Risk Averse Shortest Path Interdiction

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Network interdiction: a two-player game

Stackelberg game (two player; sequential moves) played on a network.

(a) Leader: Interdictor

(b) Follower: Operator

- Goal: maximally restrict a follower's utility gained in the network by damaging arcs or nodes.

Applications

- Smugglers (followers) evade authorities (leaders) who lead the game by placing checkpoints.
- Emergency service providers (leaders) allocate resources and fortify arcs/nodes against malicious attacks (followers).
- ...

Deterministic Network Interdiction

A shortest path network interdiction on Graph $G(V, A)$:

- $x_a \in \{0, 1\}$, $\forall a \in A$: whether or not interdict arc a
- $y_a \in \{0, 1\}$, $\forall a \in A$: whether or not arc a is on the path chosen by the follower

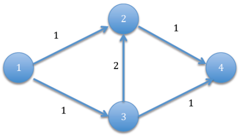
$$\max_{x \in X} \min_{y} \sum_{a \in A} (c_a + d_a x_a) y_a$$

$$\text{s.t.} \quad \sum_{a \in \delta^+(i)} y_a - \sum_{a \in \delta^-(i)} y_a = \begin{cases} 1 & \text{if } i = s \\ -1 & \text{if } i = t \\ 0 & \text{o.w.} \end{cases}, \ \forall i \in V$$

$$y_a \geq 0, \ \forall a \in A$$

where $X = \{x \in \{0, 1\}^{|A|} \mid \sum_{a \in A} r_a x_a \leq R\}$

Assume $d_a = 3, \forall a$, and we can interdict up to two arcs



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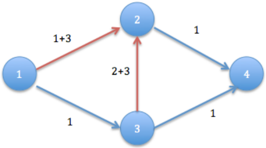

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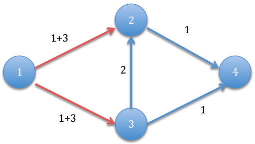

Solution Approaches (Morton 2011)

Given a relaxed interdiction $\hat{x}$, the follower chooses a shortest path using $c_a + d_a\hat{x}_a$ as the length for each arc a:

- Extended formulation: take the dual of the inner shortest path LP

$$\begin{aligned} \max_{x\in X,\pi} \ & \pi_t \\ \text{s.t. } & \pi_j - \pi_i \leq c_a + d_a x_a, \ \forall a = (i,j) \in A \\ & \pi_s = 0 \end{aligned}$$

- Benders formulation:

$$\max_{x\in X} \ \min_{P\in\mathcal{P}} \sum_{a\in P} (c_a + d_a x_a)$$

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- Benders formulation:

$$\max_{x \in X} \ \{\theta \mid \theta \leq \sum_{a \in P} (c_a + d_a x_a), \ \forall P \in \mathcal{P}\}$$

Stochastic Network Interdiction

Assume the arc lengths $\tilde{c}_a$ and interdiction effects $\tilde{d}_a$ are uncertain, and the uncertainty can be characterized by a finite set of scenarios $\{(c_a^k, d_a^k)\}_{k \in N}$

$$\max_{x \in X} \sum_{k \in N} p_k \min_{y^k} \sum_{a \in A} (c_a^k + x_a d_a^k) y_a^k$$

$$\text{s.t.} \quad \sum_{a \in \delta^+(i)} y_a^k - \sum_{a \in \delta^-(i)} y_a^k = \begin{cases} 1 & \text{if } i = s \\ -1 & \text{if } i = t \\ 0 & \text{o.w.} \end{cases}, \ \forall i \in V, \ \forall k$$

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$$\max_{x \in X} \sum_{k \in N} p_k \theta^k$$

$$\text{s.t. } \theta^k \leq \sum_{a \in P} (c_a^k + d_a^k x_a), \ \forall P \in \mathcal{P}$$

- Benders formulation is preferred, since it enables scenario decomposition
- Could be strengthened by additional valid inequalities, e.g., the step inequalities (Pan and Morton 2008)

