

PROBLEM SET 3 (POSTED ON THURSDAY, SEP 24)

(All Exercises are references to the October 21, 2025 version of *Foundations of Algebraic Geometry* by R. Vakil.)

- Problem 1.** Exercise 3.6.K (sometimes functions are determined by their values on closed points)
- Problem 2.** Exercise 3.7.G (irreducible components of $\operatorname{Spec} A$ correspond to minimal prime ideals in A)
- Problem 3.** Use the structure sheaf $\mathcal{O}_{\operatorname{Spec} A}$ to show that if $\operatorname{Spec} A$ is disconnected, then A is isomorphic to the product of two nonzero rings. (Be careful to show that the two rings are nonzero!)
- Problem 4.** Let $X = \operatorname{Spec} k[x, y, z]/(xz, yz)$ (draw a picture of X !) and let $U \subset X$ be the complement of the closed point $[(x, y, z)]$. Compute the ring $\mathcal{O}_X(U)$ along with the restriction map $\operatorname{res}_{X,U} : \mathcal{O}_X(X) \rightarrow \mathcal{O}_X(U)$. Is $\operatorname{res}_{X,U}$ isomorphic to some localization map $A \rightarrow S^{-1}A$?
- Problem 5.** Exercise 4.3.A (classifying isomorphisms of affine schemes)