Integer Programming Approaches for Appointment Scheduling with Random No-Shows and Service Durations

Siqian Shen

Department of Industrial and Operations Engineering
University of Michigan

joint work with

Ruiwei Jiang and Yiling Zhang (PhD student)

(Supported by NSF CMMI-1555983)

Outline

Outline

1. Introduction
2. DR Modeling and Optimization
   - Risk Measures and Support of No-shows
   - DR Modeling and Reformulations
3. A Less Conservative DR Approach
   - A Less Conservative D_q
   - MILP and Valid Ineq. for $D_q^{(K)}$
4. Computational Results
5. Conclusions

Problem: Appointment Scheduling on a Single Server

Decisions:

- Arrival time for each appt. $i = 1, \dots, n$ in this order.
- Examples: outpatient care/sugeries, cloud computing platforms.

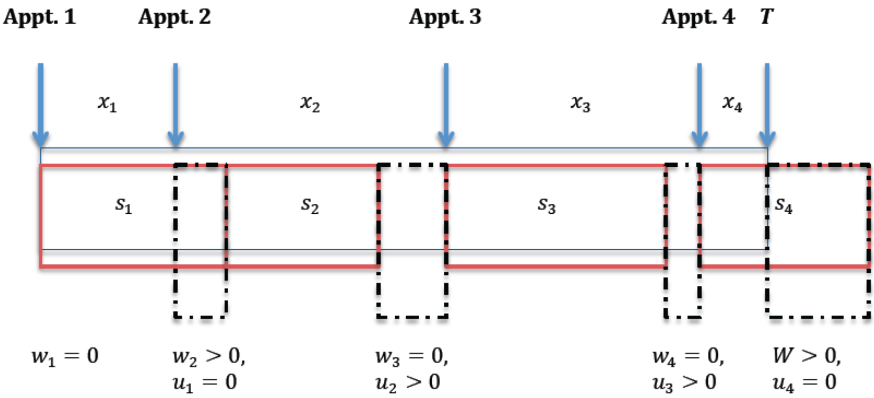
Uncertainty:

- server processing time (continuous).
- no-shows (0-1).

Scenarios:

- an appt. gets delayed $\Rightarrow$ appt. waiting time.
- an appt. finishes early $\Rightarrow$ server idle time.
- the last appt. cannot finish before the server's time limit $\Rightarrow$ overtime.

Appointment Scheduling Illustration



Objective:

- $\downarrow$ appointments' waiting time $+$ server's idle time and overtime.

Literature Review

- Under random service durations: Denton and Gupta (2003), Gupta and Denton (2008), Pinedo (2012), ...
- Near-optimal scheduling policy: Mittal et al. (2014), Begen and Queyranne (2011), Begen et al. (2012), Ge et al. (2013), ...
- Under random no-shows (mainly heuristics and approx. algorithms): Muthuraman and Lawley (2008), Zeng et al. (2010), Cayirli et al. (2012), Lin et al. (2011), Luo et al. (2012), LaGanga and Lawrence (2012), Zacharias and Pinedo (2014), ...
- Distributionally Robust (DR) appointment scheduling:
  - Distributional ambiguity; lack of reliable data.
  - Kong et al. (2014): cross moments (mean & covariance) of service durations.
  - Mak et al. (2015): marginal moments of service durations.

Research Outline

- Ambiguous no-shows & service durations.
- Ambiguity set based on the means & supports of no-shows and service durations.
- Flexible in risk preferences:
  - Risk-neutral: Expectation of waiting, idleness, and overtime.
  - Risk-averse: CVaR of waiting, idleness, and overtime.
  - Incorporated as objective and/or constraints.
  - This talk: expectation objective functions.
- DR models $\Rightarrow$ equivalent MINLP reformulations $\Rightarrow$ MILP
  - Integer programming approaches help handle 0-1 no-shows and accelerate computation.
  - Important special cases: MILP $\Leftrightarrow$ LP (deriving the convex hulls).

Outline

Notation

Parameters:

$\{1, \ldots, n\}$	the set of appointments to schedule
T	the server's operating time limit
c_i^{w}, c_i^{u}, c^{o}	unit penalty of waiting, idleness, and overtime
$s_i \in \mathbb{R}_+$	random service duration of appointment i
$q_i \in \{0, 1\}$	show ($q_i = 1$) or no-show ($q_i = 0$) of appointment i

Decision Variables:

x_i	scheduled time between appointments i and $i+1$, $\forall i = 1, \ldots, n-1$ (That is, appt. 1 arrives at time 0; appt. 2 arrives at x_1; appt. 3 arrives at $x_1 + x_2$,...)
w_i	waiting time of appointment i
W	server overtime
u_i	server idle time after finishing appointment i

