Optimizing the Profitability and Quality of Service in Carshare Systems

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Outline

Introduction

Problem formulation

Solution algorithm

Computational results

Applications and extensions

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Carsharing

- Short term car rental
- Consumer benefits
  - Private vehicle
  - No ownership responsibility
- Societal & environmental benefits
  - Reduced congestion (6 cars replaced per shared vehicle)
  - Reduced fuel consumption (vehicle mileage ↘ 44% per carshare user)
  - Reduced greenhouse gas emissions
- Becoming more popular
  - Over 1,000 cities worldwide have adopted carsharing
  - Over 1 million individuals sharing over 20,000 vehicles in the U.S.

Types of carshare

- Reservation-based vs. free-floating
    - Contracted paid parking lots for reservation-based users
    - Free-float parking permits for free float users
- One-way vs. round-trip rentals
    - One-way more desirable for consumers
        - Flexibility of using vehicles
        - Potentially save on rental fees by splitting trips
    - One-way trips problematic for companies
        - Management complexities
        - Unbalanced demand requires redistribution of fleet

Carshare design and optimization

- Strategic decisions to consider, e.g.
  - Implement one-way or not?
  - Offset cost with price differentiation?
    (Zipcar charges \$7.50-\$8.50 per hour round-trip and \$12 per hour one-way in Boston)
- Evaluate the impact
  - Field testing (Zipcar's ONE>WAY beta program)
  - **Mathematical modeling** (focus of this talk)
- Optimize profitability and quality of service via models that
  - Incorporate round-trip & one-way uncertain demands
  - Understand customer behavior in response to decisions
  - Optimize & evaluate strategic decisions

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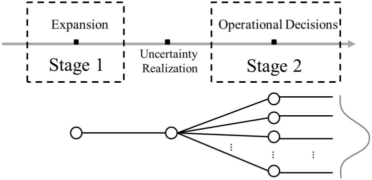
Framing the problem

How does the proportion of one-way demand vs round-trip demand affect profitability and QoS?

Assumptions:

- A set of service zones and a finite number of service periods
- Serve random one-way and round-trip rentals
  - Vehicle movement and demand aggregated by zone
  - Vehicles travel between zones at different periods
  - Demand uncertainty with known distribution
- Cars can be relocated, to balance vehicle distributions
- Unsatisfied demand is immediately lost

A two-stage stochastic programming framework



- Strategic decisions:
  - zone-based # of parking lots to buy (for reservation-based)
  - zone-based # of free-float permits to buy (for free float)
  - # of cars initially allocated in each zone (for both)
- Recourse decisions:
  - Movement of cars (from car users and also relocation)
- Objective: To minimize
  - costs of allocating cars and purchasing parking lots/permits
  - (uncertain) costs of operating and relocating cars
  - (uncertain) penalty of undesirable quality of service (related to unsatisfied demand and denied trips)

1^{st}-stage master problem

$$\begin{aligned} \min_{w,x} \quad & \sum_{i\in I}\left(c_i^{\text{lot}} w_i + \left(c^{\text{ffp}} + c_i^{\text{loc}}\right)x_i\right) + Q(w,x) \\ \text{s.t.} \quad & (w,x) \in X = \left\{ w \in \mathbb{Z}_+^{|I|}, x \in \mathbb{Z}_+^{|I|} \;:\; \sum_{i\in I} x_i \le S, \quad x_i \le w_i,\ \forall i \in I \right\} \end{aligned}$$

- I: Set of zones
- S: Maximum # cars
- c_i^{lot}, $\forall i \in I$: cost for locating one car in zone i.
- $c^{\text{ffp}} = 0$ for reservation-based, and $c_i^{\text{lot}} = 0$, $\forall i \in I$ for free-float.
- $w \in \mathbb{Z}^{|I|}$: # parking lots to purchase in each zone
- $x \in \mathbb{Z}^{|I|}$: # cars to initially allocate to each zone
- $Q(w,x)$ models the 2^{st}-stage recourse problem by using spatial-temporal networks to model realized one-way & round-trip demands.

