Chapter 4

Field and Dosimetric Quantities, Radiation Equilibrium – Definitions and Inter-relations

4.1 Introduction

We have discussed so far the charged and uncharged particles of interest in ionizing radiation dosimetry and the interactions of these particles with matter at the atomic level. We now consider radiation fields in which large numbers of particles are present and the particle interactions follow well-defined probability distributions. Metrology in ionizing radiation requires the definition of physical quantities that describe the relevant parameters of the radiation field, the interaction properties of the material under irradiation and the resulting effect on the material. We are interested in particular in the transfer of energy to the material (energy *conversion* and energy *deposition*). The biological and chemical effects that result from the energy transferred are also of interest, but are beyond the scope of the present text. Most of the relevant quantities and coefficients are defined in ICRU 85a (2011), which forms the basis of much that follows in the first part of the present chapter.

The relationships between field, more particularly *fluence*, and dosimetric quantities are then considered, which leads naturally on to a consideration of various types of *radiation equilibrium* and finally to how, given the existence of particular equilibrium conditions, it is possible to relate *absorbed dose* to particle fluence. These relationships are the starting point for the development of *cavity theory* (Chapter 9).

4.2 Stochastic and non-stochastic quantities

The nature of many physical processes is fundamentally random and this is certainly the case for the interaction of radiation with matter. We cannot predict at what position in space and at what moment in time a given particle will interact. Such processes are termed *stochastic* and mathematical descriptions of these involve quantities whose values vary discontinuously in space and time; it is not meaningful to speak of gradients or rate of change.

For most applications, however, radiation interactions are well approximated by *non-stochastic* descriptions in terms of mean or expectation values. This is because many particles are present and the spatial and temporal dimensions of interest are such that a very large number of interactions take place. From this we can also deduce the contrary; when few particles are present, or even when many are present but either the volume of interest is microscopic or the time-scale of interest is very short, non-stochastic descriptions are inappropriate. The stochastic nature of radiation interactions was introduced in Sec-

tion 1.3.

We employ the convention adopted in ICRU 85a (2011), namely, that a quantity normally expressed in terms of a mean or expectation value, and whose underlying distribution at the microscopic level is generally of little interest, is not defined explicitly as stochastic. One exception that is relevant to the present discussion is the description of energy deposition, defined at the basic level through stochastic quantities to permit its use in *microdosimetry*, that is, energy deposition at the cellular level in radiobiological studies. Fig. 4.1 shows schematically the increasing influence of statistical fluctuations as the detecting mass is decreased for a constant mean energy deposition per unit mass.

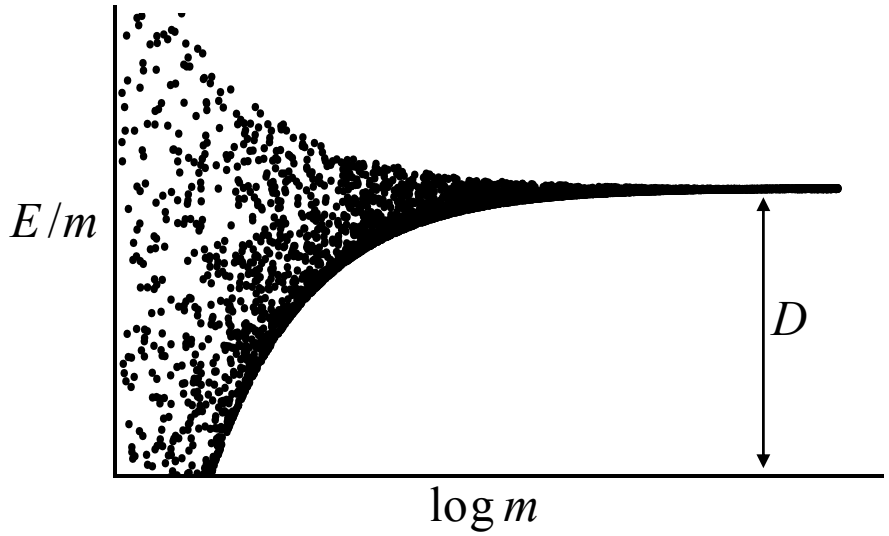


Figure 4.1: **A schematic representation of the behaviour of the energy deposition E in a mass m of material as a function of m. As the mass is decreased, the statistical fluctuations become significant and the stochastic nature of the basic interactions becomes evident. (Adapted from Rossi 1968).**

4.3 Radiation field quantities and units

There is some ambiguity in the terminology associated with the general description of fields. For example, the term flux is used in some domains to describe the number of particles per unit time and in others the number of particles per unit area per unit time. We follow here the convention adopted in ICRU 85a (2011). Radiation field quantities fall naturally into pairs referring either to the *number* of particles or the *energy transported* by these particles.

4.3.1 Particle number and radiant energy

The *particle number*, N, is the number of particles (emitted, transferred or received, as appropriate). Following the preceding discussion on stochastic quantities, we note that although it is a discrete quantity, N is usually very large and is treated as continuous.

The *radiant energy*, R, is the energy (excluding rest energy) of the particles. Thus for particles of energy E

$$R = N\,E \tag{4.1}$$

The unit of radiant energy is J.

4.3.2 Flux and energy flux

The *flux*, $\dot{N}$, is the quotient of $\mathrm{d}N$ by $\mathrm{d}t$

$$\dot{N} = \frac{\mathrm{d}N}{\mathrm{d}t} \tag{4.2}$$

where $\mathrm{d}N$ is the increment[1] in the particle number in the time interval $\mathrm{d}t$. The unit of flux is s^{-1}.

Similarly, the *energy flux*, $\dot{R}$, is the quotient of $\mathrm{d}R$ by $\mathrm{d}t$

$$\dot{R} = \frac{\mathrm{d}R}{\mathrm{d}t} \tag{4.3}$$

where $\mathrm{d}R$ is the increment in the radiant energy in the time interval $\mathrm{d}t$ and has the unit $\mathrm{J\ s}^{-1}$, which is equivalent to the watt, W.

4.3.3 Fluence and energy fluence

The *fluence*, Φ, is the quotient of $\mathrm{d}N$ by $\mathrm{d}a$

$$\Phi = \frac{\mathrm{d}N}{\mathrm{d}a} \tag{4.4}$$

where $\mathrm{d}N$ is the number of particles incident on a sphere of cross-sectional area $\mathrm{d}a$. The unit of fluence is m^{-2}. The concept of a sphere expresses the idea that one considers an area perpendicular to the direction of each particle (see Fig. 4.2); in other words, fluence is a *scalar* quantity.

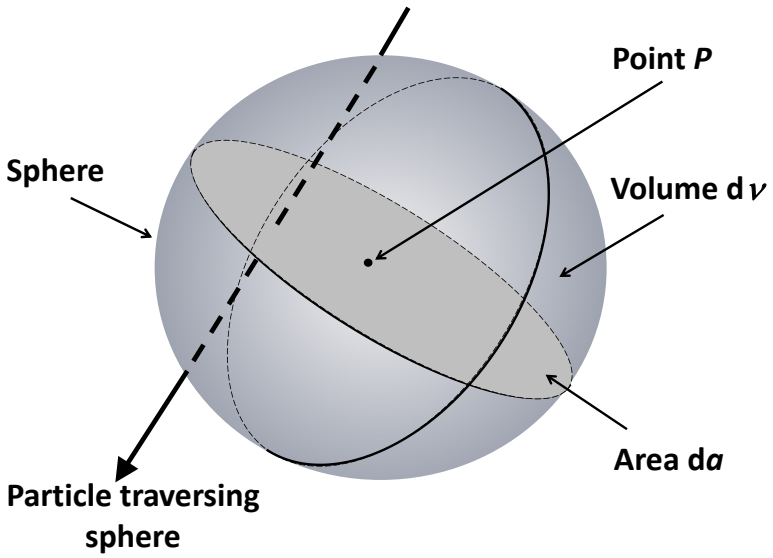


Figure 4.2: **Characterizing the radiation field at a point P in terms of the radiation traversing a sphere of cross section $\mathrm{d}a$ centred on P.**

In dosimetric calculations, the average fluence for a volume $\mathrm{d}\nu$ can be expressed in terms of the sum of the lengths of the particle trajectories $\mathrm{d}l$ in the volume (Kellerer 1971; Chilton 1978; Papiez and Battista 1994)

$$\bar{\Phi} = \frac{\sum(\mathrm{d}l)}{\mathrm{d}\,\nu} \tag{4.5}$$

[1]The use of *increment* in a mathematical context refers to a small positive or negative change in the value of a quantity.

The *energy fluence*, Ψ, is the quotient of $\mathrm{d}R$ by $\mathrm{d}a$

$$\Psi = \frac{\mathrm{d}R}{\mathrm{d}a} \tag{4.6}$$

where $\mathrm{d}R$ is the radiant energy incident on the sphere of cross-sectional area $\mathrm{d}a$. The unit of energy fluence is J m^{-2}.

4.3.4 Fluence rate and energy-fluence rate

The *fluence rate*, $\dot{\Phi}$, is the quotient of $\mathrm{d}\Phi$ by $\mathrm{d}t$

$$\dot{\Phi} = \frac{\mathrm{d}\Phi}{\mathrm{d}t} \tag{4.7}$$

where $\mathrm{d}\Phi$ is the increment in the fluence in the time interval $\mathrm{d}t$. The unit of fluence rate is m^{-2} s^{-1}.

The *energy-fluence rate*, $\dot{\Psi}$, is the quotient of $\mathrm{d}\Psi$ by $\mathrm{d}t$

$$\dot{\Psi} = \frac{\mathrm{d}\Psi}{\mathrm{d}t} \tag{4.8}$$

where $\mathrm{d}\Psi$ is the increment in the energy fluence in the time interval $\mathrm{d}t$. The unit of energy fluence rate is J m^{-2} s^{-1}, that is, W m^{-2}.

4.3.5 Planar fluence

The *planar fluence*, Φ^{P}, is the number of particles crossing a fixed plane in either direction (i.e., summed by scalar addition) per unit area of the plane. It is defined as

$$\Phi^{\mathrm{P}} = \frac{\mathrm{d}N}{\mathrm{d}a}\,\overline{\cos\theta_\Phi} = \Phi\,\overline{\cos\theta_\Phi} \tag{4.9}$$

where $\overline{\cos\theta_\Phi}$ is the fluence-weighted mean value of the directional cosine, θ being the angle between the direction of the exiting particles and their initial direction (c.f. Andreo and Brahme 1983); the right-hand side of the equation relates the planar fluence to the fluence, see below. The unit of planar fluence is m^{-2}.

The concept of *net flow* was first put forward in the context of radiological physics by Whyte (1959). He dealt with the flow of energy carried by particles, and applied the name 'plane intensity' to the vector sum of the energy flowing through a fixed plane. Whyte's illustrative diagram is reproduced in Fig. 4.3, which will be used here to discuss fluence vs. planar fluence.

A plane homogeneous beam of radiation is shown perpendicularly incident upon a flat scattering (but non-absorbing) foil. All particles are shown for simplicity being scattered through the same angle θ, at any azimuthal angle φ. A spherical and a flat detector of equal cross-sectional area are shown positioned above and again below the foil. The flat detector is oriented parallel to the foil, and thus is perpendicular to the beam of incident radiation. The number of incident particles striking each detector above the foil is clearly the same, and the planar fluence with respect to the plane of the flat detector is identical to the fluence in the same field. This can only be true in a plane-parallel beam, orthogonal to the flat detector, as shown.

In contrast, the number of scattered particles striking the spherical detector below the foil is $|1/\cos\theta|$ times the number striking the flat detector, which in turn is the same as the number it received above the foil. Thus the fluence (which is defined by Eq. (4.4) in terms of such a sphere) is $|1/\cos\theta|$ times the planar fluence, as indicated in Eq. (4.9). This increase in fluence contributes to an effect sometimes seen in broad-beam geometry, for example in a medium irradiated by a (quasi-monoenergetic) electron beam; multiple Coulomb scattering causes an increase in fluence and hence in dose over at least the first half of the electron range (see Chapter 6), whereas the planar fluence remains almost constant.

