Thermal Conductivity of Electrons and Muons in Neutron Star Cores

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Abstract

Electron and muon thermal conductivity of dense matter ($\rho \gtrsim 10^{14}\,\mathrm{g\,cm^{-3}}$) in neutron star cores with various nuclear compositions (n, p, e, μ, and possibly hyperons) is calculated. The effects of nucleon (and/or hyperon) superfluidity are taken into account. In a non-superfluid matter, the main scattering mechanism of electrons and muons is the Coulomb scattering on charged baryons. If the baryon component of matter is highly superfluid, ee, $e\mu$, and $\mu\mu$ collisions dominate. The results are described by simple analytic expressions valid for a wide class of models of dense matter.

1 Introduction

Thermal conductivity of matter of nuclear and hypernuclear density ($\rho \gtrsim 10^{14}$ g cm^{-3}) in neutron star cores is required to study cooling of young neutron stars when thermal relaxation of stellar interiors is not achieved. The non-relaxed stage lasts $t \sim 10^1 - 10^3$ yr (Van Riper [1]), Umeda *et al.* [2]), depending on model of dense matter.

So far the studies of thermal relaxation of neutron star interiors have made use of the thermal conductivity obtained by Flowers and Itoh [3−5]). These authors proposed an analytic fit valid for non-superfluid matter, and then discussed qualitatively the effects of superfluidity. Now it is generally recognized that matter of neutron star cores can be in a superfluid state (see, e.g., Pines [6]), Page and Applegate [7]). Moreover Flowers and Itoh [5]) calculated the thermal conductivity for a specified model of dense matter. It would be desirable to obtain the expressions valid for a wide class of models.

In this paper we reconsider the problem of thermal conductivity in neutron star cores. The main heat carriers are neutrons, electrons (Flowers and Itoh [4]), and muons if available. We will analyse the thermal conductivity of electrons and muons determined by Coulomb collisions with any charged particles present. The neutron contribution will be studied elsewhere. It should be added directly to the electron + muon conductivity. If the neutrons are strongly superfluid, their thermal conductivity is negligible, and our present results give the total conductivity of matter.

We will obtain simple expressions for the electron and muon thermal conductivity taking into account possible superfluidity of baryons. The final expressions are presented in the form valid for a wide class of models of dense matter.

2 Statement of the Problem

Consider the thermal conductivity of matter in neutron star cores, at densities ρ from about $0.5\rho_0$ to (3 - 10)ρ_0, where $\rho_0 = 2.8 \times 10^{14}$ g cm^{-3} is the standard nuclear matter density. The lower density corresponds to the core-crust transition (see, e.g., Lorenz *et al.* [8]), and the upper density is the typical central density of a neutron star.

Matter of the outer part of the neutron star core consists of neutrons (n) with a few percent admixture of protons and electrons (p, e); see, e.g., Shapiro and Teukolsky [9]). Deeper in the core, other particles may appear, including muons (μ), Σ, Λ, and other hyperons, pion condensates, free quarks, etc. We will restrict ourselves to the standard *npe* composition of matter with the possible presence of muons and hyperons. All these particles are strongly degenerate fermions. The electrons form free and ultra-relativistic gas. The muons are also free; they are non-relativistic near the threshold density, at which they appear, and they become relativistic at larger ρ. The baryons constitute a strongly coupled, non-relativistic fluid. Chemical potentials

μ_i (including rest-mass parts) of various particle species i are interrelated due to nuclear equilibrium conditions (Shapiro and Teukolsky [9])

$$\mu_n = \mu_p + \mu_e, \quad \mu_\mu = \mu_e, \quad \ldots \tag{1}$$

The thermal conductivity κ in question is a sum of electron and muon parts:

$$\begin{aligned} \kappa &= \kappa_e + \kappa_\mu, \\ \kappa_e &= \frac{\pi^2 k_B^2 T n_e \tau_e}{3m_e^*}, \; \kappa_\mu = \frac{\pi^2 k_B^2 T n_\mu \tau_\mu}{3m_\mu^*}, \end{aligned} \tag{2}$$

where T is temperature, k_B is the Stefan-Boltzmann constant, $m_e^* = \mu_e/c^2$ and $m_\mu^* = \mu_\mu/c^2$ are the effective masses of electrons and muons, respectively; n_e and n_μ are the electron and muon number densities, while τ_e and τ_μ are the effective electron and muon relaxation times to be determined below.

3 General Formalism

3.1 Kinetic equation

The effective (thermal-conductivity) relaxation times τ_c of the heat carriers c (electrons, $c = e$, and muons, $c = \mu$) due to Coulomb collisions with all charged particles i (leptons, $i = c$, and baryons, $i = b$) can be determined from the system of Boltzmann kinetic equations for the distribution functions F_c (see, e.g., Ziman [10]). In the presence of a weak stationary temperature gradient, the equations read

$$\boldsymbol{v}_c \, \frac{\partial F_c}{\partial \boldsymbol{r}} = \sum_i I_{ci}, \tag{3}$$

where $\boldsymbol{v}_c$ is the velocity of the heat carriers. The collision integral

$$\begin{aligned} I_{12} &= (2\pi\hbar)^{-9} \sum_{\sigma_1' \sigma_2 \sigma_2'} \int \mathrm{d}\boldsymbol{p}_2 \, \mathrm{d}\boldsymbol{p}_1' \, \mathrm{d}\boldsymbol{p}_2' \, w(12 \mid 1'2') \\ &\times \; [F_1' F_2' (1 - F_1)(1 - F_2) - F_1 F_2 (1 - F_1')(1 - F_2')] \end{aligned} \tag{4}$$

takes into account direct collisions $(1, 2 \to 1', 2') = (\boldsymbol{p}_1 \sigma_1, \, \boldsymbol{p}_2 \sigma_2 \to \boldsymbol{p}_1' \sigma_1', \boldsymbol{p}_2' \sigma_2')$, as well as inverse collisions $(1', 2' \to 1, 2)$. The non-primed quantities refer to particles before a collision, while the primed ones to particles after the collision; $\boldsymbol{p}$ is the particle momentum and σ denotes spin state; $w(12 \mid 1'2') = w(1'2' \mid 12)$ is the differential transition rate. We adopt the quasiparticle approach (e.g., Botermans and Malfliet [11]) to describe the baryons. The many-body effects are included into the dispersion relation (momentum dependence of particle energy, $\varepsilon_b(\boldsymbol{p})$). In this section we will treat the baryons as non-superfluid. The effects of superfluidity will be studied in sect. 5.

The distribution function F_i can be written as (e.g., Ziman [10])

$$F_i = f_i - \Phi_i \frac{\partial f_i}{\partial \varepsilon_i} = f_i + \frac{1}{k_B T} \Phi_i f_i (1 - f_i), \tag{5}$$

where

$$f_i = \left[1 + \exp\left(\frac{\varepsilon_i - \mu_i}{k_B T} \right) \right]^{-1} \tag{6}$$

is the Fermi-Dirac equilibrium distribution, and Φ_i is an unknown function which determines a small non-equilibrium correction induced by the temperature gradient; ε_i denotes the particle energy. Since the baryons ($i = b$) do not transport a noticeable fraction of the heat flux, we can set $F_b = f_b$, $\Phi_b = 0$. Using the standard linearization procedure (Ziman [10]) of eq. (3), we obtain the equations for $\Phi_c = \Phi_c(\boldsymbol{p}_c)$:

$$\begin{aligned} \frac{\partial f_c}{\partial \varepsilon_c} (\varepsilon_c - \mu_c)\, k_B\, \boldsymbol{v}_c \nabla T = \sum_i \sum_{\sigma_c' \sigma_i \sigma_i'} (2\pi\hbar)^{-9} \int \mathrm{d}\boldsymbol{p}_i\, \mathrm{d}\boldsymbol{p}_c'\, \mathrm{d}\boldsymbol{p}_i'\, w(ci \mid c'i') \\ \times f_c f_i (1 - f_c')(1 - f_i')(\Phi_c' + \Phi_i' - \Phi_c - \Phi_i). \end{aligned} \tag{7}$$

It is well known (e.g., Ziman [10]) that the functions Φ_c have the form

$$\Phi_c(\boldsymbol{p}_c) = -\tau_c(\varepsilon_c)\, (\varepsilon_c - \mu_c)\, \boldsymbol{v}_c \frac{\nabla T}{T}, \tag{8}$$

where $\tau_c(\varepsilon)$ is a function of ε to be found.