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Limit: the risk aversion of the players are not considered

Risk Averse Shortest Path Interdiction (RASPI)

Model risk aversion by chance constraint: risk averse interdictor (leader) targets on high probability of enforcing a long distance for the traveler

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Two settings:

- Wait-and-see follower: make optimal response after observing the random outcome
  - We do not need the follower's risk attitude in this case
  - Traditional stochastic shortest path interdiction problem assumes a risk neutral leader
- Here-and-now follower: must make a decision before the observation of the random outcome
  - We assume the follower is risk neutral in the here-and-now setting: choose a path that has the shortest expected distance

Outline

Chance-constrained RASPI with Wait-and-see Follower

Chance-constrained RASPI with Here-and-now Follower

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Risk averse interdictor with wait-and-see follower: a chance-constrained model

Idea: Ensure that the follower's shortest possible traveling distance from s to t exceeds a given length ϕ with high probability

$$\begin{aligned}
\min_{x,z} \ & r^\top x \\
\text{s.t.} \ & \sum_{a \in A} (c_a^k + d_a^k x_a) y_a^k(x) \geq \phi z_k, \ \forall k \in N, \\
& \sum_{k \in N} p_k z_k \geq 1 - \epsilon, \\
& z_k \in \{0,1\}, \ \forall k \in N, \ x_a \in \{0,1\}, \ \forall a \in A
\end{aligned}$$

where $y^k(x) \in \arg\min_{y \in Y} \sum_{a \in A} (c_a^k + d_a^k x_a) y_a$, Y : flow balance equations

- $z_k \in \{0,1\}$: whether or not scenario k is satisfied

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$$

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Standard Benders decomposition

Given a relaxation solution $\hat{x}, \hat{z}$ of the master problem (with a subset of paths)

- Solve a shortest path problem for each scenario k using $c_a^k + d_a^k \hat{x}_a$ as the arc length, and get the shortest path P^k
- Check if inequality $\sum_{a \in P^k} (c_a^k + d_a^k \hat{x}_a) \geq \phi \hat{z}_k$ is violated, and add a Benders cut if so
- Could be applied for both integer and fractional solutions

Implicit covering structure

Scenario-based path inequality:

$$\sum_{a \in P} (c_a^k + d_a^k x_a) \geq \phi z_k, \forall P \in \mathcal{P}$$

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$$\sum_{a \in P} d_a^k x_a \geq (\phi - l_P^k) z_k, \forall P \in \mathcal{P}$$

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Structure: exponentially many covering constraints

- Related to Song and Luedtke (2013), $\sum_{a \in C_k} x_a \geq z_k$, "scenario-based graph cut inequalities"
- Related to Song, Luedtke, and Kücükyavuz (2014), multi-dimensional binary packing problems with a small (non-exponential) number of constraints

Pack-based formulation: Motivation

Fix a scenario k, given a set of arcs C, if none is interdicted in C, we cannot achieve the target $\Rightarrow$ C is a pack in that scenario k!

$$\exists P \in \mathcal{P} : \sum_{a \in P \cap C} c_a^k + \sum_{a \in P \setminus C} (c_a^k + d_a^k) < \phi$$

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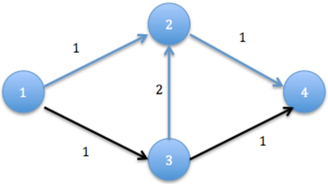
So, we must interdict enough arcs in the pack:

$$\sum_{a \in C} x_a \geq \psi(C),$$

$\psi(C)$: the minimum number of arcs in C to interdict



- $d_a = 3, \forall a \in A$
- Need the shortest path to be at least 5
- $\psi(C) = 1$

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$$\sum_{a \in C} x_a \geq \psi(C) z_k, \ \forall \text{ pack } C,$$