Computing Waiting, Idleness, Overtime

For decision x, we consider a feasible region:

$$X := \left\{ x : x_i \geq 0,\ \forall i = 1, \ldots, n,\ \sum_i^n x_i = T \right\}$$

Given $x \in X$ and realizations of parameter (q, s),

$$\begin{aligned} Q(x, q, s) := \min_{w,u,W} \quad & \sum_{i=1}^n (c_i^{\mathrm{w}} w_i + c_i^{\mathrm{u}} u_i) + c^{\mathrm{o}} W \\ \text{s.t.} \quad & w_i - u_{i-1} = q_{i-1} s_{i-1} + w_{i-1} - x_{i-1}\ \forall i = 2, \ldots, n \\ & W - u_n = q_n s_n + w_n + \sum_{i=1}^{n-1} x_i - T \\ & w_i \geq 0,\ w_1 = 0,\ u_i \geq 0,\ W \geq 0\ \forall i = 1, \ldots, n. \end{aligned}$$

Valid if $c_{i+1}^{\mathrm{u}} - c_i^{\mathrm{u}} \leq c_{i+1}^{\mathrm{w}}$ (work conserving; Ge et al. (2013)).

Ambiguity Set

- DR appointment scheduling model: $\min_{x \in X} \sup_{\mathbb{P}_{q,s} \in \mathcal{F}(D,\mu,\nu)} \mathbb{E}_{\mathbb{P}_{q,s}}[Q(x,q,s)]$.
- Ambiguity set:

$$\mathcal{F}(D,\mu,\nu) := \left\{ \mathbb{P}_{q,s} \geq 0 : \quad \begin{array}{l} \int_{D_q \times D_s} d\mathbb{P}_{q,s} = 1 \\ \int_{D_q \times D_s} s_i \; d\mathbb{P}_{q,s} = \mu_i \quad \forall i = 1,\ldots,n \\ \int_{D_q \times D_s} \left(\sum_{i=1}^n q_i\right) \; d\mathbb{P}_{q,s} = \nu \end{array} \right\},$$

where

- $\mu = [\mu_1, \ldots, \mu_n]^\mathsf{T}$: mean service duration $\mathbb{E}[s_i]$, $i = 1, \ldots, n$.
- $\nu = \mathbb{E}[\sum_{i=1}^n q_i]$: mean of # show-up appointments.
- $D = D_q \times D_s$: support of (q, s) with

$$\begin{aligned} D_q &:= \{0,1\}^n, \\ D_s &:= \left\{ s \geq 0 : s_i^{\mathrm{L}} \leq s_i \leq s_i^{\mathrm{U}}, \; \forall i = 1,\ldots,n \right\}. \end{aligned}$$

Reformulations of the DR Model

The inner $\sup_{\mathbb{P}_{q,s} \in \mathcal{F}(D,\mu,\nu)} \mathbb{E}_{\mathbb{P}_{q,s}}[Q(x,q,s)]$ is a functional linear program.

$$
\begin{aligned}
& \max_{\mathbb{P}_{q,s}} && \int_{D_q \times D_s} Q(x,q,s) d\mathbb{P}_{q,s} \\
(\mathrm{P}) \quad & \text{s.t.} && \int_{D_q \times D_s} s_i \; d\mathbb{P}_{q,s} = \mu_i \quad \forall i = 1, \ldots, n \\
& && \int_{D_q \times D_s} \left(\sum_{i=1}^{n} q_i \right) \; d\mathbb{P}_{q,s} = \nu \\
& && \int_{D_q \times D_s} d\mathbb{P}_{q,s} = 1.
\end{aligned}
$$

Reformulations of the DR Model

- By duality theory, problem (P) is equivalent to (D) as follows:

$$\min_{\rho \in \mathbb{R}^n, \gamma \in \mathbb{R}} \left\{ \sum_{i=1}^{n} \mu_i \rho_i + \nu \gamma + \max_{(q,s) \in D_q \times D_s} \left\{ Q(x, q, s) - \sum_{i=1}^{n} (\rho_i s_i + \gamma q_i) \right\} \right\}$$

- A min-max problem with a (potentially challenging) inner problem.
- Recall: $Q(x, q, s)$ is convex in (q, s).
- $\max_{(q,s) \in D_q \times D_s} \left\{ Q(x, q, s) - \sum_{i=1}^{n} (\rho_i s_i + \gamma q_i) \right\}$ is in general intractable.