Building spatial-temporal network

- Example:
  - Zones A, B
  - Time periods, $0, 1, 2, 3$
  - Travel times between zones, A↔B $= 2$
  - n_{it}: Zone i at time t

Type	Volume	Origin	Destination	Start	End
One-way	4	A	B	0	–
Round-trip	2	B	–	1	3

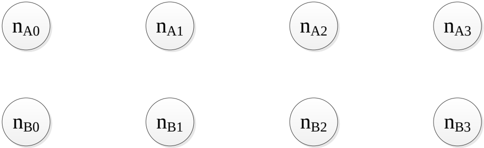


Figure: Spatial-temporal nodes

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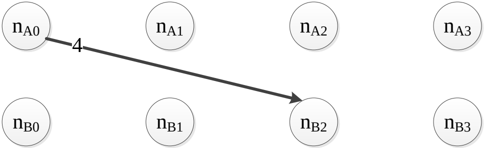

Figure: One-way arcs

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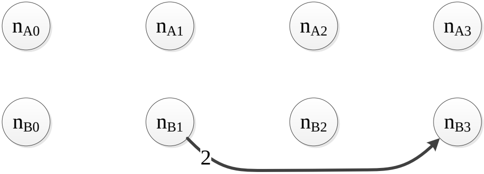

Figure: Round-trip arcs

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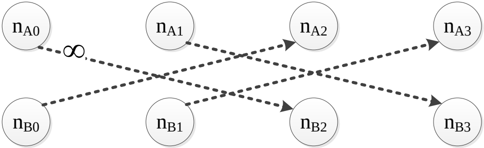

Figure: Relocation arcs

Building spatial-temporal network

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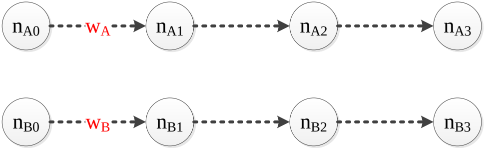

Figure: Idling arcs

Building spatial-temporal network

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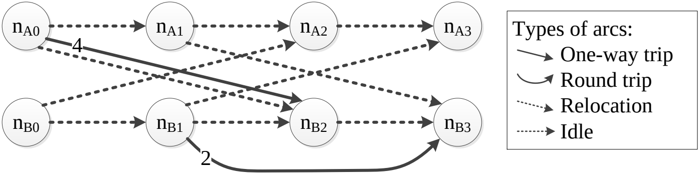

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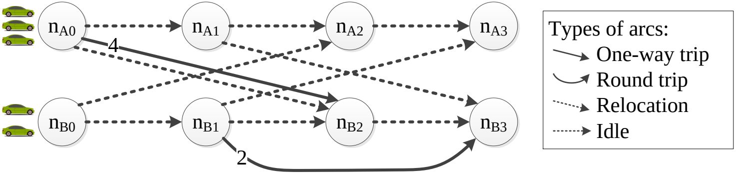


Figure: Spatial-temporal network

Spatial-temporal network

- Random demand
  - d^{one}_{ijts}: # of one-way rentals from zone i starting at period t and returned to zone j at period s.
  - d^{two}_{its}: # of round-trip rentals from zone i starting at period t and returned to zone i at period s.
- Original cost parameters
  - r^{one}: Revenue per one-way rental per period
  - r^{two}: Revenue per round-trip rental per period
  - c^{mnt}: Maintenance cost per car per period
  - c^{rel}: Relocation cost per car per period
- Network arc parameters

Type of arc	Cost per unit flow f_a	Capacity u_a
One-way arc (n_{it}, n_{js})	$-(r^{\text{one}} - c^{\text{mnt}})(s-t)$	d^{one}_{ijts}
Round-trip arc (n_{it}, n_{is})	$-(r^{\text{two}} - c^{\text{mnt}})(s-t)$	d^{two}_{its}
Relocation arc $(n_{it}, n_{j,t+\ell_{ij}})$	$(c^{\text{rel}} + c^{\text{mnt}})\ell_{ij}$	$+\infty$
Idle arc $(n_{it}, n_{i,t+1})$	c^{idle}	w_i

2^{nd}-stage minimum cost flow problem

$$Q(w,x) =$$

$$\min_{y^1,\ldots,y^{|K|}} \quad \sum_{k\in K} p^k \sum_{a\in A} f_a y_a^k + g(y^1,\ldots,y^{|K|})$$

$$\text{s.t.} \quad y^k \in Y(w,x,u^k) = \{y^k \in \mathbb{R}_+^{|A|} :$$

$$\sum_{a\in\delta^+(n_{it})} y_a^k - \sum_{a\in\delta^-(n_{it})} y_a^k = \begin{cases} x_i & \text{if } t=0 \\ 0 & \text{if } t = 1,2,...,T-1 \\ -x_i & \text{if } t = T \end{cases} \qquad \forall i \in I$$

