PROBLEM SET 12 (POSTED ON THURSDAY, NOV 20)

(All Exercises are references to the September 8, 2024 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Exercise 12.3.G (closed subvarieties of $\mathbb{P}^n$ intersect when suggested by dimensions - you will want to use (and maybe also solve, or just use the statements as black boxes) some combination of Exercises 12.3.F, 12.3.D, 12.2.F as well as the notion of an affine cone from section 9.3.11.)

Problem 2. Exercise 12.2.N (most surfaces of degree $d > 3$ have no lines - Vakil gives a detailed outline of how to do this, and you might also find it helpful to read ahead to his proof of Bertini's Theorem (13.4.2) for a similar argument, but here is an additional note: if you are unfamiliar with the Grassmannian $\mathbb{G}(1,3)$, you can replace it in this proof with a single affine chart $\mathbb{A}^4$, where the closed point $(x_1, x_2, x_3, x_4) \in \mathbb{A}^4$ corresponds to the line between $[1 : 0 : x_1 : x_2]$ and $[0 : 1 : x_3 : x_4]$ in $\mathbb{P}^3$. You will conclude that "most" degree d surfaces have no lines of this form, and then you can finish by noting that the set of lines in $\mathbb{P}^3$ can be covered by finitely many charts of this type.)