Reformulations of the DR Model

- More specifically, rewrite $Q(x,q,s)$ in its dual form:

$$
\begin{aligned}
Q(x,q,s) \;=\; \max_{y} \quad & \sum_{i=1}^{n} (q_i s_i - x_i) y_i && \text{(1a)} \\
\text{s.t.} \quad & y_{i-1} - y_i \le c_i^{\mathrm{w}} \quad \forall i = 2,\ldots,n && \text{(1b)} \\
& -y_i \le c_i^{\mathrm{u}} \quad \forall i = 1,\ldots,n && \text{(1c)} \\
& y_n \le c^{\mathrm{o}}, && \text{(1d)}
\end{aligned}
$$

where (1b)–(1d) form a feasible region Y of y. The inner problem becomes

$$
(\mathrm{D}') \qquad \max_{(q,s)\in D_q \times D_s} \; \max_{y \in Y} \left\{ \sum_{i=1}^{n} (q_i s_i - x_i) y_i - \sum_{i=1}^{n} (\rho_i s_i + \gamma q_i) \right\}.
$$

- This is a MINLP.
    - $q_i s_i y_i$ is a product of one binary and two continuous variables.

Reformulations of the DR Model

- Observation: the objective function is decomposable.

$$\max_{y \in Y} \max_{(q,s)} \left\{ \sum_{i=1}^{n} (q_i s_i - x_i) y_i - \sum_{i=1}^{n} (\rho_i s_i + \gamma q_i) \right\}$$
$$= \max_{y \in Y} \left\{ \sum_{i=1}^{n} \max_{(q_i, s_i)} \{q_i s_i - x_i\} y_i - \sum_{i=1}^{n} (\rho_i s_i + \gamma q_i) \right\}.$$

- Y is a well-studied polytope in lot-sizing (Zangwill (1966, 1969), Mak et al. (2015)).
  - Extreme points of Y can be fully characterized.

- Key idea from Mak et al. (2015): binary encoding of the extreme points of Y:

$$t_{kj} \in \{0, 1\}, \quad \forall 1 \leq k \leq j \leq n+1 \;\Leftrightarrow\; \text{extreme points of } Y$$

Reformulations of the DR Model

- (D') is equivalent to

$$\begin{aligned} \max_{t} \quad & \sum_{k=1}^{n+1} \sum_{j=k}^{n+1} \left(\sum_{i=k}^{j} \max_{(q_i,s_i)} \{q_i s_i - x_i\} \pi_{ij} \right) t_{kj} - \sum_{i=1}^{n} (\rho_i s_i + \gamma q_i) \\ \text{s.t.} \quad & \sum_{k=1}^{i} \sum_{j=i}^{n+1} t_{kj} = 1 \quad \forall i = 1, \ldots, n+1 \\ & t_{kj} \in \{0,1\}, \quad \forall 1 \le k \le j \le n+1. \end{aligned}$$

  - π_{ij} are constants about c^{w}, c^{u}, c^{o}.
  - The constraints matrix of t is TU: $t_{kj} \in \{0,1\}$ can be relaxed!
  - (D') can now be solved as a LP.

Reformulations of the DR Model

Proposition

The DR model with $D_q = \{0,1\}^n$ is equivalent to the following LP:

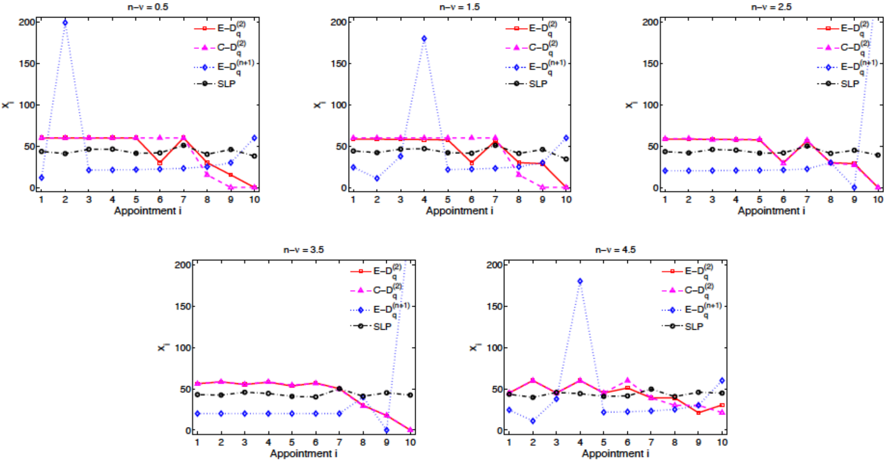
$$
\begin{aligned}
\min_{x,\rho,\gamma,\alpha,\beta} \quad & \sum_{i=1}^{n} \mu_i \rho_i + \nu\gamma + \sum_{i=1}^{n+1} \alpha_i \\
s.t. \quad & \sum_{i=k}^{j} \alpha_i \geq \sum_{i=k}^{j} \beta_{ij}, \quad \forall 1 \leq k \leq j \leq n+1 \\
& \beta_{ij} \;\geq\; -\pi_{ij} x_i - s_i^L \rho_i, \quad \forall 1 \leq i \leq n, \quad \forall i \leq j \leq n+1 \\
& \beta_{ij} \;\geq\; -\pi_{ij} x_i - s_i^U \rho_i, \quad \forall 1 \leq i \leq n, \quad \forall i \leq j \leq n+1 \\
& \beta_{ij} \;\geq\; -\pi_{ij} x_i - s_i^L \rho_i - \gamma + s_i^L \pi_{ij}, \quad \forall 1 \leq i \leq n, \quad \forall i \leq j \leq n+1 \\
& \beta_{ij} \;\geq\; -\pi_{ij} x_i - s_i^U \rho_i - \gamma + s_i^U \pi_{ij}, \quad \forall 1 \leq i \leq n, \quad \forall i \leq j \leq n+1 \\
& \sum_{i=1}^{n} x_i = T, \quad \beta_{n+1,n+1} = 0, \quad x_{n+1} = 0, \quad x_i \geq 0, \quad \forall i = 1, \ldots, n.
\end{aligned}
$$