We will adopt the simplest approximation that τ_c is independent of ε_c. Then τ_c has meaning of the effective relaxation time in question. This can easily be proved by calculating the heat flux density

$$\boldsymbol{q} = \sum_c 2(2\pi\hbar)^{-3} \int \mathrm{d}\boldsymbol{p}_c\, \boldsymbol{v}_c\, (\varepsilon_c - \mu_c) F_c \equiv -(\kappa_e + \kappa_\mu) \nabla T, \tag{9}$$

which immediately yields eq. (2). The approximation adopted is sufficiently accurate since only a narrow energy range near the Fermi level ($| \varepsilon_c - \mu_c | \lesssim k_B T$) is responsible for heat conduction of strongly degenerate particles.

Our approximation is equivalent to the variational approach with the simplest trial function widely used in the kinetic theory of terrestrial metals (e.g., Ziman [10]). It is also equivalent to keeping the main term in the expansion of $\tau(\varepsilon)$ over appropriate polynomials (Lampe [12]). Note that the thermal conductivity can be calculated exactly using the formalism developed by Sykes and Brooker [13]), Flowers and Itoh [4]), and Anderson *et al.* [14]). In sect. 8 we will discuss the corrections to our results which follow from the exact theory.

3.2 Relaxation times

Let us substitute (8) into (7), multiply (7) by $(\varepsilon_c - \mu_c)\boldsymbol{v}_c/(2\pi\hbar)^3$, integrate over $\mathrm{d}\boldsymbol{p}_c$, and sum over spin states σ_c. Then we get the system of two equations

$$\begin{aligned} 1 &= \nu_e \tau_e + \nu'_{e\mu}\tau_\mu, \\ 1 &= \nu_\mu \tau_\mu + \nu'_{\mu e}\tau_e. \end{aligned} \tag{10}$$

In this case

$$\nu_e = \sum_i \nu_{ei}, \quad \nu_\mu = \sum_i \nu_{\mu i} \tag{11}$$

are the total effective electron and muon collision frequencies, which are determined by the partial collision frequencies

$$\nu_{12} = \frac{3\hbar^3}{v_{F1} p_{F1}^2 (k_B T)^3}\, S_{12}, \tag{12}$$

where p_{Fi} is the Fermi momentum of particle species i, $v_{Fi} = \partial\varepsilon_i/\partial p_i$ (at $p_i = p_{Fi}$) is the Fermi velocity, and

$$\begin{aligned} S_{12} &= (2\pi\hbar)^{-12} \int \mathrm{d}\boldsymbol{p}_1\, \mathrm{d}\boldsymbol{p}_2\, \mathrm{d}\boldsymbol{p}'_1\, \mathrm{d}\boldsymbol{p}'_2\, W(12 \mid 1'2') f_1 f_2 (1 - f'_1)(1 - f'_2) \\ &\times\; (\varepsilon_1 - \mu_1)\, [v_1^2(\varepsilon_1 - \mu_1) - \boldsymbol{v}_1 \boldsymbol{v}'_1(\varepsilon'_1 - \mu_1)]. \end{aligned} \tag{13}$$

In addition, eqs. (10) contain the 'crossed' effective $e\mu$ and μe collision frequencies which couple the electron and muon heat transport

$$\nu'_{12} = \frac{3\hbar^3}{v_{F1} p_{F1}^2 (k_B T)^3}\, S'_{12}, \tag{14}$$

$$\begin{aligned} S'_{12} &= (2\pi\hbar)^{-12} \int \mathrm{d}\boldsymbol{p}_1\, \mathrm{d}\boldsymbol{p}_2\, \mathrm{d}\boldsymbol{p}'_1\, \mathrm{d}\boldsymbol{p}'_2\, W(12 \mid 1'2') f_1 f_2 (1 - f'_1)(1 - f'_2) \\ &\times\; (\varepsilon_1 - \mu_1)\, [\boldsymbol{v}_1 \boldsymbol{v}_2(\varepsilon_2 - \mu_2) - \boldsymbol{v}_1 \boldsymbol{v}'_2(\varepsilon'_2 - \mu_2)]. \end{aligned} \tag{15}$$

The quantities S_{12} and S'_{12} characterize the corresponding energy exchange rates (erg^2 cm^{-1} s^{-3}), and

$$W(12 \mid 1'2') = \sum_{\sigma_1\sigma_2\sigma'_1\sigma'_2} w(12 \mid 1'2') \tag{16}$$

is the spin trace of the differential transition probability.

Eqs. (10) yield

$$\tau_e = \frac{\nu_\mu - \nu'_{e\mu}}{\nu_e\nu_\mu - \nu'_{e\mu}\nu'_{\mu e}}, \quad \tau_\mu = \frac{\nu_e - \nu'_{\mu e}}{\nu_e\nu_\mu - \nu'_{e\mu}\nu'_{\mu e}}. \tag{17}$$

Now the thermal conductivity (2) is expressed in terms of the effective collision frequencies (12) and (14) to be calculated in subsequent sections.

3.3 Screening

Charged particles interact via Coulomb forces screened by plasma polarization. Generally, the screening is determined by all charged plasma particles. Under typical conditions in the neutron star cores, the screening is static. This is so since $\hbar\omega \ll v_{Fi} q$, where $\hbar\omega \sim k_B T$ and $q \sim q_0$ determine, respectively, the typical energy and momentum transfers in the collisions of study. The squared static screening momentum reads

$$q_0^2 = 4\pi\hbar^2 \sum_i e_i^2 \frac{\partial n_i}{\partial \mu_i}, \tag{18}$$

where n_i is the number density of particle species i, and sum is over all charged particles. If all particles are in non-superfluid states and possess one elementary charge $e_i^2 = e^2$, we obtain

$$q_0^2 = \frac{4e^2}{\pi\hbar} \sum_i m_i^* p_{Fi}. \tag{19}$$

The effective masses of baryons m_b^* can differ from the bare masses due to many-body effects (e.g., Botermans and Malfliet [11]).