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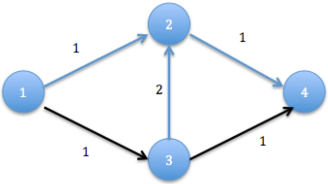

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Pack-based formulation

We can just focus on the minimal packs ($\forall a \in C, C \setminus \{a\}$ is not a pack) $\Rightarrow$ in this case $\psi(C) = 1$:

$$\begin{aligned}
\min_{x,z}\ & r^\top x \\
\text{s.t. } & \sum_{a \in C} x_a \geq z_k,\ \forall k \in N,\ \forall \text{ minimal pack } C \\
& \sum_{k \in N} p_k z_k \geq 1 - \epsilon \\
& z_k \in \{0, 1\},\ \forall k \in N
\end{aligned}$$

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- We can perform lifting to strengthen the "base" pack inequality $\sum_{a \in C} x_a \geq 1$ similarly to the 0-1 knapsack problem

Review: Lifting

1. Sequential lifting: (Gu et al., 1997)
   - Efficient, especially when combined with Zemel's Algorithm (1989)
2. Sequence independent lifting: (Gu et al., 1998)
   - Even more efficient, obtain approximate lifting coefficients simultaneously
3. Multidimentional knapsack: (Kaparis and Letchford, 2009)
   - Sequential lifting, and solve the LP relaxation of the exact lifting problem

Review: sequential lifting

1. Downlifting
   - Suppose $\sum_{j \in L} \alpha_j x_j \geq \beta$ is valid with $x_t = 1, t \in N \setminus L$
   - Downlift on x_t so that $\sum_{j \in L} \alpha_j x_j + \alpha_t x_t \geq \beta + \alpha_t$
   - Downlifting strengthens the starting valid inequality
2. Uplifting
   - Suppose $\sum_{j \in L} \alpha_j x_j \geq \beta$ is valid with $x_t = 0, t \in N \setminus L$
   - Lift on x_t so that $\sum_{j \in L} \alpha_j x_j + \alpha_t x_t \geq \beta$
   - Uplifting is necessary for the validity of the inequality

The lifting problem

- Suppose we start with the base pack inequality $\sum_{a \in C_1} x_a \geq 1$
- Assume $x_a = 0, a \in C_2,\ x_a = 1, a \in F$
- Now we downlift $x_0 \in F$ to strengthen the basic pack inequality:

Exact lifting problem:

$$\begin{aligned} \pi_0 := \min \ & \sum_{a \in C_1} x_a \\ \text{s.t.} \ & \sum_{a \in P} d_a^k x_a \geq \phi - l_P^k,\ \forall P \in \mathcal{P} \\ & x_a = 0,\ \forall a \in C_2, x_0 = 0, x_{F \setminus \{0\}} = 1 \\ & x_a \in \{0,1\},\ \forall a \in A \end{aligned}$$

Lifting coefficient: $\beta_0 := \max\{0, \pi_0 - 1\}$

$\Rightarrow$ Lifted inequality: $\sum_{a \in C_1} x_a + \beta_0 x_0 \geq 1 + \beta_0$

Approximate lifting

Motivation: relaxation of the lifting problem will also give a valid lifting coefficient

- LP relaxation of the lifting problem
- Restrict to a single path: the path that defines the pack
  - Lifting for 0-1 knapsack problems with a single knapsack constraint could be applied
  - Gu et al (1998): "default" lifting sequence

Lifted pack inequality:

$$\sum_{a \in C_1} x_a + \sum_{a \in F} \beta_a x_a + \sum_{a \in C_2} \gamma_a x_a \geq (1 + \sum_{a \in F} \beta_a) z_k$$

Preliminary results: benefit of combinatorial information

- We generate grid network instances with random length $\{c_a^k\}_{k\in N, a\in A}$ and random interdiction effects $\{d_a^k\}_{k\in N, a\in A}$
- We solve 5 replications for each setting and show the average results