Performance of the DR Schedules

Statistics	Model	$n-\nu=0.5$			$n-\nu=1.5$			$n-\nu=2.5$			$n-\nu=3.5$			$n-\nu=4.5$		
		WaitT	OverT	IdleT	WaitT	OverT	IdleT	WaitT	OverT	IdleT	WaitT	OverT	IdleT	WaitT	OverT	IdleT
Mean	DR	110.56	156.78	29.10	123.81	0.74	18.09	168.44	0.00	22.48	145.65	0.00	27.00	80.70	0.02	31.39
	SLP	0.34	0.00	13.43	0.46	0.00	18.01	0.25	0.00	22.48	0.42	0.00	27.00	0.31	0.00	31.39
50%	DR	116.42	169.62	28.91	128.32	0.00	16.74	171.68	0.00	21.45	146.93	0.00	26.18	81.56	0.00	30.51
	SLP	0.21	0.00	12.07	0.37	0.00	16.73	0.14	0.00	21.45	0.29	0.00	26.18	0.17	0.00	30.51
75%	DR	120.57	178.35	29.18	142.94	0.00	20.90	195.61	0.00	25.76	174.89	0.00	31.94	102.53	0.00	37.02
	SLP	0.48	0.00	15.64	0.65	0.00	20.90	0.39	0.00	25.76	0.60	0.00	31.94	0.46	0.00	37.02
95%	DR	125.85	187.92	31.16	149.13	6.17	28.08	223.88	0.00	33.60	210.69	0.00	38.44	129.81	0.00	43.03
	SLP	1.17	0.00	20.11	1.29	0.00	28.08	0.86	0.00	33.60	1.31	0.00	38.44	1.11	0.00	43.03

- Comparing DR schedules with perfect-information schedules obtained from SLP.
- Out-of-sample simulations.
- DR schedules perform poor in all three metrics, even when no-shows are low.

Optimal Schedule Patterns



- Blue curves for DR schedules: Batch arrivals.
- Intuition: DR schedules are avoiding the extreme cases with consecutive no-shows.
- These scenarios are possible, but very unlikely.

Outline

Remedy of the DR Model: A Less Conservative D_q

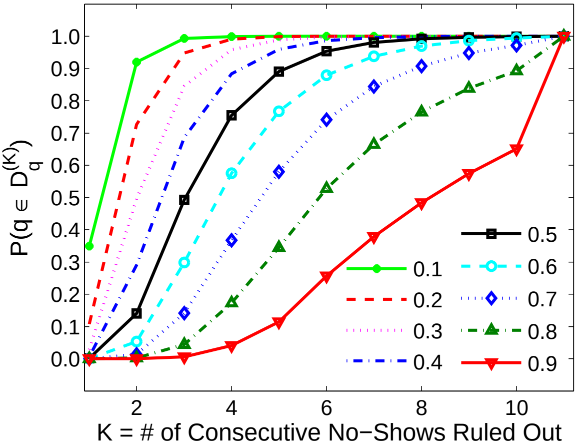
- For given $K \in \{2, \ldots, n+1\}$, define support

$$D_q^{(K)} = \left\{ q \in \{0,1\}^n : \sum_{j=i}^{i+K-1} q_j \geq 1, \ \forall i = 1, \ldots, n-K+1 \right\}$$

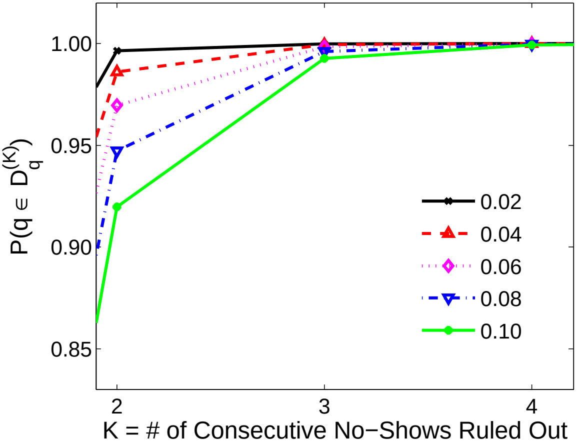
- No K-consecutive no-shows.
  - A spectrum of D_q supports.
  - $K = 2$: no consecutive no-shows (least conservative).
  - $K = n+1$: arbitrary no-shows (most conservative, $D_q^{(n+1)} = \{0,1\}^n$).
- Low no-show (e.g., inpatient surgery): $D_q^{(2)}$ is more reasonable.