3.4 Transition probability

The differential transition probability (16) for Coulomb collisions of charged particles of different species 1 and 2 in the Born approximation is (Akhiezer and Berestetskii [15])

$$W(12 \mid 1'2') = 4(2\pi\hbar)^6 \, \delta(\varepsilon_1' + \varepsilon_2' - \varepsilon_1 - \varepsilon_2) \, \delta(\boldsymbol{p}_1' + \boldsymbol{p}_2' - \boldsymbol{p}_1 - \boldsymbol{p}_2) \, \mathcal{M}, \tag{20}$$

where

$$\mathcal{M} = \frac{e^4 c^4}{\varepsilon_1 \varepsilon_2 \varepsilon_1' \varepsilon_2'} \frac{1}{t^2} \left[\frac{s^2 + u^2}{2} + (m_1^2 c^2 + m_2^2 c^2)(2t - m_1^2 c^2 - m_2^2 c^2) \right], \tag{21}$$

and s, u, and t are the kinematic invariants:

$$\begin{aligned} s &= (\mathcal{P}_1 + \mathcal{P}_2)^2 = m_1^2 c^2 + m_2^2 c^2 + 2\mathcal{P}_1 \mathcal{P}_2, \\ u &= (\mathcal{P}_1 - \mathcal{P}_2')^2 = m_1^2 c^2 + m_2^2 c^2 - 2\mathcal{P}_1 \mathcal{P}_2', \\ t &= (\mathcal{P}_1 - \mathcal{P}_1')^2 - q_0^2 = 2(m_1^2 c^2 - \mathcal{P}_1 \mathcal{P}_1') - q_0^2. \end{aligned} \tag{22}$$

In this case $\mathcal{P}_i$ is the 4-momentum of a particle i. We have added the squared screening momentum q_0^2, eq. (18), to the standard kinematic invariant t to take into account the plasma screening effect.

Since the colliding particles are strongly degenerate, we can set $\varepsilon_1' = \varepsilon_1 = \mu_1$, $\varepsilon_2' = \varepsilon_2 = \mu_2$ in slowly varying functions of energies. Then we can rewrite (21) as

$$\mathcal{M} = \frac{2e^4}{(q^2+q_0^2)^2} \left[\left(1 - \frac{\boldsymbol{p}_1 \boldsymbol{p}_2 c^2}{\mu_1 \mu_2}\right)^2 + \left(1 - \frac{\boldsymbol{p}_1 \boldsymbol{p}_2' c^2}{\mu_1 \mu_2}\right)^2 - \frac{(m_1^2 + m_2^2) c^6 q^2}{2\mu_1^2 \mu_2^2} \right], \tag{23}$$

where $q^2 = (\boldsymbol{p}_1' - \boldsymbol{p}_1)^2 = (\boldsymbol{p}_2 - \boldsymbol{p}_2')^2$ is the squared momentum transfer in a collision event. We have neglected a small term of the order of $q_0^2 c^2/\mu_1^2 \ll 1$ in the expression in the square brackets.

Note that $\mathcal{M}$ should be modified for collisions of identical particles (see sect. 7).

4 Electron and Muon Collisions with Charged Baryons

Consider collisions of electrons or muons (particles 1) with charged baryons (particles 2). The differential transition probability is given by (20) and (23). Since the baryons are heavy and non-relativistic, one has

$$\mathcal{M} = \frac{4e^4}{(q^2+q_0^2)^2} \left(1 - \frac{q^2 c^2}{4\mu_1^2}\right). \tag{24}$$

All particles are strongly degenerate, and we can place $\boldsymbol{p}_i$ at the appropriate Fermi surface, $p_i = p_{Fi}$, in all smooth functions of particle momenta. Thus $\mathrm{d}\boldsymbol{p}_i = m_i^* p_{Fi}\, \mathrm{d}\varepsilon_i\, \mathrm{d}\Omega_i$, where $\mathrm{d}\Omega_i$ is a solid angle element in direction of $\boldsymbol{p}_i$. Then eq. (13) can be rewritten as

$$\begin{aligned} S_{12} &= \frac{4 p_{F1}^2 p_{F2}^2 (m_2^*)^2}{(2\pi\hbar)^6} \int_{-\infty}^{+\infty} \mathrm{d}\varepsilon_1 \int_{-\infty}^{+\infty} \mathrm{d}\varepsilon_1' \int_{-\infty}^{+\infty} \mathrm{d}\varepsilon_2 \int_{-\infty}^{+\infty} \mathrm{d}\varepsilon_2'\, (\varepsilon_1 - \mu_1) \\ &\times\ f_1 f_2 (1 - f_1')(1 - f_2')\, \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_1' - \varepsilon_2')\, I_\Omega, \end{aligned} \tag{25}$$

where I_Ω is the angular integral

$$\begin{aligned} I_\Omega = &\int \mathrm{d}\Omega_1 \int \mathrm{d}\Omega_1' \int \mathrm{d}\Omega_2 \int \mathrm{d}\Omega_2'\ [p_1^2(\varepsilon_1 - \mu_1) - \boldsymbol{p}_1 \boldsymbol{p}_1' (\varepsilon_1' - \mu_1)] \\ &\times \mathcal{M}\ \delta(\boldsymbol{p}_1 + \boldsymbol{p}_2 - \boldsymbol{p}_1' - \boldsymbol{p}_2'). \end{aligned} \tag{26}$$

It is clear from eqs. (20) and (26) that the integrand of I_Ω depends on relative orientations of the momenta involved, $\boldsymbol{p}_1$, $\boldsymbol{p}_2$, $\boldsymbol{p}_1'$, and $\boldsymbol{p}_2'$. We can immediately integrate over $\mathrm{d}\Omega_1$ and over the azimuthal angle ϕ which specifies azimuthal position of $\boldsymbol{p}_2$ with respect to $\boldsymbol{p}_1$. Such an integration reduces to replacing $\mathrm{d}\Omega_1\, \mathrm{d}\phi \to 8\pi^2$. For further integration, we may direct $\boldsymbol{p}_1$ along the z-axis and place $\boldsymbol{p}_2$ in the (x, z)-plane.

Let (ϑ_1, φ_1) be polar angles of $\boldsymbol{p}_1'$, (ϑ_2, φ_2) be polar angles of $\boldsymbol{p}_2'$, and θ be an angle between $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$. The integration over ϑ_2, φ_1, and φ_2 is removed by the momentum conserving delta-function (see, e.g., Yakovlev and Shalybkov [16]). Then

$$I_\Omega = \frac{8\pi^2}{p_1 p_1' p_2 p_2'} \int_0^\pi \sin\theta \, \mathrm{d}\theta \int_0^{q_{max}} \mathrm{d}q \, \mathcal{M} \, \frac{2p_1^2(\varepsilon_1 - \mu_1) - (p_1^2 + {p_1'}^2 - q^2)(\varepsilon_1' - \mu_1)}{\sqrt{a - (\cos\theta + b)^2}}, \quad (27)$$

where

$$\begin{aligned} a &= \frac{[(p_1 + p_1')^2 - q^2]\,[(p_1 - p_1')^2 - q^2]\,[(p_2 + p_2')^2 - q^2]\,[(p_2 - p_2')^2 - q^2]}{16\, p_1^2\, p_2^2\, q^4}, \\ b &= \frac{\left[p_1^2 - {p_1'}^2 + q^2\right]\,\left[p_2^2 - {p_2'}^2 + q^2\right]}{4\, p_1\, p_2\, q^2}, \end{aligned} \quad (28)$$

and q_{max} is the maximum momentum transfer which corresponds to $a = 0$. It is equal to the minimum of $2p_{F1}$ and $2p_{F2}$. In these equations we keep the differences between p_1 and p_1', and between p_2 and p_2'. We will check that these differences are negligible in the final expressions, so that one can set $p_1' = p_1 = p_{F1}$ and $p_2' = p_2 = p_{F2}$ without any loss of accuracy.