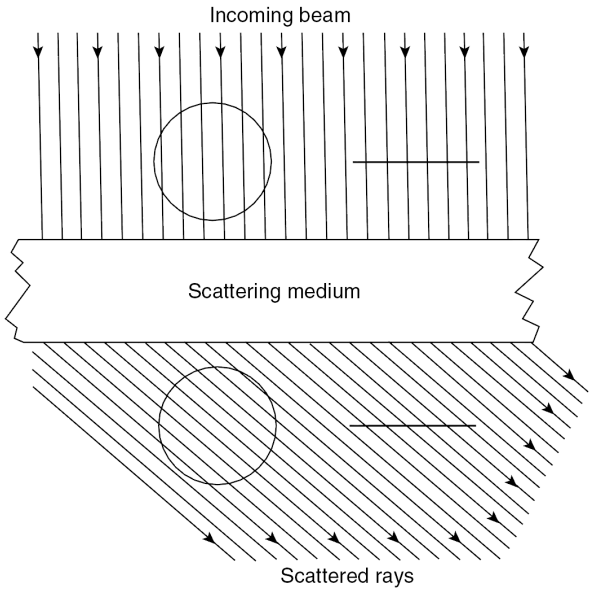


Figure 4.3: **Particles scattered through an angle θ in a non-absorbing foil. (After Whyte, 1959.)**

4.4 Distributions of field quantities

We now consider the distributions of the above quantities in terms of energy and angle.

4.4.1 Energy distributions

The distribution of the particle number N with respect to energy E is denoted N_E and is commonly referred to as the *energy spectrum*

$$N_E = \frac{\mathrm{d}N}{\mathrm{d}E} \tag{4.10}$$

where $\mathrm{d}N$ is the number of particles with energy between E and $E + \mathrm{d}E$. Similarly, R_E describes the distribution of the radiant energy with respect to energy

$$R_E = \frac{\mathrm{d}R}{\mathrm{d}E} \tag{4.11}$$

We might also consider energy distributions for the flux and energy flux.

The distribution of the fluence Φ with respect to energy is denoted Φ_E

$$\Phi_E = \frac{\mathrm{d}\Phi}{\mathrm{d}E} \tag{4.12}$$

and is commonly referred to as the *fluence spectrum*. Similarly, Ψ_E denotes the *energy-fluence spectrum*. From the relation between N and R it is evident that

$$\Psi_E = E\,\Phi_E \tag{4.13}$$

When expressing quantities, care must be taken with units. For example, a practical unit for Ψ_E might be J m^{-2} keV^{-1}. However, if the energy is expressed in keV rather than in J, then by cancellation the unit for Ψ_E becomes m^{-2}, which is also the unit of the fluence Φ. This potential for confusion when different physical quantities can have the same units highlights the importance of clearly stating the quantity and of using unambiguous symbols. Examples related to radiation spectra are given in Chapters 6 and 7.

4.4.2 Angular distributions – particle radiance and energy radiance

The specification of direction requires two variables. In a spherical coordinate system with polar angle θ and azimuthal angle φ, the solid angle $\mathrm{d}\Omega$ is equal to $\sin\theta\ \mathrm{d}\theta\ \mathrm{d}\varphi$.

The *particle radiance*, $\dot{\Phi}_\Omega$, is the quotient of $\mathrm{d}\dot{\Phi}$ by $\mathrm{d}\Omega$

$$\dot{\Phi}_\Omega = \frac{\mathrm{d}\dot{\Phi}}{\mathrm{d}\Omega} \tag{4.14}$$

where $\mathrm{d}\dot{\Phi}$ is the fluence rate of particles propagating within a solid angle $\mathrm{d}\Omega$ around a specified direction. The unit of $\dot{\Phi}_\Omega$ is $\mathrm{m}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1}$.

The *energy radiance*, $\dot{\Psi}_\Omega$, is the quotient of $\mathrm{d}\dot{\Psi}$ by $\mathrm{d}\Omega$

$$\dot{\Psi}_\Omega = \frac{\mathrm{d}\dot{\Psi}}{\mathrm{d}\Omega} \tag{4.15}$$

where $\mathrm{d}\dot{\Psi}$ is the energy-fluence rate of particles propagating within a solid angle $\mathrm{d}\Omega$ around a specified direction. The unit of $\dot{\Phi}_\Omega$ is $\mathrm{W\ m}^{-2}\ \mathrm{sr}^{-1}$.

4.4.3 Distributions in energy and angle

For a given particle type, the radiation field is completely described by the distribution of particle radiance with respect to energy, $\dot{\Phi}_{\Omega,E}$, as this defines the number, energy, local density and rate of particles propagating in a given direction. This quantity, as well as the corresponding energy distribution of the energy radiance, $\dot{\Psi}_{\Omega,E}$, can be considered a fundamental descriptor of a radiation field. A summary of the radiometric quantities used to describe radiation fields is given in Table 4.1.

4.5 Quantities describing radiation interactions

As discussed in the previous chapters, for a given interaction process the probability of an interaction depends on the incident particle type and energy and on the target material. This probability is described in terms of the cross section for a particular interaction. As noted in the preceding chapters, charged and uncharged particles undergo very different interaction types, giving rise to interaction coefficients that are different in nature. A summary of key concepts and quantities is presented in this section.

4.5.1 Cross section

For a given particle type, charged or uncharged, with a particular energy and incident on a given target material, the *cross section*, σ_j, for an interaction of type j is the quotient of N_j by Φ

$$\sigma_j = \frac{N_j}{\Phi} \tag{4.16}$$

where N_j is the mean number of interactions of type j per target entity and Φ is the incident particle fluence. The unit of σ_j is m^2 and may also be expressed in terms of barns (b), where $1\ \mathrm{b} = 10^{-28}\ \mathrm{m}^2$.

Interactions normally result in the emission of secondary particles, whose distributions are described by differential cross sections obtained by the differentiation of the cross section with respect to the energy and angle of emission of the secondary particles.

The *total cross section*, σ, is the sum of the component cross sections

$$\sigma = \frac{N}{\Phi} = \frac{1}{\Phi}\sum_j N_j \tag{4.17}$$

Table 4.1: **The quantities used to describe radiation fields, following the definitions of scalar radiometric quantities given in ICRU 85a (2011).**

Quantity	Symbol	Components
Particle number	N	–
Radiant energy	R	–
Energy distribution of particle number	N_E	$\mathrm{d}N/\mathrm{d}E$
Energy distribution of radiant energy	R_E	$\mathrm{d}R/\mathrm{d}E$
Flux	$\dot{N}$	$\mathrm{d}N/\mathrm{d}t$
Energy flux	$\dot{R}$	$\mathrm{d}R/\mathrm{d}t$
Energy distribution of flux	$\dot{N}_E$	$(\mathrm{d}N/\mathrm{d}t)\,/\mathrm{d}E$
Energy distribution of energy flux	$\dot{R}_E$	$(\mathrm{d}R/\mathrm{d}t)\,/\mathrm{d}E$
Fluence	Φ	$\mathrm{d}N/\mathrm{d}a$
Energy fluence	Ψ	$\mathrm{d}R/\mathrm{d}a$
Energy distribution of fluence	Φ_E	$(\mathrm{d}N/\mathrm{d}a)\,/\mathrm{d}E$
Energy distribution of energy fluence	Ψ_E	$(\mathrm{d}R/\mathrm{d}a)\,/\mathrm{d}E$
Fluence rate	$\dot{\Phi}$	$(\mathrm{d}N/\mathrm{d}a)\,/\mathrm{d}t$
Energy-fluence rate	$\dot{\Psi}$	$(\mathrm{d}R/\mathrm{d}a)\,/\mathrm{d}t$
Energy distribution of fluence rate	$\dot{\Phi}_E$	$[(\mathrm{d}N/\mathrm{d}a)\,/\mathrm{d}t]\,/\mathrm{d}E$
Energy distribution of energy-fluence rate	$\dot{\Psi}_E$	$[(\mathrm{d}R/\mathrm{d}a)\,/\mathrm{d}t]\,/\mathrm{d}E$
Particle radiance	$\dot{\Phi}_\Omega$	$[(\mathrm{d}N/\mathrm{d}a)\,/\mathrm{d}t]\,/\mathrm{d}\Omega$
Energy radiance	$\dot{\Psi}_\Omega$	$[(\mathrm{d}R/\mathrm{d}a)\,/\mathrm{d}t]\,/\mathrm{d}\Omega$
Energy distribution of particle radiance	$\dot{\Phi}_{\Omega,E}$	$\{[(\mathrm{d}N/\mathrm{d}a)\,/\mathrm{d}t]\,/\mathrm{d}\Omega\}\,/\mathrm{d}E$
Energy distribution of energy radiance	$\dot{\Psi}_{\Omega,E}$	$\{[(\mathrm{d}R/\mathrm{d}a)\,/\mathrm{d}t]\,/\mathrm{d}\Omega\}\,/\mathrm{d}E$

where N is the mean total number of interactions per target entity. As introduced in Chapter 2, this defines what is often referred to as the 'microscopic' total cross section. When expressed in m^2/kg (multiplying by N_A/M as in Eq (4.19) below), we obtain a macroscopic equivalent of the total cross section termed the mass attenuation coefficient.

4.5.2 Interaction coefficients for uncharged particles

As discussed in Chapter 3, the interactions of uncharged particles follow probability distributions (cross sections), which are conveniently expressed in terms of attenuation coefficients. The *linear attenuation coefficient*, μ, of a material is the quotient of $\mathrm{d}N/N$ by $\mathrm{d}l$

$$\mu = \frac{1}{\mathrm{d}l}\frac{\mathrm{d}N}{N} \tag{4.18}$$

where $\mathrm{d}N/N$ is the mean fraction of the particles that experience interactions in traversing the distance $\mathrm{d}l$ in the material. The unit of μ is m^{-1}. The reciprocal of μ, which has the unit m, is called the *mean free path* (MFP). Clearly, μ will depend strongly on the density of the material. This dependence is largely removed by defining the *mass attenuation coefficient*, μ/ρ, of a material as

$$\frac{\mu}{\rho} = \frac{1}{\rho\,\mathrm{d}l}\frac{\mathrm{d}N}{N} \tag{4.19}$$

where ρ is the material density. The unit of μ/ρ is $\mathrm{m}^2\ \mathrm{kg}^{-1}$.

The relationship between the mass attenuation coefficient and the total cross section is

$$\frac{\mu}{\rho} = \frac{N_\mathrm{A}}{M}\,\sigma = \frac{n_t}{\rho}\,\sigma \tag{4.20}$$

where N_{A} is the Avogadro constant, M is the molar mass of the material and n_t is the number density of target entities (that is, the number of target entities per volume). It is evident from this relation that when several interaction types j are possible, the mass attenuation coefficient can be written as

$$\frac{\mu}{\rho} = \frac{N_{\mathrm{A}}}{M} \sum_j \sigma_j \tag{4.21}$$

For a compound, the component entities i are normally considered to be independent, from which it follows that for a given interaction type

$$\frac{\mu}{\rho} = \frac{1}{\rho} \sum_i n_{t,i}\, \sigma_i \tag{4.22}$$

where $n_{t,i}$ is the number density of component entities i and σ_i is the cross section for component i and the given interaction type. The limitations of this approximation are discussed in ICRU 85a (2011) and the references therein.

It was noted earlier that radiation field quantities fall naturally into pairs referring either to the number of particles or the energy transported by them. This pairing is also relevant to the attenuation coefficient and we can define the *mass energy-transfer coefficient*, μ_{tr}/ρ, for a material of density ρ and for incident uncharged particles of a given type and energy, as

$$\frac{\mu_{\mathrm{tr}}}{\rho} = \frac{1}{\rho \mathrm{d}l} \frac{\mathrm{d}R_{\mathrm{tr}}}{R} \tag{4.23}$$

where $\mathrm{d}R_{\mathrm{tr}}$ is the mean energy transferred to kinetic energy of charged particles by interactions of the uncharged particles of incident radiant energy R in traversing the distance $\mathrm{d}l$ in the material. For a series of component cross sections j, the equivalent of equation (4.20) is

$$\frac{\mu_{\mathrm{tr}}}{\rho} = \frac{N_{\mathrm{A}}}{M} \sum_j f_j\, \sigma_j \tag{4.24}$$

where the coefficient f_j represents, for each interaction type j, the mean fraction of the energy k of the incident uncharged particle that is transferred to the kinetic energy of charged particles. Writing the expression for μ_{tr} in terms of partial attenuation coefficients, see Chapter 3, we have

$$\mu_{\mathrm{tr}} = \bar{f}_{\mathrm{ph}}\, \mu_{\mathrm{ph}} + \bar{f}_{\mathrm{C}}\, \mu_{\mathrm{C}} + \bar{f}_{\mathrm{pp}}\, \mu_{\mathrm{pp}} + \bar{f}_{\mathrm{tp}}\, \mu_{\mathrm{tp}} \tag{4.25}$$

where

$$\begin{aligned} \bar{f}_{\mathrm{ph}} &= 1 - \frac{\bar{k}_{\mathrm{x}}}{k} \\ \bar{f}_{\mathrm{C}} &= 1 - \frac{\bar{k}' + \bar{k}_{\mathrm{x}}}{k} \\ \bar{f}_{\mathrm{pp}} &= 1 - \frac{2 m_{\mathrm{e}} c^2}{k} \\ \bar{f}_{\mathrm{tp}} &= 1 - \frac{2 m_{\mathrm{e}} c^2 + \bar{k}_{\mathrm{x}}}{k} \end{aligned} \tag{4.26}$$

$\bar{k}'$ is the average energy of the Compton-scattered photon and the $\bar{k}_{\mathrm{x}}$ are average energies of the characteristic x-rays emitted in atomic relaxation. It should be noted that in the case of pair and triplet production the rest energies of the particles created, $2\, m_{\mathrm{e}} c^2$, do not contribute to $\mu_{\mathrm{tr}}(k)$.