Instances			Extended		Benders		Pack-based	
Instance	ϵ	N	AvgT	AvgN	AvgT	AvgN	AvgT	AvgN
nodearc-5	0.1	100	15.9	1738	15.6	45k	0.2	58
(25,80)		1000	22%(0)	>18k	30.5%(0)	>607k	6.9	252
	0.2	100	25.4	2181	72.6	178k	0.3	81
		1000	34%(0)	>16k	M	M	22.1	642

- Extended formulation and simple Benders are competitive
- Useful to exploit the combinatorial structure using pack-based formulation

Preliminary results: Benefit of doing lifting

Instances			No Lifting			Lifting		
Instance	ϵ	N	AvgT	AvgN	AvgR	AvgT	AvgN	AvgR
nodearc-5	0.1	100	0.2	58	17%	0.2	56	16%
(25,80)		1000	6.9	252	18%	5.8	189	17%
	0.2	100	0.3	81	22%	0.3	88	21%
		1000	22.1	642	28%	20.9	554	27%
nodearc-8	0.1	100	1378.0	58k	30%	384.8	17k	25%
(64,224)		1000	19%(0)	$>$31k	37%	15%(0)	$>$ 23k	32%
	0.2	100	4%(3)	$>$57k	36%	857.3	24k	31%
		1000	30%(0)	$>$13k	46%	25%(0)	$>$10k	43%

- We perform lifting based on a single path, and use the "default" sequence of lifting from Gu (1998)
- Lifting is more beneficial in the harder instances

Lifting based on a single path vs. LP-based lifting

Instances			LP-based Lifting			single path Lifting		
Instance	ϵ	N	AvgT	AvgN	AvgR	AvgT	AvgN	AvgR
nodearc-5	0.1	100	7.5	47	15%	0.2	56	16%
(25,80)		1000	299.6	194	18%	5.8	189	17%
	0.2	100	15.0	107	20.4%	0.3	88	20.5%
		1000	838.3	730	27.7%	20.9	554	27.1%
nodearc-8	0.1	100	8%(1)	>2k	25%	384.8	17k	25%
(64,224)		1000	27%(0)	>283	33%	15%(0)	>23k	32%
	0.2	100	(1)	>3k	33.4%	857.3	24k	31.4%
		1000	(0)	>283	44.2%	(0)	>10k	42.8%

- Benefit of doing LP-based lifting is not clear
- LP-based lifting too time-consuming

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Risk averse interdictor with wait-and-see follower: a bilevel optimization model

Idea: Ensure that the actual traveling distance of the traveler exceeds a given length ϕ with high probability

$$\min_{x,z} \; r^\top x \tag{1}$$

$$\text{s.t.} \; \sum_{a \in A} (c_a^k + d_a^k x_a) y_a^k(x) \geq \phi z_k, \; \forall k \in N, \tag{2}$$

$$\sum_{k \in N} p_k z_k \geq 1 - \epsilon, \tag{3}$$

$$z_k \in \{0,1\}, \; \forall k \in N, \; x_a \in \{0,1\}, \; \forall a \in A \tag{4}$$

$$\text{where } y^k(x) \in \arg\min_{y \in Y} \sum_{a \in A} (c_a^k + d_a^k x_a) y_a \tag{5}$$

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Bilevel: constraint coefficient vector of (2) is an optimal solution to another optimization problem

A trivial cutting plane method

- $y_a(x)$ is a piecewise constant function, where the discontinuity occurs only at binary integer points
- Checking the feasibility of an integer $\hat{x} \in \{0, 1\}^{|A|}$ is simple: $y(\hat{x})$ is the shortest path solution

No-good feasibility cut:

$$\sum_{a \in N_0} x_a + \sum_{a \in N_1} (1 - x_a) \geq 1,$$

where $N_0 = \{a \in A \mid \hat{x}_a = 0\}$, and $N_1 = \{a \in A \mid \hat{x}_a = 1\}$