The integration over θ can be done explicitly since $\mathcal{M}$ depends only on q. We obtain

$$I_\Omega = \frac{8\pi^3}{p_1 p_1' p_2 p_2'} \int_0^{q_{max}} \mathrm{d}q \, \mathcal{M} \left[2p_1^2(\varepsilon_1 - \mu_1) - (p_1^2 + {p_1'}^2 - q^2)(\varepsilon_1' - \mu_1)\right]. \quad (29)$$

Now we substitute eq. (24) into (29). The integration over q is easy and yields

$$I_\Omega = \frac{16\pi^4 e^4}{p_{F2}^2 q_0^3} \left[\alpha(\varepsilon_1 - \varepsilon_1') + \beta(\varepsilon_1' - \mu_1)\right], \quad (30)$$

where

$$\begin{aligned} \alpha &= \frac{2}{\pi} \left[\arctan\gamma + \frac{\gamma}{1+\gamma^2} - \frac{q_0^2 c^2}{4\mu_1^2} \left(\arctan\gamma - \frac{\gamma}{1+\gamma^2}\right)\right], \\ \beta &= \frac{q_0^2}{\pi p_{F1}^2} \left[\arctan\gamma - \frac{\gamma}{1+\gamma^2} - \frac{q_0^2 c^2}{4\mu_1^2} \left(2\gamma + \frac{\gamma}{1+\gamma^2} - 3\arctan\gamma\right)\right], \end{aligned} \quad (31)$$

and $\gamma = q_{max}/q_0$. Substituting (30) into (25), we obtain

$$S_{12} = \frac{p_{F1}^2 (m_2^*)^2 e^4}{\pi^2 \hbar^6 q_0^3} \, I_\varepsilon, \quad (32)$$

where

$$
\begin{aligned}
I_\varepsilon &= \int_{-\infty}^{+\infty} \mathrm{d}\varepsilon_1 \int_{-\infty}^{+\infty} \mathrm{d}\varepsilon_1' \int_{-\infty}^{+\infty} \mathrm{d}\varepsilon_2 \int_{-\infty}^{+\infty} \mathrm{d}\varepsilon_2' \, f_1 f_2 (1-f_1')(1-f_2') \\
&\times \ (\varepsilon_1 - \mu_1)\left[\alpha(\varepsilon_1 - \varepsilon_1') + \beta(\varepsilon_1' - \mu_1)\right] \ \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_1' - \varepsilon_2'). \qquad (33)
\end{aligned}
$$

The integration reduces to standard integrals with the result

$$
I_\varepsilon = \frac{4\pi^4}{15}(k_B T)^5 \left(\alpha + \frac{\beta}{2}\right). \qquad (34)
$$

Using eqs. (12) and (32), we come to the expression for the effective collision frequency of electrons or muons c with charged baryons b:

$$
\nu_{cb} = \frac{4\pi^2}{5}\left(\alpha + \frac{\beta}{2}\right) \frac{e^4 (m_b^*)^2 (k_B T)^2}{\hbar^3 q_0^3 v_{Fc}}. \qquad (35)
$$

According to eq. (19) the screening momentum is usually much smaller than p_{Fi}. This is violated only for newly created particles i in very narrow density ranges above the threshold density.

Outside of these ranges, almost everywhere in the neutron star core, the screening is week and one can use the weak screening approximation, $\gamma \gg 1$. In the latter approximation,

$$
\alpha \approx 1, \quad \beta \approx \frac{q_0^2}{2p_{Fc}^2} \ll \alpha, \qquad (36)
$$

and β is negligible in eq. (35). Then

$$
\nu_{cb} = \frac{4\pi^2}{5\hbar}\left(\frac{e^2}{\hbar c}\right)^2 \left(\frac{p_{Fc}}{q_0}\right)^3 \frac{\mu_c (m_b^*)^2 (k_B T)^2}{p_{Fc}^4}. \qquad (37)
$$

In sect. 10 we will calculate the density dependence of the thermal conductivity taking into account the creation of Σ^- hyperons. The conductivity obtained in the weak screening approximation is almost indistinguishable from that given by more accurate eq. (35). Thus we conclude that the weak screening result (37) can be used everywhere in the neutron star core.

Note that the collision frequencies (37) are proportional to T^2. Therefore the thermal conductivity depends on temperature as $\kappa \propto T^{-1}$. This is the general feature of highly degenerate non-superfluid particles; see, e.g., Ziman [10]).

5 Effects of Superfluidity of Charged Baryons

Consider the case when charged baryons of species b (e.g., the protons) are in a superfluid state. When T is smaller than the critical temperature T_c, the equations of sects. 3 and 4 should be modified. It is generally thought that the baryon superfluidity in the neutron star cores is of the BCS type. Since we are not interested in the superfluidity of neutrons (which suffer no Coulomb collisions) we can restrict ourselves to the simplest case of baryon pairing in the $^1S_0$ state (see, e.g., Wambach *et al.* [17]). The superfluidity creates an energy gap $\Delta(T)$ in the dispersion relation $\varepsilon(\boldsymbol{p})$ for the baryons near their Fermi surface (Lifshitz and Pitaevskii [18]), $\mid p - p_F \mid \ll p_F$,

$$\begin{aligned} \varepsilon &= \mu + \sqrt{\Delta^2 + v_F^2 (p - p_F)^2} \quad \text{for } p > p_F; \\ \varepsilon &= \mu - \sqrt{\Delta^2 + v_F^2 (p - p_F)^2} \quad \text{for } p < p_F. \end{aligned} \tag{38}$$

The dependence of Δ on T is given by the BCS theory and can be fitted as (Østgaard and Yakovlev [19], Levenfish and Yakovlev [20])

$$y \equiv \frac{\Delta(T)}{k_B T} = \sqrt{1 - \frac{T}{T_c}} \left(1.456 - 0.157 \sqrt{\frac{T_c}{T}} + 1.764 \frac{T_c}{T} \right). \tag{39}$$

The fit reproduces the asymptotes of $\Delta(T)$ at $T \to 0$ and $T \to T_c$ as well as numerical values at intermediate T. The mean fitting accuracy is $< 1\%$.

The effects of superfluidity are twofold: the superfluidity reduces the collision frequency directly, and influences the screening momentum. Note that both effects violate simple temperature dependence of the collision frequency ($\nu_{12} \propto T^2$) and the thermal conductivity ($\kappa \propto T^{-1}$) typical for the non-superfluid case (sect. 4).

The direct reduction of the collision frequency can be analysed from eq. (13). If $T < T_c$, the integration over momentum space of colliding particles should be performed with the modified baryon energies, eq. (38), in the particle distributions f_2 and f_2' and in the energy conserving delta-function, which enters the transition rate, eq. (20). Let us consider the most important week-screening case (36). A careful examination of the derivation of the energy transfer rate S_{12} (sect. 4) reveals that the main effect of the superfluidity on S_{12} is described by the factor

$$\begin{aligned} I_\varepsilon &= \int_{-\infty}^{+\infty} \mathrm{d}\varepsilon_1 \int_{-\infty}^{+\infty} \mathrm{d}\varepsilon_1' \int_{-\infty}^{+\infty} \mathrm{d}\eta_2 \int_{-\infty}^{+\infty} \mathrm{d}\eta_2' \, (\varepsilon_1 - \varepsilon_1')(\varepsilon_1 - \mu_1) \\ &\times \; f_1 f_2 (1 - f_1')(1 - f_2') \, \delta(\varepsilon_1' + \varepsilon_2' - \varepsilon_1 - \varepsilon_2), \end{aligned} \tag{40}$$

where $\eta_2 = v_{F2}(p_2 - p_{F2})$ and $\eta_2' = v_{F2}(p_2' - p_{F2})$. In the non-superfluid state (according to sect. 4), $I_{\varepsilon 0} = (4\pi^4/15)(k_B T)^5$. Thus, the effective collision frequency is