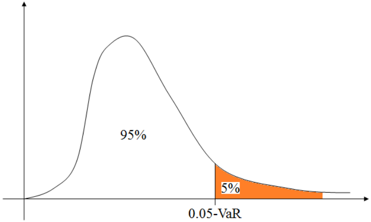
$$y_a^k \le u_a^k \qquad \forall a \in A^{\text{one}} \cup A^{\text{two}}$$

$$y_a^k \le w_i \qquad \forall i \in I, a = (n_{it}, n_{i,t+1}) \in A^{\text{idle}}\} \qquad \forall k \in K$$

- K: Set of demand scenarios (sampled from known distribution)
- A: Set of spatial-temporal arcs
- y^k: Flows on spatial-temporal network
- $Y(w,x,u^k)$: Flow balance & capacity constraints
- $Q(w,x)$ is a large-scale linear program given fixed x and w.

QoS penalty $g(x, y^1, \dots, y^K)$

- Penalize # of unserved customers
  - i.e., unused capacity on rental arcs $(u_a^k - y_a^k)$, $\forall a \in A^{\text{one}} \cup A^{\text{two}}$
- Risk-neutral model
  - $g(x, y^1, \dots, y^K) = \sum_{k \in K} p^k \sum_{a \in A^{\text{one}} \cup A^{\text{two}}} G_a(u_a^k - y_a^k)$
- Risk-averse model
  - $H_a(w, x)$: denotes the unused capacity on arc a given w and x
  - $g(x, y^1, \dots, y^K) = G_0 \text{CVaR}_{1-\epsilon} \left(\sum_{a \in A^{\text{one}} \cup A^{\text{two}}} H_a(w, x) \right)$



  - G_0 penalizes the expected value of the worst $100\epsilon\%$ scenarios.

2^{nd}-stage linear program when penalizing CVaR

$$\begin{aligned} Q_c(w,x) = \min_{y,z,v\geq 0} \quad & \sum_{k\in K} p^k \sum_{a\in A} f_a y_a^k + G_0 \left(v + \frac{1}{\epsilon} \sum_{k\in K} p^k z^k \right) \\ \text{s.t.} \quad & y^k \in Y(w,x,u^k),\ z^k \geq 0 \qquad \forall k \in K \\ & z^k \geq \sum_{a\in A^{\text{one}}\cup A^{\text{two}}} (u_a^k - y_a^k) - v \qquad \forall k \in K, \end{aligned}$$

where

- v: $\text{VaR}_{1-\epsilon}\left(\sum_{a\in A^{\text{one}}\cup A^{\text{two}}} H_a(w,x)\right)$
- $z^k = \max\left\{\sum_{a\in A^{\text{one}}\cup A^{\text{two}}}(u_a^k - y_a^k) - v, 0\right\}$

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Branch-and-cut and parallelization

- There can be many 2nd-stage subproblems
  $\Rightarrow$ many spatial-temporal networks with huge sizes due to
  - Fine division into zones
  - Fine granularity of time

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- Branch-and-cut algorithm
    - Branch on integer variables in the 1st stage
    - Use Benders decomposition to generate Benders cuts
    - Lift Benders cuts with mixed-integer rounding (MIR)
        - Follow similar ideas in Bodur and Luedtke (2014) for a two-stage stochastic integer program for call-center staffing

Branch-and-cut and parallelization

- There can be many 2nd-stage subproblems
  ⇒ many spatial-temporal networks with huge sizes due to
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- Branch-and-cut algorithm
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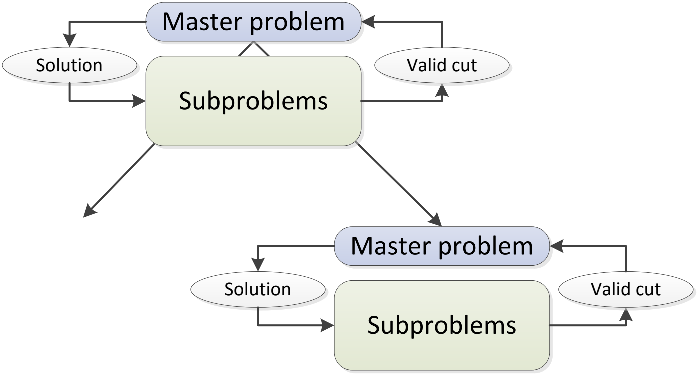
  - Use Benders decomposition to generate Benders cuts
  - Lift Benders cuts with mixed-integer rounding (MIR)
    - Follow similar ideas in Bodur and Luedtke (2014) for a two-stage stochastic integer program for call-center staffing
- Use parallel computing to speed up subproblem computation
  - Master-Worker scheme by OpenMPI 1.6
  - UM Flux HPC cluster, 20 cores each with 48GB RAM.

