Constructing Chromotopologies

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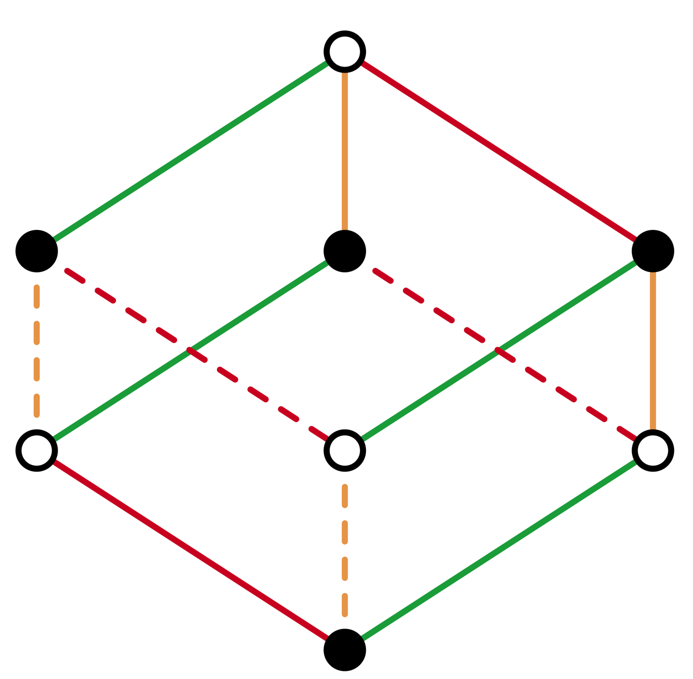
Background

We define an n-dimensional adinkra topology as a finite, connected, simple, bipartite, n-regular graph such that the nodes art partitioned into fermions (denoted with closed nodes) and bosons (denoted with open nodes).
An adinkra with 8 2^n vertices can have as few as n colors, with that minimally colored case corresponding topologically to an n-cube. For a given number of vertices, We can construct adinkras with one more colors by naturally starting with that minimally colored adinkra and connecting vertices by additional lines (associated with new supercovariant differential operators D_a) in a way which preserves the Clifford algebra

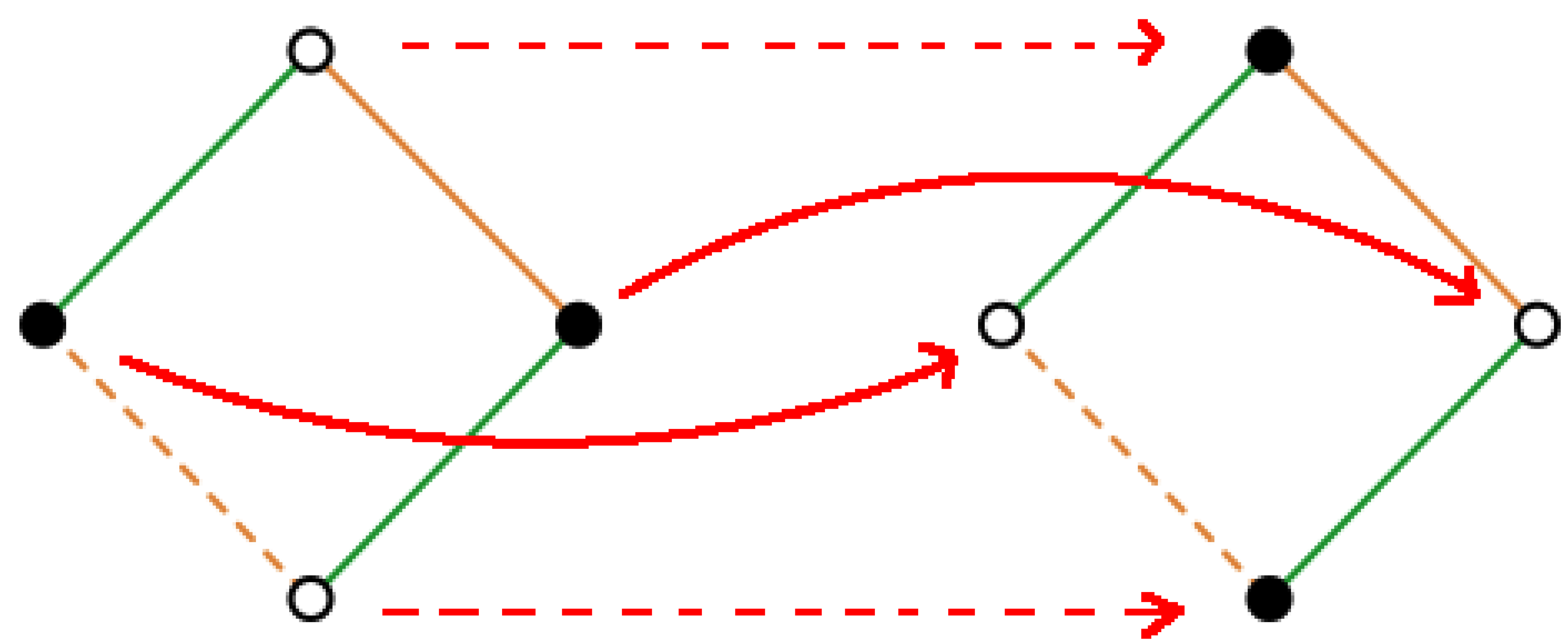
$$D_I D_J + D_J D_I = 2i\partial_\tau.$$

Base Case



An Adinkra with $n+1$ colors.

