Field Replacements on Chiral Multiplet in its 4D SUSY and 0-Brane Reductions

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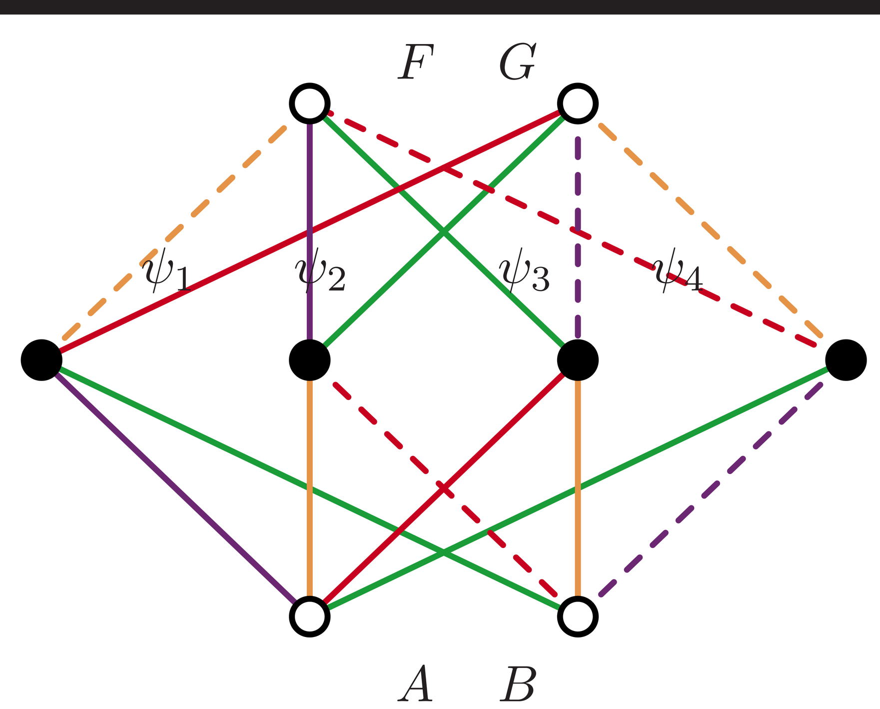
4D SUSY Chiral Multiplet (CM)

$$
\begin{aligned}
Q_a A &= \psi_a \\
Q_a B &= i(\gamma^5)_a{}^b \psi_b \\
Q_a \psi_b &= i(\gamma^\mu)_{ab}\partial_\tau A - (\gamma^5\gamma\mu)_a{}^b \partial_\tau B \\
&\quad - C_{ab}F + (\gamma^5)_{ab}G \\
Q_a F &= (\gamma^\mu)_a{}^b \partial_\tau \psi_b \\
Q_a G &= (\gamma^5\gamma^\mu)_a{}^b \partial_\tau \psi_b
\end{aligned}
$$

0-Brane Reduction of CM

$$
\begin{aligned}
Q_a A &= \psi_a \\
Q_a B &= i(\gamma^5)_a{}^b \psi_b \\
Q_a \psi_b &= i(\gamma^0)_{ab}\partial_\tau A - (\gamma^5\gamma 0)_a{}^b \partial_\tau B \\
&\quad - C_{ab}F + (\gamma^5)_{ab}G \\
Q_a F &= (\gamma^0)_a{}^b \partial_\tau \psi_b \\
Q_a G &= (\gamma^5\gamma^0)_a{}^b \partial_\tau \psi_b
\end{aligned}
$$

Adinkra



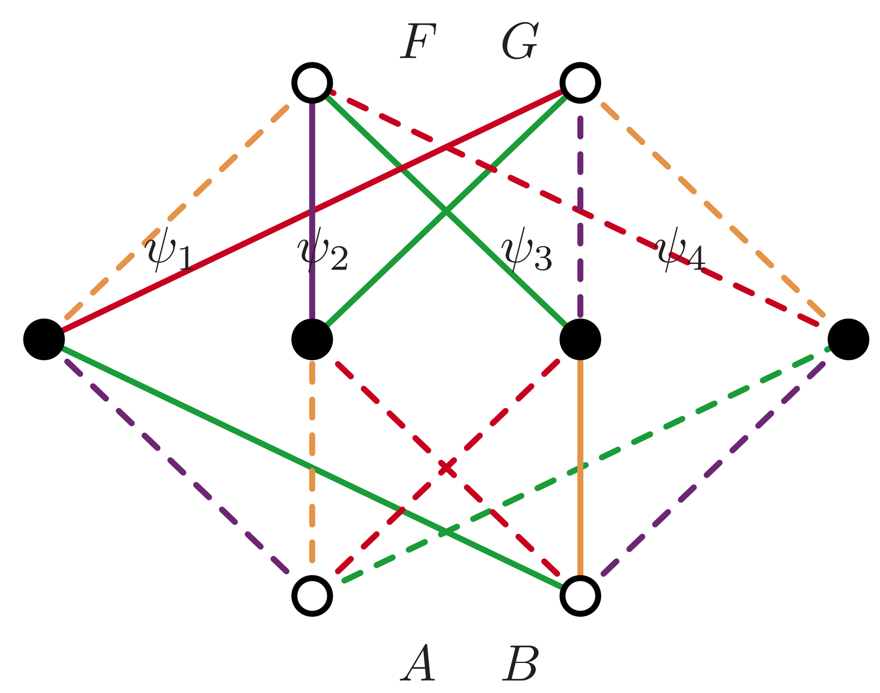
Original Adinkra (Node A is the bottom-left)

Rules for Building Adinkra

- Bipartite graph
- Two-color 4-cycles
- Each 4-cycle has an odd number of dashings
- Each type of vertex is on a different height
- Each vertex has exactly one edge of each color

Field Change

Changing the sign of field is equivalent to changing all of the dashings and non-dashing of lines connected to the field in Adinkra. The new Adinkra still satisfies the Adinkra rules.



Node $A \mapsto -A$

Question

Does Adinkra with $\widetilde{A}$ satisfy the 4D SUSY CM equations?

Conjecture: Not all field replacements that are consistent with SUSY in the 0-brane CM are consistent in the 4D CM.

Example

$A \mapsto -A = \widetilde{A}$ alone implies that $Q_a A \neq \psi_a$ because ψ_a does not flip sign. Therefore $A \mapsto \widetilde{A}$ alone is not consistent in 4D SUSY CM

Example

$B \mapsto -B = \widetilde{B}$ and $G \mapsto -G = \widetilde{G}$ implies that $\gamma^5 \mapsto -\gamma^5 = \widetilde{\gamma^5}$ because
$Q_a\widetilde{B} = i(\widetilde{\gamma^5})_a{}^b\psi_b$ and $Q_a\widetilde{G} = (\widetilde{\gamma^5}\gamma^\mu)_a{}^b\partial_\tau\psi_b$, and
$Q_a\psi_b = i(\gamma^\mu)_{ab}\partial_\tau A - (\widetilde{\gamma^5}\gamma\mu)_a{}^b\partial_\tau\widetilde{B} - C_{ab}F + (\widetilde{\gamma^5})_{ab}\widetilde{G}$
Therefore $B \mapsto \widetilde{B}$ and $G \mapsto \widetilde{G}$ is consistent in 4D SUSY CM

Example