Guideline for Choosing $K \in \{2, \ldots, n+1\}$ for $D_q^{(K)}$ I

Consider $n = 10$ and each appt. with equal no-show probability $0.1, \ldots, 0.9$.



Guideline for Choosing $K \in \{2, \ldots, n+1\}$ for $D_q^{(K)}$ II



- $\mathbb{P}\left(q \in D_q^{(2)}\right) \geq 90\%$ when $(n-\nu)/n \leq 0.1$.

Reformulations of the DR Model with $D_q^{(K)}$

- Observation: the objective function is not decomposable.

$$\max_{y\in Y} \max_{(q,s)\in D_q^{(K)}\times D_s} \left\{ \sum_{i=1}^{n} (q_i s_i - x_i) y_i - \sum_{i=1}^{n} (\rho_i s_i + \gamma q_i) \right\}$$
$$\neq \max_{y\in Y} \left\{ \sum_{i=1}^{n} \max_{(q_i,s_i)} \{q_i s_i - x_i\} y_i - \sum_{i=1}^{n} (\rho_i s_i + \gamma q_i) \right\}.$$

- The old approach cannot get through anymore.

- Alternative idea: linearize the MINLP.

$$\max_{t} \max_{(q,s)\in D_q^{(K)}\times D_s} \quad \sum_{k=1}^{n+1} \sum_{j=k}^{n+1} \left(\sum_{i=k}^{j} (q_i s_i - x_i)\pi_{ij} \right) t_{kj} - \sum_{i=1}^{n} (\rho_i s_i + \gamma q_i)$$

$$\text{s.t.} \quad \sum_{k=1}^{i} \sum_{j=i}^{n+1} t_{kj} = 1 \quad \forall i = 1, \dots, n+1 \tag{2a}$$

$$t_{kj} \in \{0,1\}, \quad \forall 1 \le k \le j \le n+1. \tag{2b}$$

Reformulations of the DR Model with $D_q^{(K)}$

- Let $p_{ikj} \equiv q_i t_{kj}$ and $o_{ikj} \equiv s_i p_{ikj} \equiv s_i q_i t_{kj}$. McCormick Ineq.:

$$p_{ikj} - t_{kj} \leq 0, \tag{3a}$$

$$p_{ikj} - q_i \leq 0,\ p_{ikj} - q_i - t_{kj} \geq -1,\ p_{ikj} \geq 0, \tag{3b}$$

$$o_{ikj} - s_i^{\mathrm{L}} p_{ikj} \geq 0,\ o_{ikj} - s_i^{\mathrm{U}} p_{ikj} \leq 0, \tag{3c}$$

$$o_{ikj} - s_i + s_i^{\mathrm{L}}(1 - p_{ikj}) \leq 0,\ o_{ikj} - s_i + s_i^{\mathrm{U}}(1 - p_{ikj}) \geq 0. \tag{3d}$$

- (D') is equivalent to a MILP:

$$\begin{aligned} \max_{t,q,s,p,o} \quad & \sum_{k=1}^{n+1} \sum_{j=k}^{n+1} \sum_{i=k}^{j} (\pi_{ij} o_{ikj} - x_i \pi_{ij} t_{kj}) - \sum_{i=1}^{n} (\rho_i s_i + \gamma q_i) \\ \text{s.t.} \quad & (2a)\text{–}(2b),\ (3a)\text{–}(3d), \\ & s_i \in [s_i^{\mathrm{L}}, s_i^{\mathrm{U}}],\ q \in D_q \subseteq \{0,1\}^n. \end{aligned}$$

Benders' Decomposition Algorithm for $D_q^{(K)}$

- A MILP-based reformulation of the DR model with $D_q^{(K)}$:

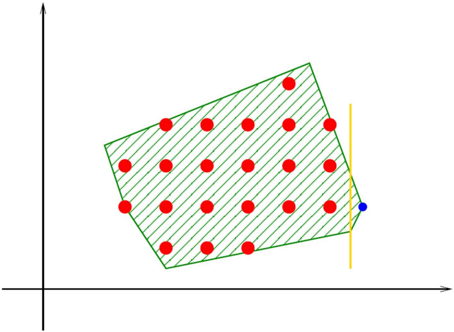
$$\min_{x\in X,\rho,\gamma,\delta} \quad \sum_{i=1}^{n} \mu_i \rho_i + \nu\gamma + \delta$$

$$\text{s.t.} \quad \delta \geq \max_{y\in Y,(q,s)\in D_q\times D_s} \left\{ \sum_{i=1}^{n} (q_i s_i - x_i) y_i - \sum_{i=1}^{n} (\rho_i s_i + \gamma q_i) \right\}. \qquad (5)$$

- A Benders' decomposition algorithm:
  - Solve a relaxed formulation without constraints (5).
  - In each iteration, solve (D') to identify violated constraints (5).
    - If no violations, done;
    - If violations found, incorporate the most violated constraint and re-solve.