Construction

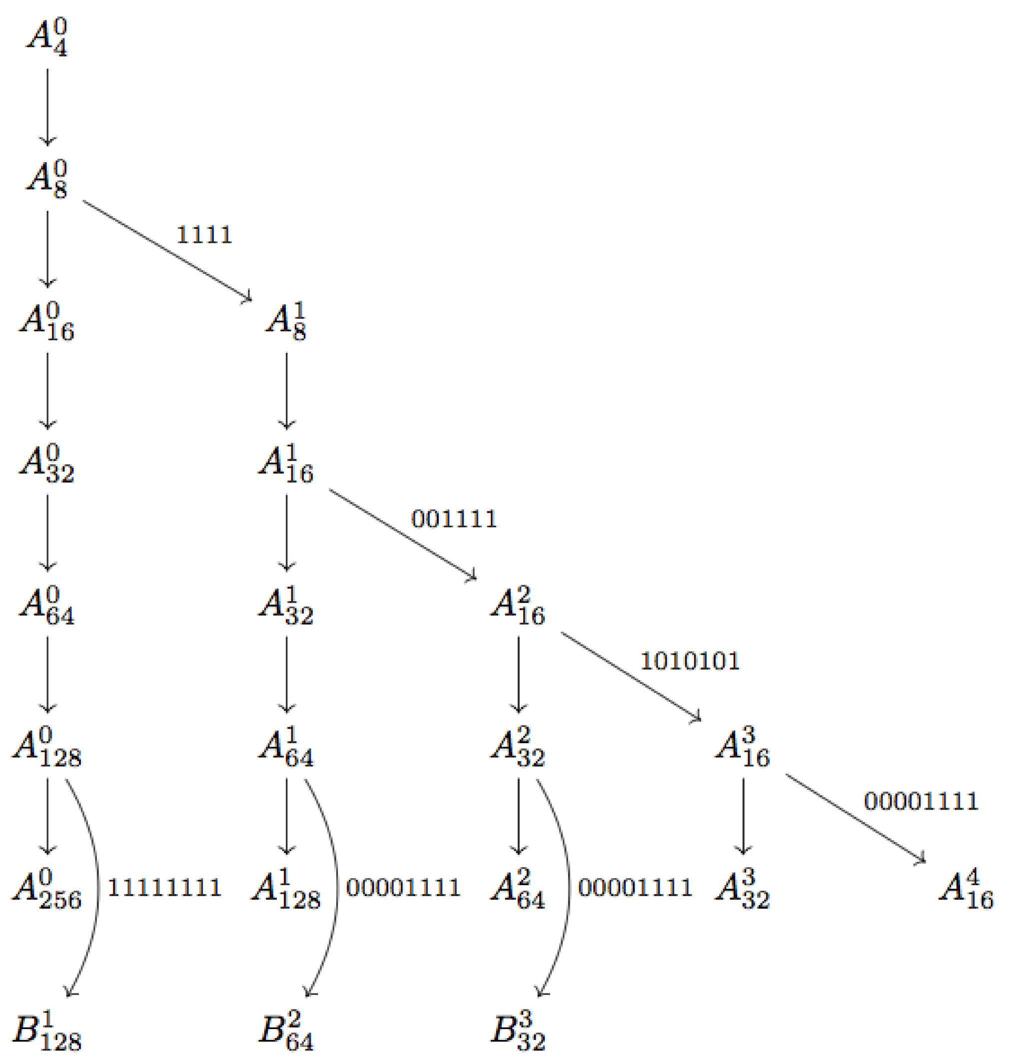


To inductively construct all connected chromatopologies with $n+1$ colors labelled $1, \ldots, n+1$

1. Label the distinct chromatopologies with n colors $T_1, \ldots, T_k$.
2. Consider an arbitrary one A with $n+1$ colors and remove all edges with color $n+1$
3. This produces l with n colors. Call them $A_1 = T_{i_1}, \ldots, A_l = T_{i_l}$
4. Consider a vertex $v \in A_1$. It had an edge of color $n+1$.
5. If this edge connects to $w \in A_1$, then the 4-cycle condition forces all edges of color $n+1$ that intersect A_1 to be contained in A_1, so by connectedness, A has only the vertices of A_1.
6. If this edge connects to $w \in A_j$, then similarly A has only the vertices of $A_1 \cup A_j$. The 4-cycle condition also forces that $i_1 = i_2$.

In conclusion, every connected chromatopology on $n+1$ colors can be produces from one on n colors in one of two ways: adding some number of edges of a new color or taking two disjoint copies, flipping fermions and bosons in one, and connecting corresponding vertices with edges of a new color.
Details about double-even codes only affect this through When and how the first case is possible.

Struggle Box



Notes

Based on data on doubly even codes producedby Robert L. Miller (`http://rlmiller.org/de_codes/`), we find that the above construction admits the only topology for an adinkra of n colors up to the case where $n = 8$ and the number of vertices is 128.