For a semi-infinite medium, the charged particles will come to rest in the material and during their slowing down a mean fraction g of their kinetic energy (and of any secondary charged particles produced by them) will be lost in radiative processes, e.g. bremsstrahlung, fluorescence and annihilation.

This *radiative fraction* g is closely related to the radiation yield (see Chapter 2), which is defined for (monoenergetic) electrons and positrons; hence g corresponds to the radiation yield averaged over the spectrum of energies of the charged particles liberated by monoenergetic photons and is characteristic of the material and the photon energy. [2] To a first approximation, this mean radiative energy fraction can be thought of as escaping the local volume and therefore does not result in *locally* absorbed energy. This gives rise to the *mass energy-absorption coefficient*, $\mu_{\rm en}/\rho$, defined as

$$\frac{\mu_{\rm en}}{\rho} = \frac{\mu_{\rm tr}}{\rho}\,(1-g) \tag{4.27}$$

Clearly, g and therefore $\mu_{\rm en}/\rho$ depend on the interaction coefficients for the charged particles, the subject of the following section. The assumption that none of the radiated energy is reabsorbed locally was tested for ^{60}Co radiation in the work of Burns (2006), who calculated a small correction factor k_g (a correction that would be greater at higher energies). However, this is a correction for a practical dosimeter and does not change the value of the interaction coefficient $\mu_{\rm en}/\rho$.

Numerous tables of attenuation coefficients and mass energy-absorption coefficients for subsets of materials and energies have been published over the years, in particular by Storm and Israel (1970), Hubbell (1982), Johns and Cunningham (1983), Attix (1986), Higgins *et al.* (1992) and Hubbell and Seltzer (1995). A widely used set of data is that available on the web-based NIST database (http://www.nist.gov/pml/data/xraycoef/), which is essentially an update of Hubbell and Seltzer (1995). The Data Tables include values for elements and compounds of dosimetric interest.

4.5.3 Interaction coefficients for charged particles

While energy deposition due to uncharged particles is indirect in the sense that it proceeds through the production of charged particles, the charged particles themselves interact in a more direct manner. As noted in Chapter 2, charged-particle interactions could be treated in the same way as uncharged particles, that is, in terms of attenuation coefficients and mean free paths. However, the very high density of charged-particle interactions lends itself to a practical treatment in terms of energy loss per unit path length.

For charged particles of a given type and energy, the *unrestricted linear stopping power*, S, is the quotient of $\mathrm{d}E$ by $\mathrm{d}l$

$$S = \frac{\mathrm{d}E}{\mathrm{d}l} \tag{4.28}$$

where $\mathrm{d}E$ is the mean energy lost by the charged particles in traversing a distance $\mathrm{d}l$ in the material. The unit of S is J m^{-1}, or eV m^{-1} if the energy loss is expressed in eV.

As for the attenuation coefficient for uncharged particles, S will depend on the density of the material and this dependence is largely removed by defining the *mass stopping power*, S/ρ, of a material as

$$\frac{S}{\rho} = \frac{1}{\rho}\frac{\mathrm{d}E}{\mathrm{d}l} \tag{4.29}$$

with unit J m^2 kg^{-1} or eV m^2 kg^{-1}.

The mass stopping power can be expressed as the sum of independent components

$$\frac{S}{\rho} = \frac{S_{\rm el}}{\rho} + \frac{S_{\rm rad}}{\rho} + \frac{S_{\rm nuc}}{\rho} \tag{4.30}$$

[2] It should be noted that for a photon spectrum, where the average is also made over all the energies in the spectrum, the symbol $\bar{g}$ is frequently used. The same criterion should strictly be applied to the different photon coefficients, i.e. $\bar{\mu}$, $\bar{\mu}_{\rm tr}$ etc. However, for simplicity in the notation, in what follows a bar over the symbol will be used only if there is a possibility for confusion between monoenergetic photons and a photon spectrum.

where $S_{\rm el}/\rho$ is the *mass electronic stopping power* (also known as the *mass collision stopping power*) due to interactions with atomic electrons resulting in ionization or excitation, $S_{\rm rad}/\rho$ is the *mass radiative stopping power* due to bremsstrahlung emission in the electric fields of atomic nuclei or atomic electrons, and $S_{\rm nuc}/\rho$ is the *mass nuclear stopping power* due to elastic Coulomb interactions in which recoil energy is imparted to atoms.

We might note that while the use of quotients S/ρ greatly reduces the dependence on density, it is not eliminated and the residual effect, referred to as the density effect, must be considered for non-gaseous materials (and even in gases beyond tens of MeV) – see Chapter 2. Tables of stopping powers are published by the ICRU, notably stopping powers for electrons and positrons in ICRU 37 (1984), for protons and alpha particles in ICRU 49 (1993) and for ions heavier than helium in ICRU 73 (2005). Values for elements and compounds of dosimetric interest are given in the Data Tables.

As we have seen in Chapter 2, the *unrestricted* linear stopping power of a particle with energy E can be expressed as

$$S_{\rm el}(E) \equiv S_{\rm el}(E, \mathcal{W}_{\rm max}) = \rho N_{\rm A} \frac{Z}{A} \int_{\mathcal{W}_{\rm min}}^{\mathcal{W}_{\rm max}} \mathcal{W} \frac{{\rm d}\sigma}{{\rm d}\mathcal{W}} \, {\rm d}\mathcal{W} \tag{4.31}$$

where $\mathcal{W}$ is the energy lost in liberating a secondary electron, with upper limit $\mathcal{W}_{\rm max}$ (which depends on the charged particle type, light e^- or e^+, or heavy) and lower limit $\mathcal{W}_{\rm min}$ (which is loosely associated with the I-value). The energy loss $\mathcal{W}$ includes the kinetic energy, $\varepsilon_{\rm ke}$, given to the secondary electron plus its binding energy $U_{\rm B}$, i.e.

$$\mathcal{W} = \varepsilon_{\rm ke} + U_{\rm B} \tag{4.32}$$

The idea of *local* energy deposition, as opposed to the *total* energy lost in electronic collisions by the primary charged particles, is embodied in the concept of *restricted* stopping power. The energy loss is limited to a maximum, Δ, i.e. it excludes energy losses between Δ and $\mathcal{W}_{\rm max}$, and therefore the restricted stopping power is obtained as

$$S_{\rm el}(E, \Delta) = \rho N_{\rm A} \frac{Z}{A} \int_{\mathcal{W}_{\rm min}}^{\Delta} \mathcal{W} \frac{{\rm d}\sigma}{{\rm d}\mathcal{W}} \, {\rm d}\mathcal{W} \tag{4.33}$$

The *Linear Energy Transfer* (LET), with symbol $L_\Delta(E)$, has traditionally been defined as being identical to $S_{\rm el}(E, \Delta)$, c.f. ICRU Report 37 (1984). However the definition in recent ICRU reports on quantities and units (c.f. ICRU Report 85a, eq. (4.5.1)), which is based on the work of Kellerer (c.f. Kellerer and Rossi 1990, Kellerer *et al.* 1992 and Chen *et al.* 1995), excludes only the kinetic energy of the secondary electron when this kinetic energy is above Δ; it does not exclude the binding energy. Therefore the definition of $L_\Delta(E)$ becomes

$$L_\Delta(E) \equiv L(E, \Delta) = \rho N_{\rm A} \frac{Z}{A} \int_{\mathcal{W}_{\rm min}}^{\mathcal{W}_{\rm max}} \mathcal{W} \frac{{\rm d}\sigma}{{\rm d}\mathcal{W}} \, {\rm d}\mathcal{W} - \varepsilon_{{\rm ke}>\Delta} = S_{\rm el}(E) - \varepsilon_{{\rm ke}>\Delta} \tag{4.34}$$

where $\varepsilon_{{\rm ke}>\Delta}$ is the total secondary electron kinetic energy for kinetic energies above Δ. [3] The usual units for L_Δ are keV $\mu{\rm m}^{-1}$.

When $E \leq \Delta$, it is clear that $L_\Delta(E) = S_{\rm el}(E)$, i.e., the two quantities coincide as there can be no energy loss above Δ. For electrons, equality of $S_{\rm el}(E)$ and $L_\Delta(E)$ occurs when $E \leq 2\,\Delta$. This is because when a 'primary' electron interacts to produce a secondary electron, the maximum energy that

[3] Readers are warned against the erroneous use of $S_{\rm el}(E, \Delta)$ instead of $S_{\rm el}(E)$ in Eq. (4.34), a potential mistake that could be made if one tries to relate the former definition of LET to the current one.

can be transferred is Δ as, by convention, the secondary is taken to be the outgoing electron with the lower energy (see Chapter 2).

Note that as a consequence of the new definition of LET, $L_\Delta(E) > S_{\rm el}(E,\Delta)$ because now $L_\Delta(E)$ includes the binding energy for *all* interactions while $S_{\rm el}(E,\Delta)$ contains the binding energy only for those interactions with energy loss below Δ [4]. Consequently, the LET as now defined is more meaningful than the restricted stopping power with regard to energy deposition, which is the quantity of interest related to absorbed dose (see Section 4.6.2).

To clarify this inequality we can write Eq. (4.31) in the simplified form

$$S_{\rm el}(E,\mathcal{W}_{\rm max}) = \sum_{\mathcal{W}_{\rm min}}^{\mathcal{W}_{\rm max}} \varepsilon + \sum_{\mathcal{W}_{\rm min}}^{\mathcal{W}_{\rm max}} U = \sum_{\mathcal{W}_{\rm min}}^{\Delta} \varepsilon + \sum_{\Delta}^{\mathcal{W}_{\rm max}} \varepsilon + \sum_{\mathcal{W}_{\rm min}}^{\Delta} U + \sum_{\Delta}^{\mathcal{W}_{\rm max}} U \tag{4.35}$$

and consider that $S_{\rm el}(E,\Delta)$, Eq. 4.33), excludes terms 2 and 4, i.e.

$$S_{\rm el}(E,\Delta) = \sum_{\mathcal{W}_{\rm min}}^{\Delta} \varepsilon + \sum_{\mathcal{W}_{\rm min}}^{\Delta} U \tag{4.36}$$

but $L_\Delta(E)$, Eq. (4.34), excludes only term 2, i.e.

$$L_\Delta(E) = \sum_{\mathcal{W}_{\rm min}}^{\Delta} \varepsilon + \sum_{\mathcal{W}_{\rm min}}^{\Delta} U + \sum_{\Delta}^{\mathcal{W}_{\rm max}} U \tag{4.37}$$

from which it is clear that $L_\Delta(E) > S_{\rm el}(E,\Delta)$. The difference between $L_\Delta(E)$ and $S_{\rm el}(E,\Delta)$ is, however, negligible for values of Δ that are large compared with the atomic ionization energies, but becomes significant when Δ is comparable or smaller than these energies. For example, in the case of water, $L_\Delta(E) = S_{\rm el}(E,\Delta)$ at $\Delta \geq 10$ keV but they differ by several percent at $\Delta \leq 1$ keV, see Fig. 4.4.

It should also be noted that as $\Delta \to 0$ (strictly, to $\mathcal{W}_{\rm min}$), $S_{\rm el}(E,\Delta) \to 0$ but $L_\Delta(E)$ does not, it tends rather to the sum of the binding energies for all interactions and its utility is based on the assumption that this energy will appear as 'local' absorbed dose.

