

PROBLEM SET 0 (POSTED ON THURSDAY, SEP 3)

(All Exercises are references to the October 21, 2025 version of *Foundations of Algebraic Geometry* by R. Vakil.)

- Problem 1.** Give a single example of a presheaf $\mathcal{F}$ that simultaneously fails both sheaf axioms (identity and gluability, as in Section 2.2.6).
- Problem 2.** Let $X = \mathbb{R}$. Let $\mathcal{F}$ be the sheaf on X of locally rational functions $f : U \rightarrow \mathbb{R}$, i.e. of functions $f : U \rightarrow \mathbb{R}$ such that for any point $p \in U$ there is an open neighborhood $p \in V \subseteq U$ and a rational function $g(t) \in \mathbb{R}(t)$ such that $f = g$ on V . Describe the stalk of $\mathcal{F}$ at the origin. (You will have to read Section 2.2.4 for stalks or wait for Tuesday's class.)