N-Dimensional Cube Dashings

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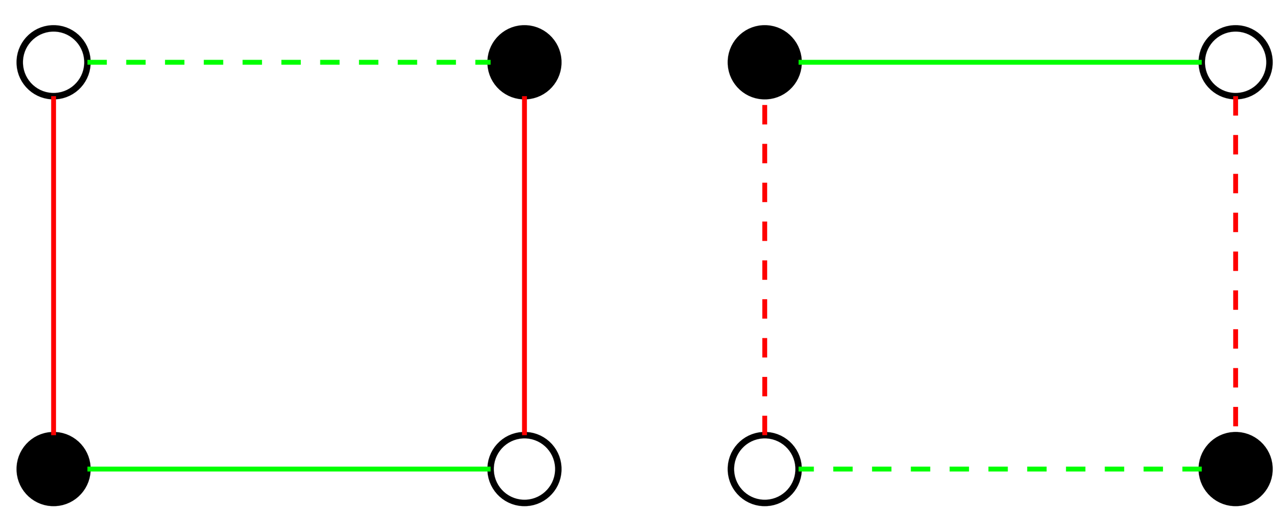
Problem

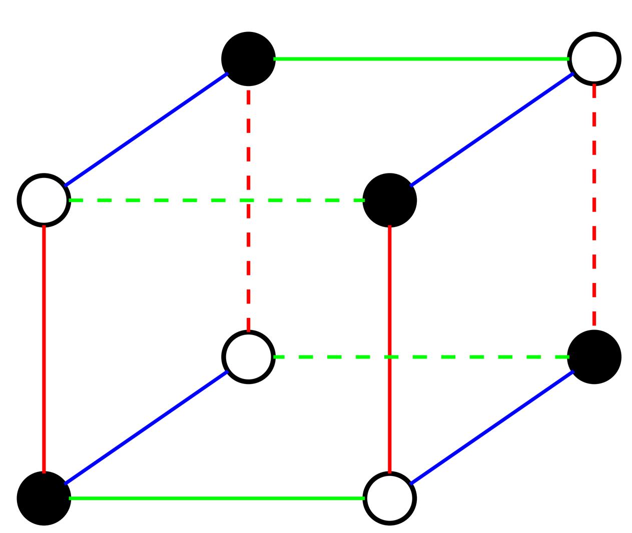
An N-dimensional adinkra topology has the property that every two-color cycle has an odd number of dashed edges. This poster proves the existence of a dashing for any N-dimensional cube by use of inductive reasoning.

Base Cases

We first begin with the case N=2. (For N=1, there is only one color.) This is a square. There exist two options for dashings, one or three dashed edges:



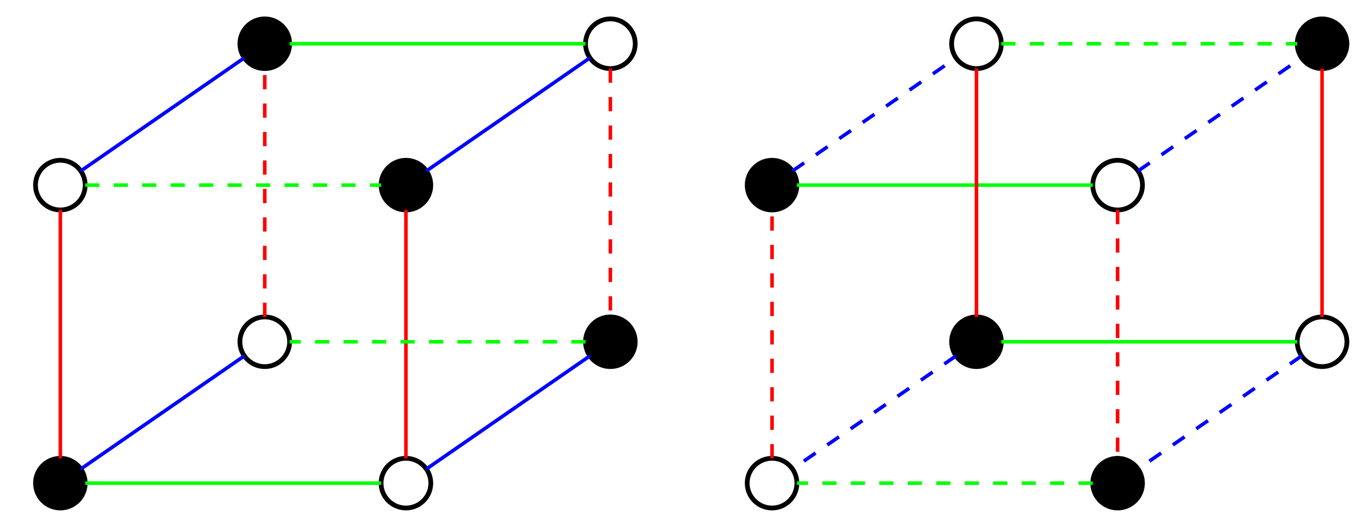
Now consider the case when N=3, a cube. One way to construct a cube is to construct two copies of a square, then draw edges connecting corresponding vertices of the squares. Similarly, one way to construct an odd dashing on the cube is to construct two copies of squares with odd dashings so that the dashings are inverted. Then, connect corresponding vertices with all solid edges of a new color:



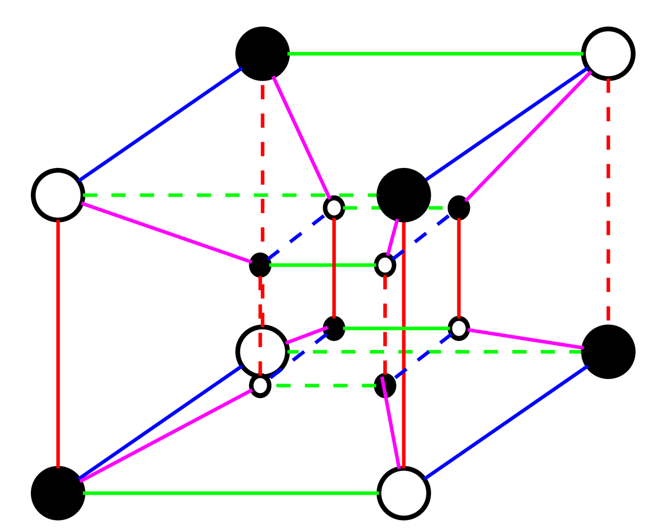
Now, any 2-color cycle corresponds to a face of the cube. Obviously, the original two squares have an odd dashing. The other four faces of the cube each have exactly one dashed edge: two solid edges formed by connecting the two original squares, and one solid and one dashed edge because the squares had inverted dashing. Therefore, each face of the cube has an odd dashing.

General Inductive Case

Now, we can generalize this process to any N-dimensional cube, given there exists an odd dashing for an (N-1)-dimensional cube.
Any N-cube can be constructed by aligning two (N-1)-cubes and connecting corresponding vertices. As with the previous construction, we take two copies of the (N-1)-cubes with inverted odd dashings so that each dotted edge in the first (N-1)-cube corresponds to a solid edge in the second and vice versa:



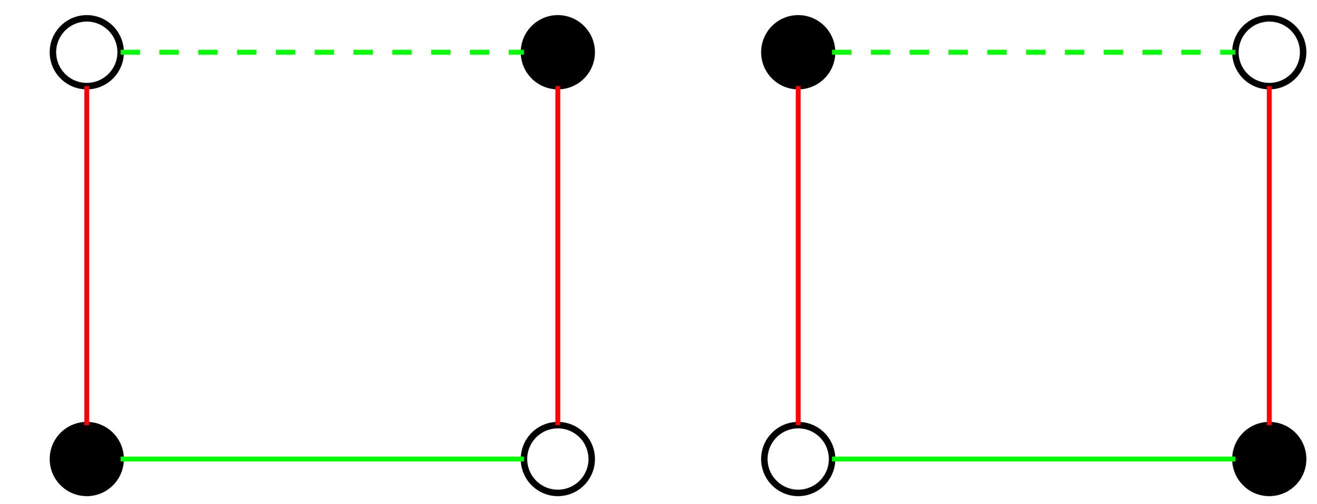
We then connect the corresponding vertices with solid edges of a new color:



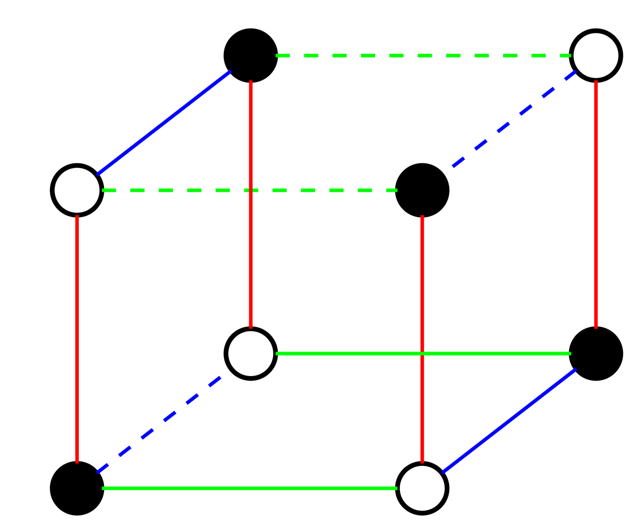
Again, given this construction, all 2-color 4-cycles correspond to faces of the N-cube. Some of the faces of the N-cube are faces from the (N-1)-cubes. These all satisfy the dashing property by our assumption. All the other faces of the N-cube are made up of a corresponding pair of edges from the (N-1)-cubes and two of the new edges. By our construction, exactly one of these four edges will be dashed, given that the pair of corresponding edges of the (N-1)-cubes have opposite dashing and the new edges are solid.

A Second Method

There is another way to construct N-cubes with odd dashings on every face. Rather than taking two (N-1)-cubes with inverted odd dashings, start with two (N-1)-cubes that have the same odd dashing:



Then, connect the corresponding vertices of the (N-1)-cubes with edges of a new color so that every other edge is dashed. That is, connect the first pair of corresponding vertices with a solid edge. Then, connect each pair of corresponding vertices that are adjacent to these with a dashed edge. Connect each pair of corresponding vertices adjacent to these with a solid edge, and so on.



Some of the faces on the N-cube will be faces of the (N-1)-cubes. By induction, these faces have an odd number of dashings. The other faces will be new faces formed by connecting corresponding edges of the (N-1)-cubes. These faces will have an odd number of dashed edges: two of the edges will come from the (N-1)-cubes and will either be both dashed or solid, and two of the edges will be new edges, one dashed and one solid. Therefore, the resulting N-cube will have an odd dashing.

Future Work

We would like to prove that it is possible to induce an odd dashing on every folded adinkra, which we believe corresponds to an N-cube with diagonals.