Branch-and-cut



Master problem

Solution

Subproblems

Valid cut

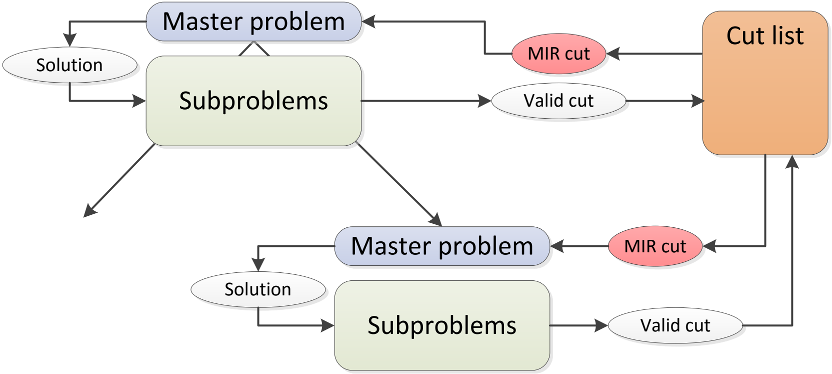
Master problem

Solution

Subproblems

Valid cut

Branch-and-cut with MIR (Bodur & Luedtke 2014)



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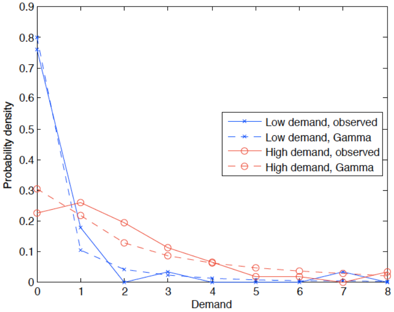
Data description

- Boston-Cambridge Zipcar; Data from Oct 1 to Dec 1, 2014
- Zipcar cost parameters; $c^{\text{rel}} = \$22$ or \$10 per hour for car relocation
- 1-hour periods, over 24 hours
- $\epsilon = 0.1$ and 0.05 for the CVaR risk-averse model
- Use Java + Gurobi 6.0.3; Intel dual-core CPU with 8GB RAM.

Demand data follows Gamma distributions



Computational efficiency

- Tests run for $|K| = 100, 200, 500, 1000$
- Across all sets of results
  - Branch-and-cut slower if subproblems solved in series, faster if solved in parallel
  - # iterations fairly constant with $|K|$
- Using MIR can improve parallel solve time

Model	Series solve time	Parallel solve time	# iterations
`Stoch`	236,207	236,207	-
`Stoch-Branch`	291,389	34,430	39
`Stoch-MIR`	160,272	27,175	37
`CVaR`	190,072	190,072	-
`CVaR-Branch`	254,038	35,301	30
`CVaR-MIR`	343,550	34,544	39

Table: Computational time (in second) for $|K| = 1000$

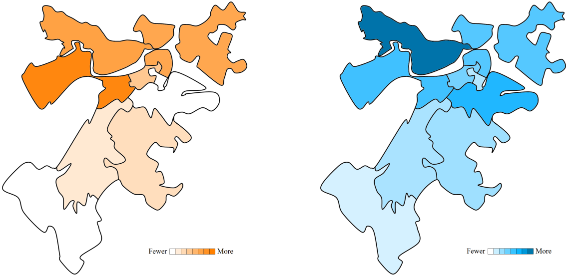
Computational results

Question: How does the proportion of one-way demand vs round-trip demand affect profitability and QoS?

- Vary proportion of one-way trips
  - 0%, 20%, 40%, 60%, 80%, and 100%
- Vary carshare systems:
  - Reservation based: $c_i^{\text{lot}} = \$9.6$ per hour in zones $i = 1, 2, 5, 6, 9$ and $c_i^{\text{lot}} = \$7.4$ per hour in other zones; $c^{\text{ffp}} = 0$
  - Free-float: $c^{\text{ffp}} = \$9.6$ per hour; $c_i^{\text{lot}} = 0, \ \forall i = 1, \dots, 9$
- Reduce relocation cost from \$22 per hour to \$10
  - U.S. average wage in 2014 ⇒ minimum hour-pay requirement