Alternatively, when $\Delta \to \mathcal{W}_{\rm max}$, $S_{\rm el}(E,\Delta) \to S_{\rm el}(E,\mathcal{W}_{\rm max})$, i.e. the restricted and unrestricted stopping powers coincide. The same applies for the LET, i.e. $L_\Delta(E) \to S_{\rm el}(E,\mathcal{W}_{\rm max})$ because the kinetic energy $\varepsilon_{{\rm ke}>\mathcal{W}_{\rm max}} \equiv 0$; this situation is usually written as $L_\infty(E) \equiv S_{\rm el}(E)$ (where clearly ∞ is not taken to mean infinity).

4.5.4 Related quantities – $G(x)$, $\mathcal{Y}$ and W

ICRU Report 85a (2011) also defines three quantities related to yield that are required for primary measurements in ionizing radiation dosimetry. The first of these is the *radiation chemical yield*, $G(x)$, of an entity x, defined as the quotient of $n(x)$ by $\bar{\varepsilon}$

$$G(x) = \frac{n(x)}{\bar{\varepsilon}} \tag{4.38}$$

where $n(x)$ is the mean amount of substance of the entity produced, destroyed or changed in a system by the mean energy imparted $\bar{\varepsilon}$ to the matter of that system. The definition of $\bar{\varepsilon}$ is given in Section 4.6.2.

[4] There is another reason why $L_\Delta(E) > S_{\rm el}(E,\Delta)$. Consider an interaction for which the secondary electron has a kinetic energy just below Δ, but the energy loss (including the binding energy) is above Δ. This event will be included in $L_\Delta(E)$ but excluded from $S_{\rm el}(E,\Delta)$. In other words, $L_\Delta(E)$ has a kinetic energy threshold Δ, while that for $S_{\rm el}(E,\Delta)$ is lower by the binding energy.

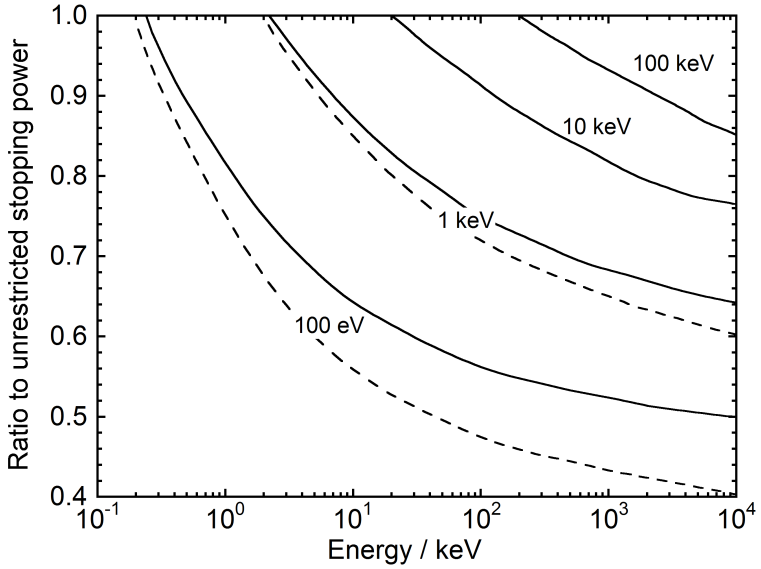


Figure 4.4: **Ratio of the LET to the unrestricted stopping power, $L_\Delta(E)/S_{\rm el}(E)$ (solid lines), and of the restricted to the unrestricted stopping power, $S_{\rm el}(E,\Delta)/S_{\rm el}(E)$ (dashed lines), in water. (Adapted from Kellerer *et al.* 1992).**

The term *amount of substance* (sometimes known as *chemical amount*) has a specific meaning in the SI system; the mole is the amount of substance of a system that contains as many elementary entities as there are atoms in 12 g of ^{12}C (this number being the Avogadro constant, $N_{\rm A}$). The unit of $G(x)$ is therefore mol J^{-1}.

A related quantity is the *G-value*, which is the mean number of entities produced, destroyed or changed for an energy imparted of 100 eV and has the curious unit (100 eV)$^{-1}$. Recalling that 1 eV= 1.6022×10^{-19} J and that $N_{\rm A} = 6.0221 \times 10^{23}$, it follows that a G-value of 1 (100 eV)$^{-1}$ corresponds to a radiation chemical yield of 0.1036 μmol J^{-1}.

In ionization chamber dosimetry, the yield of charge for a given energy imparted can be likened to the radiation chemical yield. For a charged particle of a given type with initial kinetic energy E, we define the *ionization yield in a gas*, $\mathcal{Y}$, as the quotient of N by E

$$\mathcal{Y} = \frac{N}{E} \tag{4.39}$$

where $N = q/e$ is the mean total liberated charge q of either sign, divided by the elementary charge e, when the energy of the charged particle is completely dissipated in the gas. The unit of $\mathcal{Y}$ is J^{-1} and may also be expressed in eV^{-1}.

This quantity is essentially the inverse of the *mean energy expended in a gas per ion pair formed*, W, which is the quotient of E by N (see Chapter 2)

$$W = \frac{E}{N} \tag{4.40}$$

where E and N are as above. The unit of W is J and may also be expressed in eV. An important quantity in primary standards for dosimetry, W is discussed in detail in ICRU Report 31 (ICRU 1979).

We note here several subtleties regarding W. Firstly, although the name of the quantity refers to ion pairs, and previous definitions were indeed in terms of ions, most recently in ICRU 60 (1998), the current definition is in terms of charge divided by e. This removes any ambiguity related to the production of multiply-charged ions. Note that as $N = q/e$ we can write

$$\frac{W}{e} = \frac{E}{q} \tag{4.41}$$

where W/e is expressed in J C^{-1}.

Secondly, q includes by definition not only the charge directly liberated by the incident charged particle, but also any subsequent charge produced by the interactions of bremsstrahlung or other secondary radiation emitted by the incident particle and its descendents. However, most of this 'secondary' charge will escape from a real detector. This means that to render the measured charge $q_{\rm meas}$ compatible with the definition of W, the charge must be increased by the factor $(1-g)^{-1}$, where g is the radiative fraction discussed in Section 4.5.2. It follows that

$$E = \frac{W}{e}\, q = \frac{W}{e}\, \frac{q_{\rm meas}}{(1-g)} \tag{4.42}$$

This relation will be invoked in Chapter 11 in the context of primary standards for air kerma and absorbed dose based on ionization chambers.

Thirdly, the charge of the initial charged particle is not included in q. However, as introduced by Büermann *et al.* (2006) and discussed in the review by Burns and Büermann (2009), ionization measurements will necessarily include this charge, which gives rise to a correction factor k_{ii} that is non-negligible at low energies where q is relatively small (ICRU 2016).

In certain situations, the parameter of interest is the local value at a point along the path of the particle, for which the concept of the differential value for W is required, denoted w and defined as the quotient of $\mathrm{d}E$ and $\mathrm{d}N$, where $\mathrm{d}E$ is the mean energy lost in traversing a gas layer of infinitesimal thickness and $\mathrm{d}N$ is the mean total charge liberated by this energy loss, divided by e.

The mean energy required to create an ion pair in dry air has been taken as $W_{\rm air} = 33.97$ eV with a standard uncertainty 0.15%, recommended by the Consultative Committee for Ionizing Radiation (CCEMRI 1985). The current recommendation (ICRU 2016) maintains this value for $W_{\rm air}$ but increases the uncertainty to 0.35%. By dividing $W_{\rm air}$ by the charge of the electron in coulombs and converting the energy from electron volts to joules, one obtains

$$\frac{W_{\rm air}}{e} = \frac{33.97\ \mathrm{eV/electron}}{1.6022\times10^{-19}\ \mathrm{C/electron}} \times 1.0622\times10^{-19}\ \mathrm{J\,eV^{-1}} = 33.97\ \mathrm{J\,C^{-1}}$$

We see that the conversion constants cancel so as to give $W_{\rm air}/e$ in J C^{-1}, the same numerical value as $W_{\rm air}$ expressed in eV/ion-pair (or per electron), which is convenient. Moreover $W_{\rm air}$ may be regarded as a constant, independent of photon or electron energy, for energies above a few keV; for protons and heavier particles $W_{\rm air}$ has different values, see e.g. IAEA TRS-398 (Andreo *et al.* 2000). There has been some controversy over the assumed constancy of $W_{\rm air}$ for electrons and photons in the energy range between 1 MeV and approximately 50 MeV (c.f. Svensson and Brahme 1986; Muir and Rogers 2010), but work in this domain by Burns *et al.* (2014) using the BIPM calorimetric and ionometric standards (see Chapter 11) has demonstrated the constancy of $W_{\rm air}$ between ^{60}Co radiation and accelerator photon beams up to 25 MV.

4.6 Dosimetric quantities

The interaction of a radiation field with matter can be described in terms of quantities that are essentially products of radiation field quantities and interaction coefficients for materials. The definitions of these quantities are given here and their inter-relations in terms of field quantities and interactions coefficients are discussed in Section 4.7. The process by which energy is converted to energy dissipation in matter can be conveniently divided into two stages, the transfer of the incident particle energy to secondary particle energy and the deposition of this energy in matter. We recall the discussion of Section 4.2 regarding stochastic and non-stochastic quantities. Although in radiation dosimetry at the macroscopic level the deposition of energy can be treated non-stochastically, the quantities defined by the ICRU, most

recently in ICRU 85a (2011), encompass the treatment of energy deposition for radiobiological studies at the cellular level, a field known as *microdosimetry*, and are therefore stochastic at the basic level.

4.6.1 Quantities related to the transfer of energy

The stochastic quantity *energy transferred*, $\varepsilon_{\mathrm{tr}}$ (Attix 1979, 1983), can be defined in terms of the *radiant energy R*. The energy transferred in a volume ν is

$$\varepsilon_{\mathrm{tr}} = (R_{\mathrm{in}})_{\mathrm{u}} - (R_{\mathrm{out}})_{\mathrm{u}}^{\mathrm{non-r}} + \sum \mathsf{Q} \tag{4.43}$$

where $(R_{\mathrm{in}})_{\mathrm{u}}$ is the radiant energy of uncharged particles entering ν, $(R_{\mathrm{out}})_{\mathrm{u}}^{\mathrm{non-r}}$ is the radiant energy of uncharged particles leaving ν, excluding the energy that originates from *radiative losses* (of kinetic energy) by charged particles while in ν, and $\sum \mathsf{Q}$ is the total decrease in the rest energies of the nuclei and elementary particles in the volume. By radiative losses, we refer to conversion of charged-particle kinetic energy to photon energy, through bremsstrahlung production or in-flight annihilation of positrons. Note that in the latter case only the *kinetic energy* possessed by the positron at the instant of annihilation is classed as radiative energy loss. In addition to this, the pair of annihilation photons carry away $2\,m_{\mathrm{e}}c^2$ (1.022 MeV) of rest energy, which appears not only in the radiant energy leaving ν but also in $\sum \mathsf{Q}$.

Upon consideration of Eq. (4.43) it will be seen that energy transferred is just the kinetic energy received by charged particles in ν from uncharged particles, regardless of where or how this energy is expended. We note that, according to this definition, any *kinetic energy passed from one charged particle to another* (e.g. through the creation of a knock-on electron) in ν does not enter in $\varepsilon_{\mathrm{tr}}$. Examples of the application of Eq. (4.43) are given in the Addendum to this chapter.

For uncharged particles we can define the quantity *kerma* K as the quotient of $\mathrm{d}E_{\mathrm{tr}}$ by $\mathrm{d}m$

$$K = \frac{\mathrm{d}E_{\mathrm{tr}}}{\mathrm{d}m} \tag{4.44}$$

where $\mathrm{d}E_{\mathrm{tr}}$ is the mean sum of the initial kinetic energies of all the charged particles liberated in a mass $\mathrm{d}m$ of a material of volume ν by the uncharged particles incident on ν, i.e. $\mathrm{d}E_{\mathrm{tr}}$ is the expectation value of the energy transferred in the finite volume ν. This is a non-stochastic quantity. The unit of K is the J kg^{-1}, which for kerma is given the special name gray (Gy). Note that the kerma includes the kinetic energy of charged particles produced by the relaxation of excited atoms or molecules (for example Auger electrons) and by nuclear de-excitation or disintegration. Kerma is an acronym for 'kinetic energy released per unit mass'.