$$\nu_{lb} = \nu_{lb}^{(0)} \, \mathcal{R}, \tag{41}$$

where $\nu_{lb}^{(0)}$ is the non-superfluid collision frequency given by eq. (37), while the factor

$$\begin{aligned}\mathcal{R} &= \frac{I_\varepsilon}{I_{\varepsilon 0}} = \frac{15}{4\pi^4} \int_{-\infty}^{+\infty} \mathrm{d}x_1 \int_{-\infty}^{+\infty} \mathrm{d}x_1' \int_{-\infty}^{+\infty} \mathrm{d}x_2 \int_{-\infty}^{+\infty} \mathrm{d}x_2'\, f(x_1) f(z_2) \\ &\times \; (1-f(x_1'))(1-f(z_2'))(x_1-x_1')x_1 \delta(x_1'+z_2'-x_1-z_2) \end{aligned} \tag{42}$$

describes the superfluid reduction of the collision rate. Here we have introduced the notations $x_1 = (\varepsilon_1 - \mu_1)/(k_B T)$, $x_1' = (\varepsilon_1' - \mu_1)/(k_B T)$, $x_2 = \eta_2/(k_B T)$, $x_2' = \eta_2'/(k_B T)$, $z_2 = (\varepsilon_2 - \mu_2)/(k_B T) = \mathrm{sign}(x_2)\sqrt{x_2^2 + y^2}$, $z_2' = (\varepsilon_2' - \mu_2)/(k_B T) = \mathrm{sign}(x_2')\sqrt{{x_2'}^2 + y^2}$; $f(x) = (e^x + 1)^{-1}$, and y is given by (39). The integration over x_1' is taken with the aid of the δ-function. The integral over x_1 reduces to a standard integral of the kinetic theory of Fermi systems (Ziman [10]). Then

$$\mathcal{R} = \frac{15}{4\pi^4} \int_0^{\infty} \mathrm{d}x_2 \int_0^{\infty} \mathrm{d}x_2'\, f(z_2) \left[\frac{f(z_2')(z_2+z_2')^3}{1-\exp(-z_2-z_2')} + \frac{f(-z_2')(z_2-z_2')^3}{1-\exp(-z_2+z_2')} \right]. \tag{43}$$

When the superfluidity disappears ($T \to T_c$, $y \to 0$), we obtain $\mathcal{R} \to 1$. For a strong superfluidity ($T \to 0$, $y \to \infty$), we have $\mathcal{R} \to Ay\exp(-y)$, where

$$A = \frac{15}{2\pi^4} \int_0^{\infty} \mathrm{d}u_1 \int_0^{\infty} \mathrm{d}u_2\, \frac{(u_1^2 - u_2^2)^3}{\exp(u_1^2) - \exp(u_2^2)} \approx 0.2362. \tag{44}$$

We have calculated $\mathcal{R}$ numerically for intermediate y. The results can be fitted by the expression

$$\begin{aligned}\mathcal{R} &= \Big\{ 0.7694 + \sqrt{(0.2306)^2 + (0.07207y)^2} \\ &+ \; (27.00y^2 + 0.1476y^4)\exp\left(-\sqrt{(4.273)^2 + y^2}\right) \\ &+ \; 0.5051 \left[\exp\left(4.273 - \sqrt{(4.273)^2 + y^2}\right) - 1 \right] \Big\} \\ &\times \; \exp\left(1.187 - \sqrt{(1.187)^2 + y^2}\right). \end{aligned} \tag{45}$$

Our fit satisfies also the condition $\mathcal{R} = 1$ for $y = 0$ and reproduces the above asymptote for $y \to \infty$. The mean fitting error is about 0.3 %. Eqs. (39) and (45) enable one to determine the reduction of the collision frequency for any $T < T_c$. When $T > T_c$, one naturally has $\mathcal{R} = 1$.

The superfluidity affects also the screening momentum q_0. Calculating the baryon number densities in eq. (18) for q_0, one should introduce the dispersion relation (38) into the distribution function. In this way we have

$$\frac{\partial n}{\partial \mu} = \left(\frac{\partial n}{\partial \mu}\right)_0 Z, \quad Z = v_F \int_{-\infty}^{+\infty} \mathrm{d}p \left(-\frac{\partial f}{\partial \varepsilon}\right) = 2\int_0^{\infty} \mathrm{d}x\, \frac{\exp(z)}{(\exp(z)+1)^2}, \tag{46}$$

where $z = \sqrt{x^2+y^2}$, and the subscript '0' denotes the quantity calculated in the absence of the superfluidity. Accordingly Z describes the superfluid reduction of the screening momentum for $T < T_c$. With this reduction, eq. (19) is modified:

$$q_0^2 = \frac{4e^2}{\pi\hbar}\left(m_e^* p_{Fe} + m_\mu^* p_{F\mu} + \sum_b m_b^* p_{Fb} Z_b\right), \tag{47}$$

where Z_b is the reduction factor (46) for baryon species b.

Eq. (46) yields $Z = 1$ for $y = 0$, and $Z = \sqrt{2\pi y}\exp(-y)$ for $y \to \infty$. We have calculated Z at intermediate y and proposed the fit to reproduce the asymptotes and numerical values:

$$Z = \left[0.9443 + \sqrt{(0.0557)^2 + (0.1886y)^2}\right]^{1/2} \exp\left(1.753 - \sqrt{(1.753)^2 + y^2}\right). \tag{48}$$

The mean fitting error is about 1.3 %. Eqs. (39), (47), and (48) give the screening momentum q_0 for any $T < T_c$. If $T \geq T_c$, one should formally set $Z = 1$, and then eq. (19) is restored.

Consider, for instance, the most important case of *npe* matter. The *ep* collision frequency is greatly affected by the proton superfluidity ($T_c = T_{cp}$). The superfluidity reduces the contribution of the protons into the screening. If $T \ll T_{cp}$, the screening momentum is completely determined by the electrons. This enhances the collision frequency and reduces the conductivity. However the direct effect of the superfluidity onto the collision frequency (described by the factor $\mathcal{R}$) is much stronger: it reduces ν_{ep} exponentially. The net effect is naturally to suppress the contribution of *ep* collisions into the thermal conductivity at $T \ll T_{cp}$.

6 Electron-Muon and Muon-Electron Collisions

Let us examine the most important weak-screening case (36). In this case typical momentum transfers and scattering angles are small ($\vartheta_1 \ll 1$), and one can set $\vartheta_2 \approx \theta$ in eq. (23) due to momentum conservation. Placing again the momenta of the colliding particles on the appropriate Fermi surfaces in smooth functions, we get

$$\mathcal{M} = \frac{4e^4}{(q^2+q_0^2)^2}\left(1 - \frac{p_{F1}p_{F2}c^2}{\mu_1\mu_2}\cos\theta\right)^2. \tag{49}$$

First consider $\nu_{e\mu}$ and $\nu_{\mu e}$. Calculations are similar to those in sect. 4: $S_{e\mu}$ and $S_{\mu e}$ are given by eq. (13). However, now $\mathcal{M}$ depends not only on q but on θ as well. This modifies slightly the integration in (27). In the weak screening approximation

$$I_\Omega = \frac{16\pi^4 e^4(\varepsilon_1 - \varepsilon_1')}{p_{F2}^2 q_0^3}\left[1 + \frac{1}{2}\left(\frac{p_{F1}p_{F2}c^2}{\mu_1\mu_2}\right)^2\right]. \tag{50}$$