Valid Ineq. for General $D_q^{(K)}$

- We derive valid inequalities to strengthen (D').
- Tighter LP relaxation → faster solution of (D').



Valid Ineq. for General $D_q^{(K)}$

Proposition

Valid ineq. for the MILP model (D'):

$$\sum_{k=1}^{i}\sum_{j=i}^{n+1} p_{ikj} = q_i \quad \forall i = 1,\dots,n+1, \tag{6a}$$

$$s_i - \sum_{k=1}^{i}\sum_{j=i}^{n+1}\left(o_{ikj} - s_i^L p_{ikj}\right) \geq s_i^L \quad \forall 1 \leq i \leq n+1, \tag{6b}$$

$$s_i - \sum_{k=1}^{i}\sum_{j=i}^{n+1}\left(o_{ikj} - s_i^U p_{ikj}\right) \leq s_i^U \quad \forall 1 \leq i \leq n+1, \tag{6c}$$

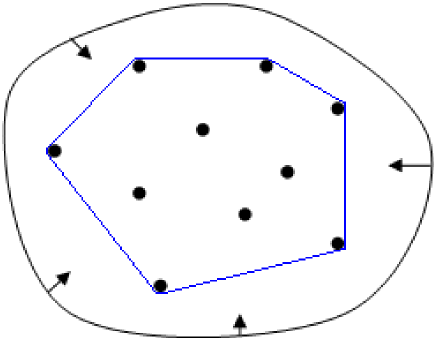
$$\sum_{\ell=i}^{i+K-1} p_{\ell kj} \geq t_{kj} \quad \forall 1 \leq k < j \leq n+1, \forall k \leq i \leq j-K+1, \tag{6d}$$

$$\sum_{k=1}^{i-K+2}\sum_{\ell=i-K+2}^{i} p_{\ell ki} + \sum_{j=i+1}^{n+1} p_{(i+1)(i+1)j} \geq \sum_{k=1}^{i-K+2} t_{ki} \quad \forall i = K-1,\dots,n, \tag{6e}$$

$$\sum_{k=1}^{i} p_{iki} + \sum_{\ell=i+1}^{i+K-1}\sum_{j=i+K-1}^{n+1} p_{\ell(i+1)j} \geq \sum_{j=i+K-1}^{n+1} t_{(i+1)j} \quad \forall i = 1,\dots,n-K+2. \tag{6f}$$

Even Better

- $K = 2$: (6a)–(6f) recover the convex hull of (D') feasible region.
- The DR model is (again) equivalent to a LP.
  - No decomposition needed.
- The LP is of size $\mathcal{O}(n^3)$.



Outline

CPU Time

Table: Average CPU time (in seconds) of solving DR models

Model	E-$D_q^{(2)}$	E-$D_q^{(n+1)}$	E-$D_q^{(K=3)}$		E-$D_q^{(K=5)}$		E-$D_q^{(K=7)}$		E-$D_q^{(K=9)}$	
			Ineq.	w/o	Ineq.	w/o	Ineq.	w/o	Ineq.	w/o
Time (s)	0.053	0.031	8.954	21.045	7.114	28.018	6.427	20.561	6.302	19.266

- E-$D_q^{(2)}$ and E-$D_q^{(n+1)}$ compute polynomial-sized LPs.
- Valid inequalities speed up the decomposition algorithm by 3 times faster, for solving E-$D_q^{(K)}$ with $3 \leq K \leq n$.