A closely related quantity, the *electronic kerma*[5], K_{el}, includes only that component of the kerma that results in local energy deposition (Attix 1979)

$$K_{\mathrm{el}} = K(1-g) \tag{4.45}$$

where g is the radiative fraction as presented in Section 4.5.2 in the context of the interaction coefficients μ_{tr} and μ_{en}. The quantity *radiative kerma*, K_{rad}, can also be defined such that

$$K = K_{\mathrm{el}} + K_{\mathrm{rad}} \tag{4.46}$$

The nature of the electronic kerma and its relation to absorbed dose is discussed in Sections 4.9 and 4.10. It will be convenient when discussing the concept of charged-particle equilibrium (CPE) in these sections to define the electronic kerma (K_{el}) in a manner corresponding to that employed for kerma K in Eqs. (4.43) and (4.44).

[5]The quantity *electronic kerma* is also known as *collision kerma*, K_{col}. The change of name from 'collision' to 'electronic' is consistent with the change from collision stopping power, S_{col}, to electronic stopping power, S_{el}, as discussed in Chapter 2 and in ICRU 85a (ICRU 2011).

Let $\varepsilon_{\rm tr}^{\rm n}$ be the related stochastic quantity *net energy transferred*, which is defined for a volume ν as

$$\varepsilon_{\rm tr}^{\rm n} = (R_{\rm in})_{\rm u} - (R_{\rm out})_{\rm u}^{\rm non-r} - R_{\rm u}^{\rm r} + \sum \mathsf{Q} = \varepsilon_{\rm tr} - R_{\rm u}^{\rm r} \tag{4.47}$$

where the new term $R_{\rm u}^{\rm r}$ is the radiant energy emitted (as radiative losses) by any charged particles liberated in ν, regardless of where the radiative-loss events occur. This equation is identical to Eq. (4.43) except for the inclusion of the term $R_{\rm u}^{\rm r}$. Thus $\varepsilon_{\rm tr}$ and the kerma K include energy that goes into radiative losses, while $\varepsilon_{\rm tr}^{\rm n}$ and the electronic kerma $K_{\rm el}$ do not.

Now in analogy with Eq. (4.43) we can define $K_{\rm el}$ as

$$K_{\rm el} = \frac{{\rm d}E_{\rm tr}^{\rm n}}{{\rm d}m} \tag{4.48}$$

where ${\rm d}E_{\rm tr}^{\rm n}$ is the expectation value of the *net* energy transferred in a mass ${\rm d}m$.

Closely related to kerma and defined specifically for photon interactions in air is the quantity *exposure*, X, defined as the quotient of ${\rm d}q$ by ${\rm d}m$

$$X = \frac{{\rm d}q}{{\rm d}m} \tag{4.49}$$

where ${\rm d}q$ is the absolute value of mean total charge of the positive or negative ions produced when all of the charged particles liberated by the photons incident on a mass ${\rm d}m$ of dry air are completely stopped in dry air. The unit of exposure is C kg^{-1}.

Although at first glance the exposure appears to be the charge equivalent of the kerma in air, there is one important difference; the exposure relates only to the ions produced by the charged particles directly emitted by the photons and therefore does not include subsequent charge produced by radiative photons. Kerma and exposure therefore differ by the factor $(1-g)$. It follows from Eq. (4.42) that exposure and *electronic kerma* are related by

$$X = (K_{\rm el})_{\rm air} \left(\frac{e}{W}\right)_{\rm air} \tag{4.50}$$

Although essentially superseded by kerma, exposure is a useful quantity in understanding the principle of operation of the free-air chamber primary standard for x rays, as presented in Chapter 11.

A quantity that might be regarded as the equivalent to the electronic kerma for charged particles is the *cema*, C, an acronym for 'converted energy per unit mass' (Kellerer and Rossi 1990; Kellerer *et al.* 1992); it is defined as the quotient of ${\rm d}E_{\rm el}$ by ${\rm d}m$

$$C = \frac{{\rm d}E_{\rm el}}{{\rm d}m} \tag{4.51}$$

where ${\rm d}E_{\rm el}$ is the mean energy lost in electronic interactions in a mass ${\rm d}m$ of a material by the primary charged particles, excluding secondary (or knock-on) electrons, incident on ${\rm d}m$. The unit of cema is J kg^{-1}, for which the special name gray (Gy) is used. Note that, according to its definition, cema includes only an electronic component, i.e., radiative energy losses are excluded.

The reference to electronic interactions pertains to the electronic (or collision) stopping power, $S_{\rm el}$, presented in Chapter 2 and in Section 4.5.3. The exclusion of secondary electrons is a subtle point that avoids 'double counting' of their energy. The fluence of charged particles incident on ${\rm d}m$, which might include primary electrons, primary or secondary protons, alpha particles, ions produced in nuclear interactions, etc, liberates secondary electrons whose kinetic energy (as well as their binding energy) comprises ${\rm d}E_{\rm el}$. So although the fluence incident on ${\rm d}m$ will also include secondary electrons liberated outside of ${\rm d}m$, these must be excluded from the fluence used to evaluate the cema in ${\rm d}m$ because their energy has already contributed to the cema at the point outside of ${\rm d}m$ where they were liberated. It follows that the cema can only be meaningfully related to the local energy deposition in situations where the secondary electron fluence entering ${\rm d}m$ is the same as that leaving ${\rm d}m$ – the charged-particle equilibrium (CPE)

discussed in Section 4.9. While not possible in practice, the distinction between primary and secondary electrons can be made, for example, in a Monte Carlo simulation (Chapter 8).

In addition to cema the concept of *restricted cema* has been proposed by Kellerer and Rossi (1990) and Kellerer *et al.* (1992). Restricted cema only includes energy lost in electronic interactions up to energy Δ, i.e. all knock-on electrons with energies greater than Δ are excluded (strictly, greater than $\Delta - U$). The connection between cema, restricted cema and absorbed dose is explored in Section 4.7.3.

4.6.2 Quantities related to the deposition of energy

The stochastic quantity *energy deposit*, ε_i, is the energy deposited in a single interaction i

$$\varepsilon_i = \varepsilon_{\mathrm{in}} - \varepsilon_{\mathrm{out}} + \mathsf{Q} \tag{4.52}$$

where $\varepsilon_{\mathrm{in}}$ is the energy of the incident ionizing particle, excluding rest energy, $\varepsilon_{\mathrm{out}}$ is the sum of the energies of all the ionizing particles leaving the interaction, again excluding rest energy, and Q is the total decrease in the rest energies of the nucleus and all elementary particles involved in the interaction (that is, $\mathsf{Q} > 0$ for a decrease in total rest energy and $\mathsf{Q} < 0$ for an increase). The unit of ε_i is J and may also be expressed in eV. The restriction of $\varepsilon_{\mathrm{out}}$ to ionizing particles means that ε_i is overestimated to the extent that non-ionizing particles can leave the interaction. In effect, the dimensions over which their energy is reabsorbed through non-ionizing processes defines a spatial region associated with a single interaction.

The *energy imparted*, ε, to the matter in a given volume is the sum

$$\varepsilon = \sum_i \varepsilon_i \tag{4.53}$$

of all the energy deposits ϵ_i in the volume. Dividing this quantity by the mass m of the volume, we obtain the *specific energy* (imparted), z

$$z = \frac{\varepsilon}{m} \tag{4.54}$$

The unit of z is the J kg^{-1} with the special name gray (Gy). Both ε and z are stochastic quantities; their non-stochastic counterparts are defined in what follows.

The *mean energy imparted*, $\bar{\varepsilon}$, in a volume is defined as

$$\bar{\varepsilon} = R_{\mathrm{in}} - R_{\mathrm{out}} + \sum \mathsf{Q} \tag{4.55}$$

where R_{in} is the mean total radiant energy of the ionizing particles entering the volume, R_{out} is the mean total radiant energy of the ionizing particles leaving the volume and $\sum \mathsf{Q}$ is the mean total decrease in the rest energies of the nuclei and elementary particles in the volume (that is, $\mathsf{Q} > 0$ for a decrease in total rest energy and $\mathsf{Q} < 0$ for an increase). Note that $\bar{\varepsilon}$ was introduced in Section 4.5.4 for the definition of radiation chemical yield.

The *mean specific energy*, $\bar{z}$, is the quotient of $\bar{\varepsilon}$ by m

$$\bar{z} = \frac{\bar{\varepsilon}}{m} \tag{4.56}$$

where $\bar{\varepsilon}$ is the mean energy imparted in the volume of mass m.

The non-stochastic quantity *absorbed dose*, D, is the quotient of $\mathrm{d}\bar{\varepsilon}$ by $\mathrm{d}m$

$$D = \frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}m} \tag{4.57}$$

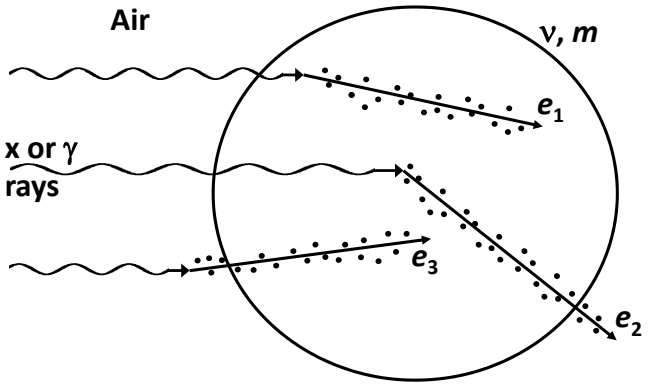


Figure 4.5: **Clarification on the distinction between absorbed dose and kerma by reference to volume ν. The kinetic energy of the secondary electron e_1 created by the uppermost photon is included in the kerma, as is that of the electron e_2 that leaves the volume ν, because this electron was liberated *inside* ν. The electron e_3 created *outside* ν does not contribute at all to the kerma. Only the energy imparted (indicated by the small dots) along the tracks of the electrons *inside* ν can contribute to the absorbed dose. It is evident that for the absorbed dose to be equal to the kerma the energy *entering* ν needs to be exactly equal to the energy *leaving* ν, i.e., charged-particle equilibrium (see Section 4.9).**

where $\mathrm{d}\bar{\varepsilon}$ is the mean energy imparted by ionizing radiation to matter of mass $\mathrm{d}m$. The unit of D is J kg^{-1} with the special name gray (Gy).[6] Mathematically, D is a point quantity representing the limit of the mean specific energy as the mass tends to zero.

From the definitions of these quantities it is clear that kerma relates to the energy given to secondary electrons at the point of liberation, while absorbed dose relates to the energy lost by electrons along their tracks. A clarification on the difference between kerma and absorbed dose is shown in Fig. 4.5.

4.6.3 Summary of the definitions of fundamental dosimetric quantities

Figure 4.6 summarizes the definitions of the fundamental radiation dosimetry quantities, covered in Sections 4.3 and 4.6, according to ICRU Report 84a (2011).

4.7 Relationships between field and dosimetric quantities

A number of important quantities have been defined in the preceding sections, in particular the radiation field quantities particle fluence, Φ, and fluence differential in energy, Φ_E, the energy-conversion quantities kerma, K, electronic kerma, K_{el}, cema, C, restricted cema, C_Δ, and the quantity concerning energy deposition, absorbed dose D. In this section mathematical relationships are developed between photon fluence and kerma, and between charged-particle fluence and cema. The final step, the establishment of relationships between fluence and absorbed dose, requires certain types of radiation equilibrium which are discussed in Sections 4.8 and 4.9.

[6]When the particle energies are given in eV, absorbed dose may be expressed in eV g^{-1}, where 1 eV g^{-1} $= 1.6022 \times 10^{-16}$ Gy.