Then

$$S_{12}=\frac{4\pi^2e^4p_{F1}^2\mu_2^2(k_BT)^5}{15\hbar^6c^4q_0^3}\left[1+\frac{1}{2}\left(\frac{p_{F1}p_{F2}c^2}{\mu_1\mu_2}\right)^2\right]. \tag{51}$$

Finally, the collision frequencies are

$$\nu_{e\mu} = \frac{4\pi^2}{5\hbar}\left(\frac{e^2}{\hbar c}\right)^2\left(\frac{p_{Fe}}{q_0}\right)^3\frac{\mu_e\mu_\mu^2(k_BT)^2}{p_{Fe}^4c^4}\left[1+\frac{1}{2}\left(\frac{p_{Fe}p_{F\mu}c^2}{\mu_e\mu_\mu}\right)^2\right], \tag{52}$$

$$\nu_{\mu e} = \frac{4\pi^2}{5\hbar}\left(\frac{e^2}{\hbar c}\right)^2\left(\frac{p_{F\mu}}{q_0}\right)^3\frac{\mu_e^2\mu_\mu(k_BT)^2}{p_{F\mu}^4c^4}\left[1+\frac{1}{2}\left(\frac{p_{Fe}p_{F\mu}c^2}{\mu_e\mu_\mu}\right)^2\right]. \tag{53}$$

Note that $\nu_{e\mu}$ is analogous to ν_{eb} (eq. (37)) when the muons are non-relativistic.

The calculation of the cross-frequencies $\nu'_{e\mu}$ and $\nu'_{\mu e}$ is similar. From (15) we have

$$\begin{aligned} S'_{12} &= \frac{4p_{F1}^2p_{F2}^2\mu_1\mu_2}{(2\pi\hbar)^6c^4}\int_{-\infty}^{+\infty}\mathrm{d}\varepsilon_1\int_{-\infty}^{+\infty}\mathrm{d}\varepsilon'_1\int_{-\infty}^{+\infty}\mathrm{d}\varepsilon_2\int_{-\infty}^{+\infty}\mathrm{d}\varepsilon'_2\; f_1f_2(1-f'_1)(1-f'_2) \\ &\times\ (\varepsilon_1-\mu_1)\,\delta(\varepsilon_1+\varepsilon_2-\varepsilon'_1-\varepsilon'_2)\,I'_\Omega. \end{aligned} \tag{54}$$

The weak screening approximation yields

$$\begin{aligned} I'_\Omega &= \int\mathrm{d}\Omega_1\int\mathrm{d}\Omega'_1\int\mathrm{d}\Omega_2\int\mathrm{d}\Omega'_2\;[p_1p_2(\varepsilon_2-\mu_2)\cos\theta-p_1p'_2(\varepsilon'_2-\mu_2)\cos\vartheta_2] \\ &\times\ \mathcal{M}\,\delta(\boldsymbol{p}_1+\boldsymbol{p}_2-\boldsymbol{p}'_1-\boldsymbol{p}'_2)\ =\ \frac{16\pi^4e^4c^2(\varepsilon'_2-\varepsilon_2)}{\varepsilon_1\varepsilon_2q_0^3}, \end{aligned} \tag{55}$$

and

$$S'_{12}=\frac{4\pi^2e^4p_{F1}^2p_{F2}^2(k_BT)^5}{15\hbar^6c^2q_0^3}. \tag{56}$$

Finally, the cross frequencies are

$$\nu'_{e\mu} = \frac{4\pi^2}{5\hbar}\left(\frac{e^2}{\hbar c}\right)^2\left(\frac{p_{F\mu}}{q_0}\right)^3\frac{\mu_e(k_BT)^2}{p_{Fe}p_{F\mu}c^2}, \tag{57}$$

$$\nu'_{\mu e} = \frac{4\pi^2}{5\hbar}\left(\frac{e^2}{\hbar c}\right)^2\left(\frac{p_{Fe}}{q_0}\right)^3\frac{\mu_\mu(k_BT)^2}{p_{Fe}p_{F\mu}c^2}. \tag{58}$$

7 Electron-Electron and Muon-Muon Collisions

The *ee* collision frequency ν_{ee} was calculated earlier by Flowers and Itoh [3]), and Yakovlev and Urpin [21]). The formalism is similar to that used in sects. 3, 4, and 6 but the collision integral is slightly modified for collisions of identical particles.

Taking into account that the electrons in the neutron star cores are ultra-relativistic, and the screening is static, from these results one has

$$\nu_{ee} = \frac{2\pi^2}{\hbar}\left(\frac{e^2}{\hbar c}\right)^2\left(\frac{p_{Fe}}{q_0}\right)^3\frac{(k_BT)^2}{\mu_e}. \tag{59}$$

The collision frequency $\nu_{\mu\mu}$ is readily obtained from the expression for ν_{ee} derived by Flowers and Itoh [3]) in the static screening approximation for degenerate electron gas of any degree of relativism. Replacing the parameters of electrons to those of muons, we have

$$\nu_{\mu\mu} = \frac{2\pi^2}{\hbar}\left(\frac{e^2}{\hbar c}\right)^2\left(\frac{p_{F\mu}}{q_0}\right)^3\frac{(k_BT)^2}{\mu_\mu}\left[1+\frac{6}{5}\left(\frac{m_\mu c}{p_{F\mu}}\right)^2+\frac{2}{5}\left(\frac{m_\mu c}{p_{F\mu}}\right)^4\right]. \tag{60}$$

This frequency is similar to ν_{ee} in the limit of relativistic muons.

8 Comparison with Exact Solution

As mentioned in sect. 3, we have used a simplified approach which corresponds to τ_c=const in eq. (8). However it is possible to construct an exact thermal conductivity, using the advanced theory developed by Sykes and Brooker [13]) for one-component Fermi-systems, and extended by Flowers and Itoh [4]) and Anderson *et al.* [14]) to multi-component systems.

The exact approach is complicated, and we shall restrict ourselves to the case when the heat is carried either by electrons or by muons colliding with all charged fermions. Then the exact thermal conductivity κ_{exact} is related to the thermal conductivity κ_{var}, calculated in our approximation, through the correction factor C:

$$\kappa_{exact} = \kappa_{var}\, C(\lambda), \quad C(\lambda) = \frac{12}{5}H(\lambda), \tag{61}$$

where

$$H(\lambda) = \frac{3-\lambda}{4}\sum_{n=0}^{\infty}\frac{4n+5}{(n+1)(2n+3)[(n+1)(2n+3)-\lambda]} \tag{62}$$

is the function introduced by Sykes and Brooker [13]). The parameter λ can be expressed in terms of integrals containing transition probabilities for various collisions and functions of angles between particle momenta before and after collisions. For the Coulomb scattering in the weak-screening approximation, we obtain

$$\lambda = 1 - 2p_{Fc}^2\left[\sum_i\left(m_i^{*2}c^2+\frac{1}{2}\frac{p_{Fi}^4}{m_i^{*2}c^2}\right)\mathcal{R}_i\right]^{-1}, \tag{63}$$

where $\mathcal{R}_i = 1$ for non-superfluid particles, and $\mathcal{R}_i$ is given by (45) in the presence of superfluidity of particle species i at $T < T_{ci}$.