Solution Performance for Different $n - \nu$

Statistics	Model	$n-\nu=0.5$			$n-\nu=1.5$			$n-\nu=2.5$			$n-\nu=3.5$			$n-\nu=4.5$		
		WaitT	OverT	IdleT	WaitT	OverT	IdleT	WaitT	OverT	IdleT	WaitT	OverT	IdleT	WaitT	OverT	IdleT
Mean	E-$D_q^{(2)}$	2.16	0.18	13.44	2.01	0.00	18.01	0.38	0.00	22.48	0.33	0.00	27.00	0.00	0.00	31.39
	SLP	0.34	0.00	13.43	0.46	0.00	18.01	0.25	0.00	22.48	0.42	0.00	27.00	0.31	0.00	31.39
50%	E-$D_q^{(2)}$	2.14	0.00	12.08	2.15	0.00	16.73	0.38	0.00	21.45	0.29	0.00	26.18	0.00	0.00	30.51
	SLP	0.21	0.00	12.07	0.37	0.00	16.73	0.14	0.00	21.45	0.29	0.00	26.18	0.17	0.00	30.51
75%	E-$D_q^{(2)}$	2.58	0.00	15.66	2.66	0.00	20.90	0.61	0.00	25.76	0.57	0.00	31.94	0.00	0.00	37.02
	SLP	0.48	0.00	15.64	0.65	0.00	20.90	0.39	0.00	25.76	0.60	0.00	31.94	0.46	0.00	37.02
95%	E-$D_q^{(2)}$	3.28	1.15	20.13	3.39	0.00	28.08	0.94	0.00	33.60	0.91	0.00	38.44	0.00	0.00	43.03
	SLP	1.17	0.00	20.11	1.29	0.00	28.08	0.86	0.00	33.60	1.31	0.00	38.44	1.11	0.00	43.03

- The DR schedules from E-$D_q^{(2)}$ is near-optimal in all metrics.

Solution Performance for Different $n - \nu$

Statistics	Model	$n-\nu=0.5$			$n-\nu=1.5$			$n-\nu=2.5$			$n-\nu=3.5$			$n-\nu=4.5$		
		WaitT	OverT	IdleT	WaitT	OverT	IdleT	WaitT	OverT	IdleT	WaitT	OverT	IdleT	WaitT	OverT	IdleT
Mean	E-$D_q^{(2)}$	2.16	0.18	13.44	2.01	0.00	18.01	0.38	0.00	22.48	0.33	0.00	27.00	0.00	0.00	31.39
	SLP	0.34	0.00	13.43	0.46	0.00	18.01	0.25	0.00	22.48	0.42	0.00	27.00	0.31	0.00	31.39
50%	E-$D_q^{(2)}$	2.14	0.00	12.08	2.15	0.00	16.73	0.38	0.00	21.45	0.29	0.00	26.18	0.00	0.00	30.51
	SLP	0.21	0.00	12.07	0.37	0.00	16.73	0.14	0.00	21.45	0.29	0.00	26.18	0.17	0.00	30.51
75%	E-$D_q^{(2)}$	2.58	0.00	15.66	2.66	0.00	20.90	0.61	0.00	25.76	0.57	0.00	31.94	0.00	0.00	37.02
	SLP	0.48	0.00	15.64	0.65	0.00	20.90	0.39	0.00	25.76	0.60	0.00	31.94	0.46	0.00	37.02
95%	E-$D_q^{(2)}$	3.28	1.15	20.13	3.39	0.00	28.08	0.94	0.00	33.60	0.91	0.00	38.44	0.00	0.00	43.03
	SLP	1.17	0.00	20.11	1.29	0.00	28.08	0.86	0.00	33.60	1.31	0.00	38.44	1.11	0.00	43.03

- The DR schedules from E-$D_q^{(2)}$ is near-optimal in all metrics.
- $K = 5, 7, 9$ yield performance in between those of E-$D_q^{(2)}$ and by E-$D_q^{(n+1)}$.

Solution Performance for Different $n - \nu$

Statistics	Model	$n-\nu=0.5$			$n-\nu=1.5$			$n-\nu=2.5$			$n-\nu=3.5$			$n-\nu=4.5$		
		WaitT	OverT	IdleT	WaitT	OverT	IdleT	WaitT	OverT	IdleT	WaitT	OverT	IdleT	WaitT	OverT	IdleT
Mean	E-$D_q^{(2)}$	2.16	0.18	13.44	2.01	0.00	18.01	0.38	0.00	22.48	0.33	0.00	27.00	0.00	0.00	31.39
	SLP	0.34	0.00	13.43	0.46	0.00	18.01	0.25	0.00	22.48	0.42	0.00	27.00	0.31	0.00	31.39
50%	E-$D_q^{(2)}$	2.14	0.00	12.08	2.15	0.00	16.73	0.38	0.00	21.45	0.29	0.00	26.18	0.00	0.00	30.51
	SLP	0.21	0.00	12.07	0.37	0.00	16.73	0.14	0.00	21.45	0.29	0.00	26.18	0.17	0.00	30.51
75%	E-$D_q^{(2)}$	2.58	0.00	15.66	2.66	0.00	20.90	0.61	0.00	25.76	0.57	0.00	31.94	0.00	0.00	37.02
	SLP	0.48	0.00	15.64	0.65	0.00	20.90	0.39	0.00	25.76	0.60	0.00	31.94	0.46	0.00	37.02
95%	E-$D_q^{(2)}$	3.28	1.15	20.13	3.39	0.00	28.08	0.94	0.00	33.60	0.91	0.00	38.44	0.00	0.00	43.03
	SLP	1.17	0.00	20.11	1.29	0.00	28.08	0.86	0.00	33.60	1.31	0.00	38.44	1.11	0.00	43.03