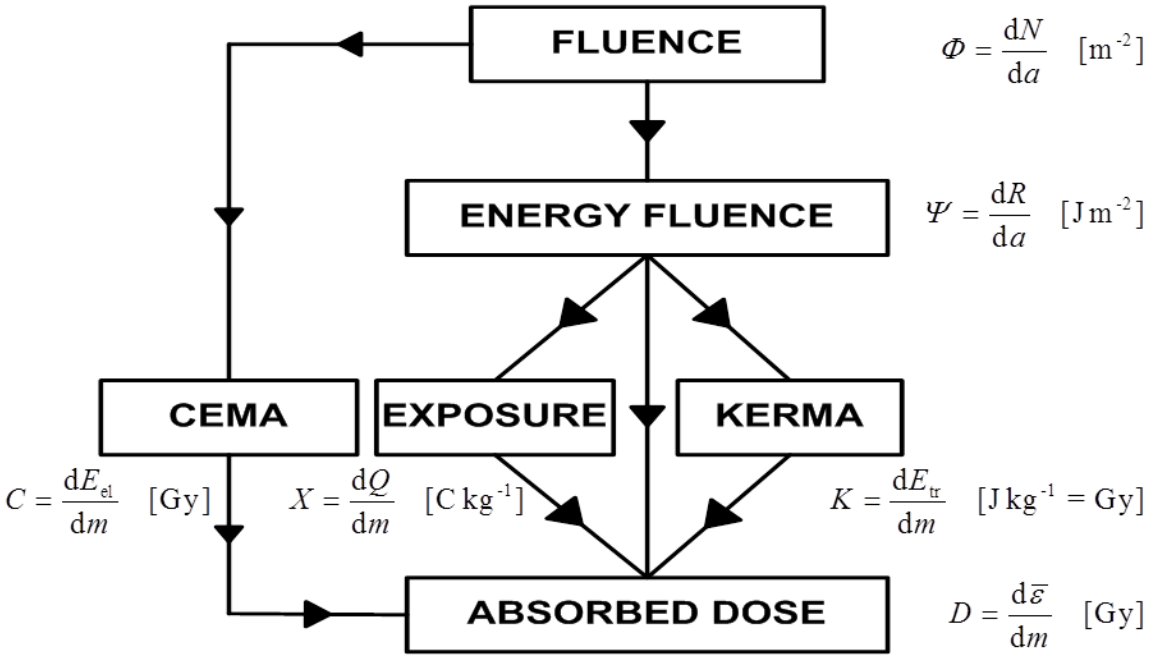


Figure 4.6: **Summary of the definitions of fundamental radiation dosimetry quantities according to ICRU Report 84a (2011).**

4.7.1 Photons

Consider a photon field, shown schematically in Fig. 4.7 by several photon tracks, each of energy k, crossing a small volume ν of material *med*, which has mass dm and density ρ.

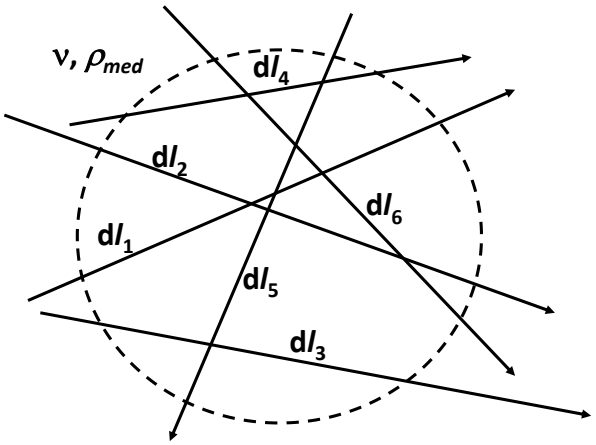


Figure 4.7: **A schematic representation of photon tracks, each of energy k, crossing a volume ν of material *med*.**

In order to calculate the total energy 'liberated' in the form of kinetic energy (E) of charged particles in this volume, we recall the definition of the mass-energy transfer coefficient, Eq. (4.22), but write this in terms of N photons of energy k transferring a mean energy $N\,\mathrm{d}E_{\mathrm{tr}}$

$$\frac{\mu_{\mathrm{tr}}}{\rho} = \frac{1}{\rho\,\mathrm{d}l}\,\frac{N\,\mathrm{d}E_{\mathrm{tr}}}{N\,k} \tag{4.58}$$

The mean energy transferred to charged particles by photon i of energy k with track length $\mathrm{d}l_i$ in the volume is therefore

$$\mathrm{d}E_{\mathrm{tr},i} = k\,\mathrm{d}l_i\,\mu_{\mathrm{tr}} \tag{4.59}$$

and the total energy transferred to charged-particle kinetic energy by all the particles in volume ν can be written

$$\sum_i \mathrm{d}E_{\mathrm{tr},i} = k\,\sum_i \mathrm{d}l_i\,\mu_{\mathrm{tr}} \tag{4.60}$$

Dividing both sides by the mass $\mathrm{d}m$ of the volume, and replacing $\mathrm{d}m$ by $\rho\,\mathrm{d}\nu$ on the r.h.s. we obtain

$$\frac{\sum_i \mathrm{d}E_{\mathrm{tr},i}}{\mathrm{d}m} = k\,\frac{\sum_i \mathrm{d}l_i\,\mu_{\mathrm{tr}}}{\rho\,\mathrm{d}\nu} = k\,\left[\frac{\sum_i \mathrm{d}l_i}{\mathrm{d}\nu}\right]\,\frac{\mu_{\mathrm{tr}}}{\rho} \tag{4.61}$$

Recognizing that firstly the quantity on the l.h.s. is kerma, secondly that the sum of track lengths divided by volume is fluence (Section 4.3.3), and adding the subscript *med* to indicate that all these quantities are specific to the material in volume ν, we have

$$K_{\mathrm{med}} = k\,\Phi_{\mathrm{med}}\left[\frac{\mu_{\mathrm{tr}}\,(k)}{\rho}\right]_{\mathrm{med}} \tag{4.62}$$

Alternatively, replacing $k\,\Phi$ by the symbol for energy fluence, Ψ

$$K_{\mathrm{med}} = \Psi_{\mathrm{med}}\,\left[\frac{\mu_{\mathrm{tr}}\,(k)}{\rho}\right]_{\mathrm{med}} \tag{4.63}$$

The above relationship applies to monoenergetic photons. It is straightforward to generalize this to a polyenergetic photon fluence spectrum, described by the fluence differential in energy Φ_k

$$K_{\mathrm{med}} = \int_0^{k_{\max}} k\,[\Phi_k]_{\mathrm{med}}\,[\mu_{\mathrm{tr}}(k)/\rho]_{\mathrm{med}}\,\mathrm{d}k \tag{4.64}$$

If the electronic kerma, K_{el}, is required, then following Eqs. (4.24) and (4.45) we simply substitute the mass energy-*transfer* coefficient for the mass energy-*absorption* coefficient, yielding

$$(K_{\mathrm{el}})_{\mathrm{med}} = \int_0^{k_{\max}} k\,[\Phi_k]_{\mathrm{med}}\,[\mu_{\mathrm{en}}(k)/\rho]_{\mathrm{med}}\,\mathrm{d}k \tag{4.65}$$

The above expressions do not involve any approximations – in this sense they are 'exact'. They refer solely to the *transfer* of energy from a photon field, characterized by the fluence Φ_k, to the kinetic energy of charged particles; the subsequent transport of this energy is not involved here and no assumptions are therefore necessary regarding any kind of equilibrium. Bouchard (2012) has derived the above expressions in a more rigorous manner, starting from the Boltzmann transport equation.

4.7.2 Neutrons

Equations (4.62)-(4.64) could be equally well applied to neutrons but generally neutron fields are described in terms of the neutron fluence (and flux), but not in terms of energy fluence as is often the case for photons. Thus, for monoenergetic neutrons one uses the following relation for kerma

$$K_{\mathrm{med}} = E_{\mathrm{n}}\, \Phi_{\mathrm{med}} \left[\frac{\mu_{\mathrm{tr}}\left(E_{\mathrm{n}}\right)}{\rho}\right]_{\mathrm{med}} \tag{4.66}$$

where E_{n} is the neutron kinetic energy in MeV and Φ_{med} is the fluence of monoenergetic neutrons of energy E_{n} in neutrons m^{-2}. Rather than providing data for neutron mass energy-transfer coefficients, $\left[\mu_{\mathrm{tr}}\left(E_{\mathrm{n}}\right)/\rho\right]_{\mathrm{med}}$, tabulated data are given for the neutron kerma-to-fluence ratio, K/Φ, a quantity known as the *fluence-to-kerma conversion coefficient*, or briefly *kerma coefficient* [7], which is defined as

$$\kappa_{\Phi} = \frac{K}{\Phi} = E_{\mathrm{n}}\, \frac{\mu_{\mathrm{tr}}}{\rho} \tag{4.67}$$

and is given in units of fGy m^2 [femto (f)= 10^{-15}; gray (Gy) = J kg^{-1}]. Tabulations of k_{Φ} for a wide range of neutron energies and materials have been published by ICRU in Report 63 (ICRU 2000), based on the compilation and analysis of neutron cross sections and kerma coefficients by Chadwick *et al.* (1999), which superseded that of Caswell *et al.* (1980, 1982). An extract of those tables is included in the Data Tables. There are two main uses of kerma coefficients in neutron dosimetry, both requiring charged-particle equilibrium (see below): (a) if the neutron fluence spectrum is known from experiment or calculation, the absorbed dose in a given material can be derived from the kerma coefficient; and (b) for a given neutron fluence spectrum, the absorbed dose determined in one material (e.g., water) can be converted to the absorbed dose in another material (e.g., tissue) using the ratio of kerma coefficients.

Equation (4.66) is therefore usually written as

$$K_{\mathrm{med}} = \Phi_{\mathrm{med}}\, \kappa_{\Phi}\left(E_{\mathrm{n}}\right)_{\mathrm{med}} \tag{4.68}$$

and, as in the case of photons, for neutrons with a fluence spectrum differential in energy $\Phi_{E_{\mathrm{n}}}$, this expression is replaced by

$$K_{\mathrm{med}} = \int_0^{E_{\mathrm{n,max}}} \Phi_{E_{\mathrm{n}}}\, \kappa_{\Phi}\left(E_{\mathrm{n}}\right)_{\mathrm{med}}\, \mathrm{d}E_{\mathrm{n}} \tag{4.69}$$

where $\Phi_{E_{\mathrm{n}}}$ is in units of neutrons m^{-2} MeV^{-1}, κ_{Φ} represents tabulated neutron kerma-coefficient values in fGy m^2 neutron^{-1}, and $\mathrm{d}E_{\mathrm{n}}$ is expressed in MeV; hence, K_{med} is in units of fGy.

Given a neutron fluence spectrum $\Phi_{E_{\mathrm{n}}}$, a fluence-weighted average value of the fluence-to-kerma conversion coefficient (or kerma coefficient) for a given medium can be determined from

$$\bar{\kappa}_{\Phi} = \frac{K}{\Phi} = \frac{\int_{E_{\mathrm{n}}} \Phi_{E_{\mathrm{n}}}\, \kappa_{\Phi}\left(E_{\mathrm{n}}\right)_{\mathrm{med}}\, \mathrm{d}E_{\mathrm{n}}}{\int_{E_{\mathrm{n}}} \Phi_{E_{\mathrm{n}}}\, \mathrm{d}E_{\mathrm{n}}} \tag{4.70}$$

It should be noted that the neutron kerma coefficient described so far should strictly be called *total kerma coefficient* and, as described by Chadwick *et al.* (1999), its theoretical determination is partitioned into *partial* contributions from the different charged particle types generated by neutron interactions, in particular from the light ($A \leq 4$) secondary charged particles, the nonelastic recoils ($A > 4$), and the

[7] Note the use of the term 'kerma coefficient', instead of the common 'kerma factor', since ISO (1992) recommends that 'coefficient' should be reserved for a multiplier having dimensions, whereas 'factor' should be used for dimensionless multipliers.

elastic recoils (A refers here to the atomic mass number or nucleon number). Thus, κ_Φ is determined from the microscopic cross sections according to

$$\kappa_\Phi = C \sum_j N_j \sum_i \bar{Q}_{ij}(E_\mathrm{n})\ \sigma_{ij}(E_\mathrm{n}) \tag{4.71}$$

where N_j is the number of target nuclides of type j per unit mass in the material (kg^{-1}), $\bar{Q}_{ij}$ is the average neutron energy transferred to kinetic energy of charged particles of type i (MeV), and σ_{ij} is the total production cross section for particle type i (barns). The factor $C = 9.64853 \times 10^{-15}/M_A$, where M_A is the atomic mass of the target in units of u, converts the kerma coefficient from units of MeV b to units of Gy m^2; it is the product of the three factors J MeV^{-1} (1.6022×10^{-13}), m^2 b^{-1} (10^{-28}) and the number of atoms per kg (6.022×10^{26}).

For each particle type, the partial kerma coefficient κ_Φ^i is the kerma coefficient for secondary particles of the type i

$$\kappa_\Phi^i = C \sum_j N_j \bar{Q}_{ij}(E_\mathrm{n}) \sigma_{ij}(E_\mathrm{n}) \tag{4.72}$$

where the symbols have the same meaning as above.

Numerical values and plots of kerma coefficients for several elements, human tissues and 'tissue-equivalent' materials are given in Chapter 19 and in the Data Tables.