Note, that for $-1 \leq \lambda \leq 3$ the correction factor can be accurately fitted by

$$C(\lambda) = 1 - 0.1146(\lambda - 3) - 0.007162(\lambda - 3)^2, \quad -1 \leq \lambda \leq 3, \tag{64}$$

and varies monotonically from $C(3) = 1$ to $C(-1) = 1.34$. For the Coulomb small-angle scattering from eq. (63) we have $-1/3 \leq \lambda \leq 1$. When the heat carriers (e or μ) collide mainly with other particles ($i \neq c$), $\lambda \approx 1$ and $C(1) = 1.2$. The same is true if the heat is mostly transported by the non-relativistic muons due to any muonic collisions, $\mu\mu$ and/or μi. However when the heat is carried by the relativistic electrons and the ee collisions dominate, we obtain $\lambda \approx -1/3$ and $C(-1/3) \approx 1.30$. Taking into account that $C(\lambda)$ is a slowly varying function of the collision probabilities, we can expect that $1.2 \lesssim C \lesssim 1.3$ for all the cases of interest. It seems to be reasonable, therefore, to adopt the constant correction factor $C = 1.2$ to the values of the thermal conductivity calculated in the preceding sections. If the muons are absent, a better correction is given by (63) and (64).

9 Formulas for Calculations

We have calculated the Coulomb thermal conductivity of electrons and muons in the neutron star cores. The conductivity is expressed through the effective electron and muon relaxation times, τ_e and τ_μ, see eq. (2):

$$\kappa_e \approx 1.70 \times 10^{24}\, C\, T_8 \left(\frac{\tau_e}{10^{-15}\,\mathrm{s}}\right) \left(\frac{n_e}{n_0}\right)^{2/3} \ \mathrm{erg\ cm^{-1}\,s^{-1}\,K^{-1}}, \tag{65}$$

$$\kappa_\mu = \kappa_e \left(\frac{n_\mu}{n_e}\right) \left(\frac{\tau_\mu}{\tau_e}\right), \tag{66}$$

where $n_0 = 0.16\,\mathrm{fm}^{-3}$ is the nucleon number density in standard nuclear matter, T_8 is temperature in units of 10^8 K, and $C \approx 1.2$ is the correction factor discussed in sect. 8. We have taken into account that $\mu_e = \mu_\mu$, see eq. (1).

The relaxation times τ_e and τ_μ are given by eq. (17). They are expressed through the collision frequencies $\nu_e = \sum_i \nu_{ei}$ and $\nu_\mu = \sum_i \nu_{\mu i}$ of the electrons and muons with all charged particles i (with leptons $i = e, \mu$ and baryons $i = b$), and also through the cross-frequencies $\nu'_{e\mu}$ and $\nu'_{\mu e}$. The collision frequencies ν_{eb} and $\nu_{\mu b}$ are given by eqs. (37) and (41):

$$\nu_{eb} \approx 1.15 \times 10^{12} \left(\frac{p_{Fe}}{q_0}\right)^3 \left(\frac{m_b^*}{m_p}\right)^2 \left(\frac{n_0}{n_e}\right) T_8^2\, \mathcal{R}_b \ \ \mathrm{s}^{-1}, \tag{67}$$

$$\nu_{\mu b} = \nu_{eb} \left(\frac{n_e}{n_\mu}\right)^{1/3}, \tag{68}$$

where the screening momentum (47) reads

$$\frac{q_0^2}{p_{Fe}^2} \approx 0.00929 \left[1 + \left(\frac{n_\mu}{n_e}\right)^{1/3} + 2.83 \sum_b \left(\frac{m_b^*}{m_p}\right) \left(\frac{n_b n_0}{n_e^2}\right)^{1/3} Z_b\right]. \tag{69}$$

In this case $\mathcal{R}_b$ and Z_b take into account possible superfluidity of the baryons. When the baryons of species b are superfluid ($T < T_{cb}$), $\mathcal{R}_b$ and Z_b should be calculated, respectively, from eqs. (45) and (48). Otherwise $\mathcal{R}_b = Z_b = 1$.

The frequencies $\nu_{e\mu}$ and $\nu_{\mu e}$ are given by eqs. (52) and (53):

$$\nu_{e\mu} \approx 1.43 \times 10^{11} \left(\frac{p_{Fe}}{q_0}\right)^3 \left(\frac{n_0}{n_e}\right)^{1/3} \left[1 + \frac{1}{2}\left(\frac{n_\mu}{n_e}\right)^{2/3}\right] T_8^2 \ \ \mathrm{s}^{-1}, \tag{70}$$

$$\nu_{\mu e} = \nu_{e\mu} \left(\frac{n_e}{n_\mu}\right)^{1/3}. \tag{71}$$

The collision frequencies ν_{ee} and $\nu_{\mu\mu}$ are calculated from (59) and (60):

$$\nu_{ee} \approx 3.58 \times 10^{11} \left(\frac{p_{Fe}}{q_0}\right)^3 \left(\frac{n_0}{n_e}\right)^{1/3} T_8^2 \ \ \mathrm{s}^{-1}, \tag{72}$$

$$\nu_{\mu\mu} = \nu_{ee} \left(\frac{n_\mu}{n_e}\right) \left[1 + \frac{6}{5}\left(\frac{n_{\mu r}}{n_\mu}\right)^{2/3} + \frac{2}{5}\left(\frac{n_{\mu r}}{n_\mu}\right)^{4/3}\right], \tag{73}$$

where $n_{\mu r} = (m_\mu c)^3/(3\pi^2\hbar^3) \approx 0.0052\ \mathrm{fm}^{-3}$ is the muon number density for which $p_{F\mu} = m_\mu c$.

Finally, the cross-frequencies $\nu'_{e\mu}$ and $\nu'_{\mu e}$ are determined by eqs. (57) and (58):

$$\nu'_{e\mu} \approx 1.43 \times 10^{11} \left(\frac{p_{Fe}}{q_0}\right)^3 \left(\frac{n_0}{n_e}\right)^{1/3} \left(\frac{n_\mu}{n_e}\right)^{2/3} T_8^2 \ \ \mathrm{s}^{-1}, \tag{74}$$

$$\nu'_{\mu e} = \nu'_{e\mu} \left(\frac{n_e}{n_\mu}\right). \tag{75}$$

All the collision frequencies depend on the screening momentum q_0 as q_0^{-3} (the weak screening approximation). The superfluidity of baryons (if available) lowers q_0 (see eq. (69)), which enhances the collision frequencies.

If the muons are absent, the electron thermal conductivity survives, with $\tau_e = 1/\nu_e$. If the muons are present but density ρ tends to the threshold density of their appearance, then $n_\mu \to 0$. In this case $\nu_\mu \to \infty$, $\nu'_{\mu e} \to \infty$, $\nu'_{e\mu} \to 0$. Accordingly, $\kappa_\mu \to 0$, and $\kappa = \kappa_e$, again with $\tau_e = 1/\nu_e$.

10 Results and Discussion

The expressions presented above can be used for a wide class of models of dense matter in the neutron star cores. For implications, one needs the number densities of charged particles, the effective masses of baryons, and the critical temperatures of baryon superfluidity, T_{cb}.

Let us discuss briefly the most important case of npe matter. If the protons are non-superfluid ($T > T_{cp}$), eqs. (37) and (59) yield

$$\frac{\nu_{ep}}{\nu_{ee}} = \frac{2}{5}\left(\frac{m_p^*}{m_e^*}\right)^2 = 3.21\left(\frac{m_p^*}{m_p}\right)^2\left(\frac{n_0}{n_e}\right)^{2/3}. \tag{76}$$

Thus the ν_{ep}/ν_{ee} ratio is large and the ep collisions dominate. The main contribution into the screening also comes from the protons. Under these conditions the electron thermal conductivity is

$$\begin{aligned} \kappa = \kappa_{ep} &= \frac{5C}{36\pi^2}\left(\frac{m_e^*}{m_p^*}\right)^2 \frac{q_0^3 c^4}{Te^4} \\ &\approx 6.3\times 10^{24}\frac{C}{T_8}\left(\frac{m_p}{m_p^*}\right)^{1/2}\left(\frac{n_e}{n_0}\right)^{7/6} \ \text{erg cm}^{-1}\,\text{s}^{-1}\,\text{K}^{-1}. \end{aligned} \tag{77}$$

Since n_e increases with mass density nearly as ρ^2 in npe matter of the outer neutron star core (see, e.g., Shapiro and Teukolsky 1983), the electron thermal conductivity grows as $\kappa_{ep} \propto \rho^{7/3}$.