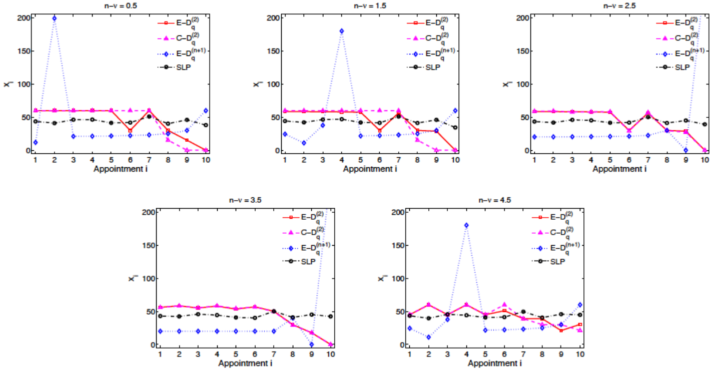
- The DR schedules from E-$D_q^{(2)}$ is near-optimal in all metrics.
- $K = 5, 7, 9$ yield performance in between those of E-$D_q^{(2)}$ and by E-$D_q^{(n+1)}$.
- E-$D_q^{(2)}$ performs better than SLP in misspecified distributions of s and/or q.

Solution Performance for Different $n - \nu$

Statistics	Model	$n-\nu=0.5$			$n-\nu=1.5$			$n-\nu=2.5$			$n-\nu=3.5$			$n-\nu=4.5$		
		WaitT	OverT	IdleT	WaitT	OverT	IdleT	WaitT	OverT	IdleT	WaitT	OverT	IdleT	WaitT	OverT	IdleT
Mean	E-$D_q^{(2)}$	2.16	0.18	13.44	2.01	0.00	18.01	0.38	0.00	22.48	0.33	0.00	27.00	0.00	0.00	31.39
	SLP	0.34	0.00	13.43	0.46	0.00	18.01	0.25	0.00	22.48	0.42	0.00	27.00	0.31	0.00	31.39
50%	E-$D_q^{(2)}$	2.14	0.00	12.08	2.15	0.00	16.73	0.38	0.00	21.45	0.29	0.00	26.18	0.00	0.00	30.51
	SLP	0.21	0.00	12.07	0.37	0.00	16.73	0.14	0.00	21.45	0.29	0.00	26.18	0.17	0.00	30.51
75%	E-$D_q^{(2)}$	2.58	0.00	15.66	2.66	0.00	20.90	0.61	0.00	25.76	0.57	0.00	31.94	0.00	0.00	37.02
	SLP	0.48	0.00	15.64	0.65	0.00	20.90	0.39	0.00	25.76	0.60	0.00	31.94	0.46	0.00	37.02
95%	E-$D_q^{(2)}$	3.28	1.15	20.13	3.39	0.00	28.08	0.94	0.00	33.60	0.91	0.00	38.44	0.00	0.00	43.03
	SLP	1.17	0.00	20.11	1.29	0.00	28.08	0.86	0.00	33.60	1.31	0.00	38.44	1.11	0.00	43.03

- The DR schedules from E-$D_q^{(2)}$ is near-optimal in all metrics.
- $K = 5, 7, 9$ yield performance in between those of E-$D_q^{(2)}$ and by E-$D_q^{(n+1)}$.
- E-$D_q^{(2)}$ performs better than SLP in misspecified distributions of s and/or q.
- A benchmark model not considering no-shows performs poorly once $n - \nu > 0$.

Optimal Schedule Patterns I



- Red curves describe the DR schedules.
- Intuition: “plateau-half-dome” shaped DR schedules.
  - More frequent arrivals towards the end.
  - Performs better if no-show rate increases across the day.

Outline

Conclusions

Technical contributions:

- DR expectation/CVaR models for appointment scheduling with two uncertainties.
- Computationally tractable reformulations via IP approaches.
- Important special cases: LP reformulations.

Conclusions

Technical contributions:

- DR expectation/CVaR models for appointment scheduling with two uncertainties.
- Computationally tractable reformulations via IP approaches.
- Important special cases: LP reformulations.

Managerial insights:

- Reasonable supports in a DR model play an important role.
- Ignoring no-shows is risky.
- "Plateau-half-dome" shaped DR schedules.

Conclusions

Technical contributions:

- DR expectation/CVaR models for appointment scheduling with two uncertainties.
- Computationally tractable reformulations via IP approaches.
- Important special cases: LP reformulations.

Managerial insights:

- Reasonable supports in a DR model play an important role.
- Ignoring no-shows is risky.
- “Plateau-half-dome” shaped DR schedules.

Further research

- Multiple servers; sequencing decisions...

Thank you!

Questions?