4.7.3 Charged particles

Consider now a charged-particle field, specifically an electron field, consisting of several electron tracks crossing a small volume ν of material *med*, which has mass $\mathrm{d}m$ and density ρ (Fig. 4.8). The difference from the photon field (Fig. 4.7) is that the electrons lose energy and change direction quasi-continuously (see Chapter 2), and therefore the volume in Fig. 4.8 is assumed to be sufficiently small that the energy loss and directional change are small as the electrons traverse the volume.

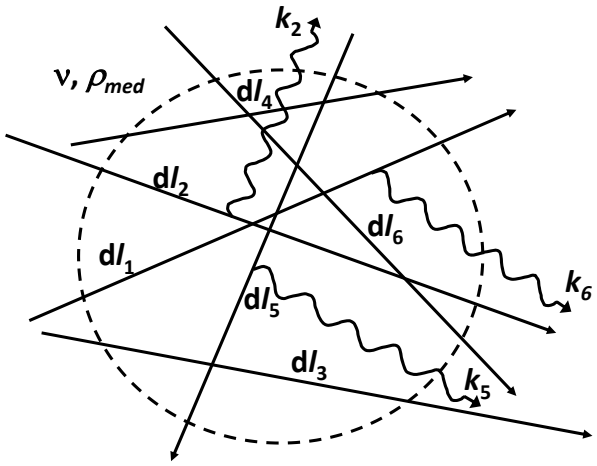


Figure 4.8: **A schematic representation of electron tracks crossing a small volume ν, of material *med* with density ρ; it is assumed that the mean chord length across the volume is small compared to the mean free path of the electrons (see Chapter 2). Electron tracks 2, 5 and 6 show the emission of bremsstrahlung photons.**

In order to calculate the total energy lost by the incident charged particles in this volume, we recall the definition of the mass electronic stopping power, Eq. (4.28), but write this in terms of N incident

electrons each losing energy $\mathrm{d}E_{\mathrm{loss}}$ in the volume

$$\frac{S_{\mathrm{el}}}{\rho} = \frac{1}{\rho\,\mathrm{d}l}\frac{N\,\mathrm{d}E_{\mathrm{loss}}}{N} \tag{4.73}$$

The energy lost by electron i with track length $\mathrm{d}l_i$ in the volume is therefore

$$\mathrm{d}E_{\mathrm{loss},i} = \mathrm{d}l_i\,S_{\mathrm{el}} \tag{4.74}$$

and the total energy lost by all the particles in volume ν can be written

$$\sum_i \mathrm{d}E_{\mathrm{loss},i} = \sum_i \mathrm{d}l_i\,S_{\mathrm{el}} \tag{4.75}$$

Dividing both sides by the mass $\mathrm{d}m$ of the volume, and replacing $\mathrm{d}m$ by $\rho\,\mathrm{d}\nu$ on the right-hand side

$$\frac{\sum_i \mathrm{d}E_{\mathrm{loss},i}}{\mathrm{d}m} = \frac{\sum_i \mathrm{d}l_i\,S_{\mathrm{el}}}{\rho\,\mathrm{d}\nu} = \left[\frac{\sum_i \mathrm{d}l_i}{\mathrm{d}\nu}\right]\frac{S_{\mathrm{el}}}{\rho} \tag{4.76}$$

Recognizing firstly that the term on the left-hand side is *cema*, secondly that the sum of track lengths divided by volume is the average fluence of incident (primary) electrons, and adding the subscript 'med' to indicate that all these quantities are specific to the material in volume $\mathrm{d}\nu$, we arrive at

$$C_{\mathrm{med}} = \Phi^{\mathrm{prim}}_{\mathrm{med}}\,[S_{\mathrm{el}}(E)/\rho]_{\mathrm{med}} \tag{4.77}$$

This relationship applies to monoenergetic electrons or charged particles in general. It is straightforward to generalize it to a charged-particle fluence spectrum, described by Φ^{prim}_E

$$C_{\mathrm{med}} = \int_0^{E_{\max}} \left[\Phi^{\mathrm{prim}}_E\right]_{\mathrm{med}} [S_{\mathrm{el}}(E)/\rho]_{\mathrm{med}}\,\mathrm{d}E \tag{4.78}$$

As noted in the context of Eq. (4.51), the charged-particle fluence Φ^{prim}_E excludes secondary electrons generated outside of the volume ν.

We consider also the *restricted cema*, corresponding to the energy loss per unit mass excluding energy losses above a certain cut-off Δ

$$C_{\mathrm{med},\Delta} = \int_\Delta^{E_{\max}} \left[\Phi^{\mathrm{tot}}_E\right]_{\mathrm{med}} [L_\Delta(E)/\rho]_{\mathrm{med}}\,\mathrm{d}E + \mathrm{TE} \tag{4.79}$$

where the primary fluence of Eq. (4.78) is replaced by the total electron fluence, Φ^{tot}_E, the electronic stopping power S_{el} is replaced by the linear energy transfer L_Δ (see Section 4.5.3), and TE is the so-called *track-end* term, which corresponds to the energy deposited 'on the spot' by electrons as they drop below Δ in energy.[8] We will return to these concepts in relation to cavity theory in Chapter 9.

Similarly to Eqs. (4.62) to (4.65) the above expressions do not involve any approximations. They refer to the *loss* of energy from a charged-particle field, characterized by fluence Φ_E, to ionizations and excitations of the medium and to the kinetic energy of secondary electrons.

4.8 Radiation equilibrium (RE)

The concepts of *radiation equilibrium* (RE) and *charged-particle equilibrium* (CPE) are extremely useful in radiation physics as a means of inter-relating certain basic quantities. RE enables absorbed dose to be

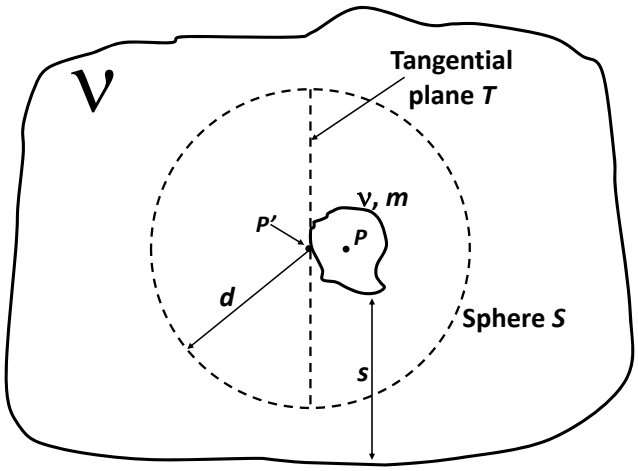


Figure 4.9: **Radiation equilibrium (RE). Extended volume $\mathcal{V}$ contains a homogeneous medium and a homogeneous isotropic source distribution. Radiation equilibrium will exist in the smaller internal volume ν if the maximum distance of penetration d of primary particles plus their secondaries is less than the minimum separation s of ν from the boundary of $\mathcal{V}$. Neutrinos are ignored.**

equated to the net rest mass that is converted to energy, per unit mass at the point of interest, while CPE (discussed in Section 4.9) enables absorbed dose D to be related to electronic (or collision) kerma.

Consider an extended volume $\mathcal{V}$, as in Fig. 4.9, containing a distributed radioactive source. A smaller internal volume ν exists about a point of interest, P. Here $\mathcal{V}$ is required to be large enough so that the maximum distance of penetration d of any emitted radiation (excluding neutrinos) and its progeny (i.e., scattered and secondary particles) is less than the minimum separation s of the boundaries of $\mathcal{V}$ and ν. The radiation is assumed to be emitted isotropically.

If the following four conditions exist throughout $\mathcal{V}$, it will be shown that *radiation equilibrium* exists for the volume ν (in the non-stochastic limit):

(a) The atomic composition of the medium is homogeneous.

(b) The density of the medium is homogeneous.[9]

(c) The radioactive source is uniformly distributed.

(d) There are no external electric or magnetic fields present to perturb the charged-particle paths.

The type of radioactive material distributed in the medium need not be specified for purposes of the present argument. However, it should be pointed out here that all radioactive sources derive their emitted energy from a reduction in the rest mass of the atoms involved, through Einstein's mass-energy equation $E = m\,c^2$, where c is the velocity of light in vacuum. In the case of positive or negative β-ray emission, a large part of this energy is carried away kinetically by the associated neutrinos. Because of their exceedingly small interaction probability, they can carry their kinetic energy through thousands of kilometres of matter. For this reason the size requirement on the volume $\mathcal{V}$ ignores neutrinos.

[8]Note that ICRU Report 85a (2011) does not include the *track-end* term in its expression for the *restricted cema*. The term was, however, included in the Kellerer *et al.* (1992) definition of this dosimetric quantity and in the ICRU Report 90 (2016).

[9]The Fano (1954) theorem (Chapter 9) states that the density of the medium need not be homogeneous if the source strength per unit mass is homogeneous. However, this theorem lacks rigour in the presence of the polarization or density effect (see Chapter 2). Hence for generality we require condition (b).

As we will see in Chapter 6, other uncharged particles (photons and neutrons) are attenuated in matter more or less exponentially; hence they do not have a true 'range' beyond which no particle will penetrate. However, one can (in principle, at least) require $\mathcal{V}$ to be large enough to achieve a negligible number of particles penetrating from its boundaries to reach ν.

Imagine now a plane T (see Fig. 4.9) that is tangential to the volume ν at a point P' on its surface, and consider the particles crossing the plane per unit area at that point. In the non-stochastic limit there will be perfect reciprocity of particles (see the pair of arrows in the figure) of each type and energy crossing both ways, since the radioactive source distribution within the sphere S of radius d about point P' is perfectly symmetrical with respect to plane T. This will be true for all possible orientations of tangent planes around the volume ν; hence one can say that, *in the non-stochastic limit, for each type and energy of particle entering ν, another identical particle leaves*. This condition is called *radiation equilibrium* (RE) with respect to ν.

Recalling Eq. (4.55), we can write a more general expression for the mean energy imparted

$$\bar{\varepsilon} = \left(\bar{R}_{\mathrm{in}}\right)_{\mathrm{u}} - \left(\bar{R}_{\mathrm{out}}\right)_{\mathrm{u}} + \left(\bar{R}_{\mathrm{in}}\right)_{\mathrm{c}} - \left(\bar{R}_{\mathrm{out}}\right)_{\mathrm{c}} + \sum \bar{\mathrm{Q}} \tag{4.80}$$

where 'u' and 'c' stand for uncharged and charged ionizing radiation, respectively, and the bars over the different quantities represent their expectation values. As a consequence of radiation equilibrium the following equalities of expectation values exist

$$(\bar{R}_{\mathrm{in}})_{\mathrm{u}} = (\bar{R}_{\mathrm{out}})_{\mathrm{u}} \tag{4.81}$$

and

$$(\bar{R}_{\mathrm{in}})_{\mathrm{c}} = (\bar{R}_{\mathrm{out}})_{\mathrm{c}} \tag{4.82}$$

that is, the energy carried in and that carried out of ν are balanced for both uncharged charged particles. The mean energy imparted, Eq. (4.80), can then be simplified to

$$\bar{\varepsilon} = \sum \bar{\mathrm{Q}} \tag{4.83}$$

which means that under RE conditions the expectation value of the mean energy imparted to the matter in the volume ν is equal to that emitted by the radioactive material in ν, excluding that given to neutrinos.

Because we are dealing with the non-stochastic case, for which the concept of radiation equilibrium is of practical interest, the volume ν may be reduced to an infinitesimal $\mathrm{d}\nu$ about point P, and RE may then be said to exist at that point. The absorbed dose at P will then be given by Eq. (4.57) , $D = \mathrm{d}\bar{\varepsilon}/\mathrm{d}m$, where $\bar{\varepsilon}$ is equal to $\Sigma\bar{\mathrm{Q}}$ in Eq. (4.83). Thus we can make the following statement:

> *If radiation equilibrium exists at a point in a radioactive material, the absorbed dose is equal to the expectation value of the energy released by the radioactive material per unit mass at that point, ignoring neutrinos.*

The concept of radiation equilibrium has practical importance especially in the field of nuclear medicine, where distributed radioactive sources may be introduced into the human body or other biological systems for diagnostic, therapeutic, or analytical purposes. The resulting absorbed dose at any given point in such circumstances depends on the size of the object relative to the radiation range and on the location of the point within the object. This will be discussed in Chapter 18, and references will be given to more detailed treatments.