Some general remarks of the results

- Similar risk neutral and CVaR penalty model results
- Similar reservation-based and free-floating system results
    - Because similar costs of parking lot and free-float permit per year
- Vehicle allocation cannot be directly inferred from demand concentration



(a) Demand heatmap (b) Vehicle allocation

Figure: Demand concentration (by starting location) vs vehicle allocation

Recommending Changes

- We tried to increase the revenue of one-way to improve profitability and quality of service (QoS)
  - Does not help when one-way proportion is high
- **Effective approach**: Decrease unit cost for car relocation $c^{\text{rel}} = \$22 \Rightarrow c^{\text{rel}} = \$10$
- Effects of change
  - Slight increase in profitability
  - Major improvement in QoS
  - Major reduction in trips denied
- More willing to relocate cars to meet demand

Change in profitability

One-way proportion	Setup cost (\$)	One-way trip revenue (\$)	Round-trip revenue (\$)	Relocation cost (\$)	Profit (\$)
60%	1,770	4,804	7,475	333	10,176
80%	1,682	6,625	3,572	454	8,061
100%	1,496	8,359	0	549	6,315

Table: Profitability before change ($c^{\text{rel}} = \$22$)

One-way proportion	Setup cost (\$)	One-way trip revenue (\$)	Round-trip revenue (\$)	Relocation cost (\$)	Profit (\$)
60%	1,749	6,761	7,491	1,986	10,517
80%	1,635	9,282	3,570	2,696	8,521
100%	1,418	11,700	0	3,389	6,894

Table: Profitability after change ($c^{\text{rel}} = \$10$)

Change in QoS (unsatisfied demand)

In general, we observe expected unserved customers ↗ as one-way proportion ↗ for both $c^{\text{rel}} = \$22$ and $c^{\text{rel}} = \$10$.

One-way proportion	Proportion of trips unfulfilled	Idle vehicle-hours
60%	18.50%	3,209
80%	23.40%	3,433
100%	29.10%	3,264

Table: QoS before change ($c^{\text{rel}} = \$22$)

One-way proportion	Proportion of trips unfulfilled	Idle vehicle-hours
60%	1.20%	2,871
80%	0.20%	2,981
100%	0.00%	2,711

Table: QoS after change ($c^{\text{rel}} = \$10$)

Change in QoS (denied trips)

Definition: Trips disallowed despite there being a car to serve demand. Expected denied trips ↗ as one-way proportion ↗.

One-way proportion	Mean	99th percentile
60%	11.64%	15.47%
80%	15.30%	20.40%
100%	19.71%	25.03%

Table: Denied trips before change ($c^{\text{rel}} = \$22$)

One-way proportion	Mean	99th percentile
60%	0.08%	0.82%
80%	0.00%	0.21%
100%	0.00%	0.21%

Table: Denied trips after change ($c^{\text{rel}} = \$10$)

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Extensions of for-profit carshare

- Not-for-profit car sharing
  - City Carshare (Carma): nonprofit carsharing program in Bay Area since 2001
  - Function: provide last-miles solutions for public transit
  - Mission: Reduce cars; improve traffic; reduce emissions

- $>$ 80% of Carma's vehicles are electric powered $\Rightarrow$
  - Project 1: Composition of shared car fleet design for meeting not-for-profit goals and encouraging carsharing

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    - Project 2: Locating charging stations for shared electric vehicles (EVs)
    - Project 3: Joint management of shared EVs in coupled power and transportation networks

Carshare for underserved communities

Underserved communities: Disabled, elderly, and low-income

- limited access to personal vehicles
- limited access to high techs used for booking vehicles
- limited access to services other than transportation need

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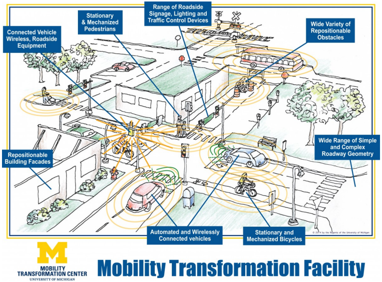
A vehicle-and-service-sharing system (V3S):

- allocate vehicles to communities shared by households
- match vehicle sharing with service sharing needs
- encourage service sharing by waiving/decreasing drivers' vehicle usage fee
- periodically relocates vehicles across multiple communities by learning service sharing behavior

Take a bigger step?

Next: Autonomous vehicle (AV) sharing?



- Obtain data about AV speed and connectivity from M-city.
- Design AV-based carshare system and V3S.
- Develop stochastic dynamic program for AV control.

Thank you!

Questions?