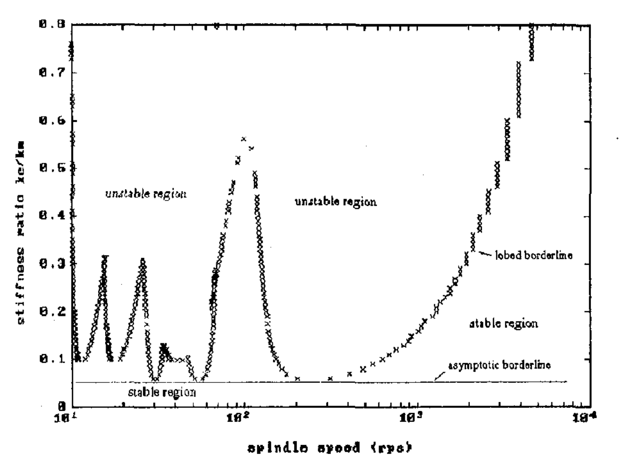

Fig. 6 Typical stability chart for multiple degree-of-freedom models

stability margin. When Max (gap (v)) is smaller than zero, there is no solution for the stability boundary, which implies that the process is under the tangent borderline, (e.g., see Figs. 5 and 6). As Max (gap (v)) approaches zero, the specified process also approaches the tangent borderline. When Max (gap (v)) is greater than zero, the process is above the tangent borderline, which indicates the possibility of chatter at certain spindle speeds [Merrit, 1965].

4 Summary and Conclusions

A general method has been proposed for absolute and relative stability analysis against machine-tool chatter:

(1) Lengthy algebraic (symbolic) manipulations in deriving a characteristic equation are avoided. Only the system matrices are needed for the computation of the stability boundary. Thus the efficiency in formulating machine tool chatter problems is significantly improved. A case-independent solver can be developed based on the proposed algorithm to compute the stability chart.

(2) The computational scheme is robust since the stability onset calculation is simplified to solving a single variable equation whose variable is well bounded.

(3) An asymptotic stability index is proposed to indicate the stability margin.

(4) The method can also be applied to other machining processes, as long as the system equations can be expressed as a set of linear time invariant difference-differential equation.

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