In the opposite case, when the protons are highly superfluid ($T \ll T_{cp}$), the ep collisions are strongly reduced, and the ee collisions dominate. Then the main contribution into the screening comes from the electrons, and we obtain

$$\kappa = \kappa_{ee} = \frac{C}{18\pi^2}\frac{q_0^3 c^4}{Te^4} \approx 4.3\times 10^{24}\frac{C}{T_8}\left(\frac{n_e}{n_0}\right) \ \text{erg cm}^{-1}\,\text{s}^{-1}\,\text{K}^{-1}. \tag{78}$$

The electron thermal conductivity in this regime is generally smaller, than for the normal protons, and grows with density approximately as $\kappa_{ee} \propto \rho^2$. The smallness of κ_{ee} results from strong reduction of the screening momentum.

Fig. 1 shows the density dependence of κ at $T = 10^8$ K for two cases: when the protons are normal ($T_{cp} = 0$; curves N), and when they are strongly superfluid ($T_{cp} = 10^9$ K; curves S). Dashes correspond to the model of free particles (e.g., Shapiro and Teukolsky [9]), Yakovlev and Shalybkov [16]) - the simplest model of dense matter. Solid lines refer to a more realistic phenomenological model of matter proposed by Prakash *et al.* [22]). We have used the same set of parameters of the latter model as Page and Applegate [7]), and we have adopted the value of the proton effective mass $m_p^* = 0.7\,m_p$.

Many-body effects which are important in the realistic model of matter are seen to enhance the thermal conductivity by about one order of magnitude, with respect to the model of free particles. The enhancement comes mainly from the increase of n_e when nucleon-nucleon interaction is taken into account.

Fig. 2 demonstrates the effects of superfluidity. Solid line shows the thermal conductivity for the same model of matter (Prakash *et al.* [22]) at $\rho = 2\rho_0$ versus temperature for $T_{cp} = 5 \times 10^8$ K. One can clearly observe the transition from the non-superfluid regime (dashes N) to the highly superfluid regime (dashes S) with the decrease of temperature. Note that in both cases, N and S, the electron thermal conductivity depends on temperature as $\kappa \propto T^{-1}$.

Finally, fig. 3 shows density dependence of the thermal conductivity for a model of dense matter proposed by Glendenning [23]) (case 3 of his work) at $T = 10^8$ K. In this model, matter consists of neutrons, protons, and electrons for densities $\rho < 0.7\rho_0$. For higher densities, μ mesons appear, and for $\rho > 2\rho_0$, Σ^- and Λ^0 hyperons are created. We consider again two cases (concerned with the proton superfluidity): $T_{cp} = 0$ (N) and $T_{cp} = 10^9$ K (S). The hyperons are assumed to be non-superfluid. We also show the partial conductivities κ_e and κ_μ (dashed curves e and μ) for case S. The contribution of muons is seen to be noticeable. When the hyperons are absent, but the muons present, the thermal conductivity is mainly determined by the ep and μp collisions for non-superfluid protons, and by the ee, $e\mu$, and $\mu\mu$ collisions for strongly superfluid protons. When the Σ^- hyperons appear, the thermal conductivity is mainly governed by the ep, μp, $e\Sigma^-$, and $\mu\Sigma^-$ collisions for normal protons, and by the $e\Sigma^-$ and $\mu\Sigma^-$ collisions for the superfluid protons. In any case the appearance of Σ^- hyperons produces sharp decrease of the total thermal conductivity as well as of the partial conductivities. The sharp dips are concerned with very rapid growth of n_Σ with ρ just above the threshold (Glendenning [23]).

Our results enable one to calculate the Coulomb part of the thermal conductivity for various models of matter in neutron star cores. Using the equations obtained above, one can easily determine the electron and muon conductivities produced by scattering of electrons and muons on charged particles (including hyperons) and take into account possible superfluidity of charged scatterers (protons or hyperons). In this sense our results represent extension of the well known results of Flowers and Itoh [5]) obtained for one particular model of non-superfluid dense matter composed of *npe*. However Flowers and Itoh [5]) included also the conductivity of neutrons (produced by strong-interaction collisons with the nucleons) while we have not considered the neutron conductivity in the present work. Flowers and Itoh [5]) presented an analytic fit ($\kappa = 10^{23}\rho_{14}/T_8$ erg s^{-1} cm^{-1} K^{-1}) to the total thermal conductivity which cannot be compared directly with our results (since the fit includes the conductivity of the neutrons). However the conductivity given by the fit does not deviate drastically from the conductivities presented in fig. 1. It would be desirable to evaluate the thermal conductivity of neutrons for a set of models of neutron star matter including the effects of superfluidity.

The present results can be used for numerical simulation of neutron star cooling (e.g., Van Riper [1], and Umeda *et al.* [2]). The thermal conductivity in the neutron star interiors is especially important at early cooling stages, $t \lesssim 10^2 - 10^3$ years, when the thermal relaxation in the stellar core and the crust is not yet achieved. Exact values of the thermal conductivity are required to study the temperature profiles in the course of this relaxation. The neutrino energy losses in the core of a young star can be strongly non-uniform. For instance the critical temperatures of the appearance of superluidity of neutrons and protons can be strongly density dependent and produce density dependent superfluid suppression of the neutrino energy losses. Then the large temperature gradients can appear which can be sufficient to initiate convection (Gnedin and Yakovlev [24])). The convection onset is sensitive to the core thermal conductivity. When the convection is generated, the thermal relaxation in the neutron star interior (and hence the effective surface temperature of the star) can be different from that in the non-convective core. This effect has to be studied in details further.

Note that if protons (and charged hyperons) are normal, the neutron star core may possess large-scale magnetic fields. Even not very large fields magnetize motion of charged particles. Under these conditions, the thermal conductivity across the magnetic field can be essentially smaller, and it can be produced by heavy charged particles rather than by the electrons or muons. Similar effects in the electric conductivity have been studied by Yakovlev and Shalybkov [16]).

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Figure Captions

Figure 1

Coulomb thermal conductivity of npe matter in a neutron star core at $T = 10^8$ K vs density for the cases when the protons are normal (curves N) or highly superfluid (S). Solid lines correspond to a realistic model of matter (Prakash *et al.* [22]) while dashed lines are for the model of free particles.

Figure 2

Coulomb thermal conductivity vs temperature for the same model of matter (Prakash *et al.* [22]) as in fig. 1 at $\rho = 2\rho_0$. Solid curve is for $T_{cp} = 5 \times 10^8$ K. Dashed curve N corresponds to non-superfluid protons, and dashed curve S is for highly superfluid protons.

Figure 3

Density dependence of the thermal conductivity for the model of dense $npe\mu\Sigma^-\Lambda^0$ matter (Glendenning [23]) at $T = 10^8$ K. Solid curves show total $e + \mu$ thermal conductivity for the non-superfluid (N), and strongly superfluid (S) protons. Dashes show the partial electron and muon conductivities for case S. Vertical arrows mark the threshold densities of μ and Σ^- creation.