Reformulation using strong LP duality

The follower's problem is an LP. Apply strong duality:

$$
\begin{aligned}
\min_{x,z,y,u} \quad & r^\top x \\
\text{s.t.} \quad & \sum_{a \in A} (c_a^k + d_a^k x_a) y_a \geq \phi z_k, \ \forall k \in N \\
& \sum_{a \in A} (\bar{c}_a + \bar{d}_a x_a) y_a = u_s - u_t \\
& u_i - u_j \leq \bar{c}_a + \bar{d}_a x_a, \ \forall a = (i,j) \in A \\
& \sum_{k \in N} p_k z_k \geq 1 - \epsilon \\
& z_k \in \{0,1\}, \ \forall k \in N, \ x_a \in \{0,1\}, \ y \in Y.
\end{aligned}
$$

- An MINLP model, could apply the standard linearization trick to linearize the bilinear term
- Bad news: not decomposable by scenario, since decision variables $\{x_a, y_a\}_{a \in A}$ and $\{u_i\}_{i \in V}$ are independent of scenario.

An alternative "primal" formulation

An alternative way to model the shortest path:

$$
\begin{aligned}
\min_{x,z,y,w} \quad & r^\top x \\
\text{s.t.} \quad & \sum_{a \in A} (c_a^k y_a + d_a^k w_a) \geq \phi z_k, \ \forall k \in N \\
& \sum_{a \in A} (\bar{c}_a + \bar{d}_a x_a) y_a \leq \sum_{a \in P} (\bar{c}_a + \bar{d}_a x_a), \forall P \in \mathcal{P} \ (*) \\
& \sum_{k \in N} p_k z_k \geq 1 - \epsilon \\
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& \sum_{a\in A}(\bar{c}_a y_a + \bar{d}_a w_a) \leq \sum_{a\in P}(\bar{c}_a + \bar{d}_a x_a), \forall P \in \mathcal{P}\ (*) \\
& \sum_{k\in N} p_k z_k \geq 1-\epsilon \\
& z_k \in \{0,1\},\ \forall k \in N,\ x_a \in \{0,1\},\ y \in Y \\
& w_a = x_a y_a,\ \forall a \in A
\end{aligned}
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$$\min_{x,z,y,w} \; r^\top x$$

$$\text{s.t.} \; \sum_{a \in A} (c_a^k y_a + d_a^k w_a) \geq \phi z_k, \; \forall k \in N$$

$$\sum_{a \in A} (\bar{c}_a y_a + \bar{d}_a w_a) \leq \sum_{a \in P} (\bar{c}_a + \bar{d}_a x_a), \forall P \in \mathcal{P} \; (*)$$

$$\sum_{k \in N} p_k z_k \geq 1 - \epsilon$$

$$z_k \in \{0,1\}, \; \forall k \in N, \; x_a \in \{0,1\}, \; y \in Y$$

$$w_a = x_a y_a, \; \forall a \in A$$

Preliminary experiments: given a relaxation solution $\hat{x}$:

- Separate inequality (*)
- Look for no-good cuts based on an integer solution by rounding $\hat{x}$

Very preliminary results

Instances			MIP		Cutting plane	
Instance	ϵ	N	AvgT	AvgN	AvgT	AvgN
nodearc-5	0.2	100	0.9	367	1.7	415
(25,80)		1000	17.9	744	29.1	897
nodearc-8	0.2	100	317.5	30525	106.1	34201
(64,224)		1000	588.0	5462	795.0	22715

The two formulations are competitive

Summary

We investigate:

- Two type of risk averse (chance-constrained) shortest path interdiction problem (RASPI)
- Wait-and-see follower:
  - Take advantage of the combinatorial information
  - Lifted pack inequalities are effective
- Here-and-now risk neutral follower:
  - A bilevel problem formulation

Summary

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Ongoing research

- Investigate other variants of network interdiction problems: maximum flow, minimum cost flow, etc.
- Strong valid inequalities for bilevel programming formulation