A few additional comments about the above condition (d) for the existence of RE are called for. The absence of electric and magnetic fields from $\mathcal{V}$ allows the use of the simplest symmetry argument for proving that RE occurs, since radioactive point sources emit radiation isotropically. The presence of a homogeneous, constant magnetic and/or electric field throughout $\mathcal{V}$ makes the symmetry argument more difficult to visualize, since the flow of charged particles past a point such as P' will no longer be isotropic.

However, isotropicity is not a requirement for RE in the volume ν; it is merely necessary that the inward and outward flow of identical particles of the same energy be balanced for all particles present. Even if all the particles flow in one side of ν and out the other side, RE will still be obtained so long as the in- *versus* out-flow is balanced. Any source anisotropy, or distortion of charged-particle tracks, that is homogeneously present everywhere throughout $\mathcal{V}$, will have no perturbing effect on the existence of RE in ν.

This can be seen with the aid of Fig. 4.10. Consider an elemental volume dν at the point of interest P, and two other elemental volumes dν' and dν'' that are symmetrically positioned with respect to dν. We assume dν is located at a distance s from the boundary of volume $\mathcal{V}$ that is greater than the maximum range of radiation penetration, d. Throughout $\mathcal{V}$ both the medium and the distributed source are homogeneous, as in Fig. 4.9, but now we allow the presence of a homogeneous electric and/or magnetic field.

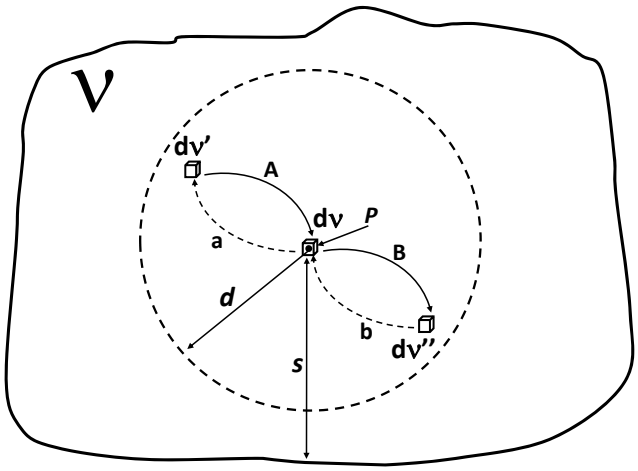


Figure 4.10: **Radiation equilibrium in homogeneous but anisotropic fields of radiation (see text).**

Assuming that radiation moves preferentially from left to right in Fig. 4.10, homogeneity and symmetry considerations require that the particles (A) travelling from dν' to dν are identical to those (B) travelling from dν to dν'', in the expectation-value limit. Likewise the lesser flow (b) of particles from dν'' to dν is identical to that (a) from dν to dν'. Consequently a+B = A+b, that is, the flow of particles from dν to dν' + dν'' is identical to that from dν' + dν'' to dν.

The pair of volumes dν' and dν'' can be moved to all possible symmetrical locations within $\mathcal{V}$, and their particle flows can be integrated. Locations outside the sphere of radius d about point P of course neither receive particles from nor contribute particles to dν. One may conclude from such an argument that each particle flowing out of dν is replaced by an identical particle flowing in. Thus RE exists at P.

This simple proof can also be viewed as an extension of Mayneord (1945) statement of the *reciprocity theorem* to homogeneous but *anisotropic* sources in infinite homogeneous media: The integral dose (i.e., expectation value of the energy imparted) in dν due to the sources in dν' + dν'' equals the integral dose in dν' + dν'' resulting from the source in dν. Thus the integral dose throughout $\mathcal{V}$ due to the source in dν equals the integral dose in dν resulting from the source throughout $\mathcal{V}$. The reciprocity theorem for unsealed radionuclide dosimetry is discussed in more detail in the addendum to Chapter 18.

Although it is obvious that Eq. (4.81) would not be perturbed by the presence of a magnetic or electric field, since photons and neutrons are not influenced, the dose resulting from uncharged particles is delivered by secondary charged particles and these *will* be affected. For example, Galbraith *et al.*

(1984), Mattsson and Svensson (1984), Thwaites (1984) and Anthony *et al.* (1986) have demonstrated that when electrons (from a beam) stop in an insulating medium, the resulting non-homogeneous electric field created by the trapped charges can distort the absorbed dose distribution deposited by a subsequent irradiation, whether of photons or electrons.

Finally it should not be forgotten that charged particles acquire kinetic energy from motion under the influence of an electric field, or a time-varying magnetic field. This extra energy, which appears as absorbed dose, is not a component of the true absorbed dose that would exist in the absence of the perturbing field. Fortunately, such spurious dose contributions are rarely large enough to be significant except for the case of electron multiplication in a gas counter or an ion chamber biased at excessive voltage (see Chapter 12).

4.9 Charged-particle equilibrium (CPE)

Charged-particle equilibrium exists for the volume ν if each charged particle of a given type and energy leaving ν is balanced by a particle of the same type and energy entering, in terms of expectation values (Attix, 1983). If CPE exists, Eq. (4.82) is naturally satisfied. Evidently, where radiation equilibrium exists, so does CPE. In other words, the existence of RE is a *sufficient* condition for CPE to exist. However, the practical importance of CPE stems from the fact that under certain conditions it can be adequately approximated even in the absence of RE. Two important cases will be considered in the following subsections.

4.9.1 CPE for distributed radioactive sources

(a) Let us first consider the trivial case where only charged particles are emitted and radiative losses are negligible. Again referring to Fig. 4.9, the dimension s is taken to be greater than the maximum range d of the particles. If the same four conditions (a–d) are satisfied throughout the volume $\mathcal{V}$ as required for RE in Section 4.8, then RE and CPE will both exist for the volume ν, since they are identical in this case. Fig. 4.10 applies likewise to CPE.

(b) Consider now the more interesting case where both charged particles and relatively more penetrating indirectly ionizing radiation are emitted. In Fig. 4.9 let the distance d be the maximum range of the *charged particles only*, and let $\mathcal{V}$ be just large enough so the minimum distance s separating $\mathcal{V}$ from ν exceeds d. If the indirectly ionizing particles are penetrating enough to escape from $\mathcal{V}$ without interacting significantly with the medium, then they will produce practically no secondary charged particles. Only the primary charged particles then need be considered in the symmetry argument as before, where again we assume conditions (a–d) throughout $\mathcal{V}$, as stated for radiation equilibrium. Since the passage of identical charged particles in and out of ν is thus seen to be balanced, CPE exists with respect to the primary charged particles, and Eq. (4.82) is satisfied.

However, RE is not attained, and consequently Eq. (4.81) is not satisfied, since $(\bar{R}_{\mathrm{out}})_{\mathrm{u}} > (\bar{R}_{\mathrm{in}})_{\mathrm{u}}$ for the volume ν This is evident from the fact that the indirectly ionizing particles that originate in ν and escape from $\mathcal{V}$ are not balanced by incoming particles, because there is no source outside of $\mathcal{V}$. The equation for the expectation value of the energy imparted in this case becomes, from Eq. (4.80)

$$\bar{\varepsilon} = (\bar{R}_{\mathrm{in}})_{\mathrm{u}} - (\bar{R}_{\mathrm{out}})_{\mathrm{u}} + \sum \bar{\mathrm{Q}} \tag{4.84}$$

$\Sigma\bar{\mathrm{Q}}$ is the energy emitted by the radioactive material, as charged and uncharged particles. The uncharged particles don't interact and therefore escape as part of $(\bar{R}_{\mathrm{out}})_{\mathrm{u}}$. The other part of $(\bar{R}_{\mathrm{out}})_{\mathrm{u}}$ is the bremsstrahlung produced by the charged particles. Because of the geometry, there is no $(\bar{R}_{\mathrm{in}})_{\mathrm{u}}$. So finally $\bar{\varepsilon} = \Sigma\bar{\mathrm{Q}} - (\bar{R}_{\mathrm{out}})_{\mathrm{u}}$. The average absorbed dose in ν is thus equal to $\bar{\varepsilon}$ from Eq. (4.84) divided by the mass in ν, for CPE conditions.

Suppose now that the size of the volume $\mathcal{V}$ occupied by the source is expanded so that distance s in Fig. 4.9 gradually increases from being merely equal to the charged-particle range to being greater than the effective range of the indirectly ionizing particles and their secondaries. That transition will cause the $(\bar{R}_{\rm in})_{\rm u}$ term in Eq. (4.84) to increase until it equals $(\bar{R}_{\rm out})_{\rm u}$ in value. Thus RE will be restored, according to the symmetry argument applied to all particles in Section 4.8. The mean energy imparted, as expressed by Eq. (4.84) for the CPE case, would hence be transformed into Eq. (4.83) for RE.

The calculation of the absorbed dose is evidently straightforward for either of these limiting cases (CPE or RE), but intermediate situations are more difficult to deal with, i.e., when the volume $\mathcal{V}$ is larger than necessary to achieve CPE in ν, but not large enough for RE. In that case some fraction of the energy of the indirectly ionizing radiation component will be absorbed, and it is relatively difficult to determine what that fraction is. This problem will be discussed in Chapter 18.

(c) Achieving CPE for the case of a distributed source of only indirectly ionizing radiation requires that RE also be present; hence the discussion in Section 4.8 applies.

4.9.2 CPE for external sources of uncharged particles

In Fig. 4.11 a volume $\mathcal{V}$ is shown, again containing a smaller volume ν. The boundaries of ν and $\mathcal{V}$ are required in this case to be separated by at least the maximum distance of penetration of any secondary charged particle present. If the following conditions are satisfied throughout $\mathcal{V}$, CPE will exist for the volume ν (in the non-stochastic limit):

(a) The atomic composition of the medium is homogeneous.

(b) The density of the medium is homogeneous[10].

(c) There exists a uniform *field* of indirectly ionizing radiation (i.e. the particles must be only negligibly attenuated by their passage through the medium).

(d) No inhomogeneous electric or magnetic fields are present.

It will be evident that the above assumed conditions are similar to those listed for radiation equilibrium in Section 4.8, except that a uniform field of uncharged particles throughout $\mathcal{V}$ replaces the uniform radioactive *source* in condition (c), and the separation of the boundary of $\mathcal{V}$ from that of ν now only has to exceed the secondary charged particle range rather than having to be larger than the range of the most penetrating radiation, which is usually the uncharged particle radiation present. Condition (d) was demonstrated by means of Fig. 4.10 to be a sufficient substitute for requiring the absence of electric or magnetic fields.

Because of the uniformity of the radiation field of uncharged particles and of the medium throughout $\mathcal{V}$, one can say that the number of charged particles produced per unit volume in each energy interval and element of solid angle will be uniform everywhere in $\mathcal{V}$ (in the non-stochastic limit). Here, however, the particles are not emitted isotropically unlike in the case of radioactive point sources. Neutron and photon interactions generally result in anisotropic angular distributions of secondary and scattered radiations, as seen in Chapter 3 dealing with such interactions. However this anisotropy will be homogeneous throughout $\mathcal{V}$. This condition, together with a uniform medium in which the charged particles can slow down throughout $\mathcal{V}$ (as guaranteed by conditions (a) and (b)) is sufficient to produce CPE for the volume ν, as shown by the reciprocity argument illustrated in 4.10.

This is further demonstrated in Fig. 4.11 for the simplified case of straight charged-particle tracks, all emitted at angle θ with respect to the mono-directional primary particles. Consider first the track of charged particle e_1, generated by the total absorption of an uncharged particle at a point P_1 just inside

[10]The remarks about the Fano theorem in the footnote in Section 4.8 also apply here, replacing 'source strength' by 'number and type of interactions.'

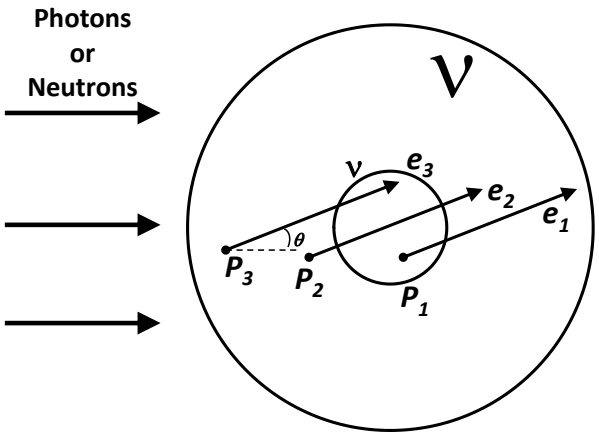


Figure 4.11: **Charged-particle equilibrium conditions for an external source. The large volume $\mathcal{V}$ contains a homogeneous medium, uniformly irradiated throughout by indirectly ionizing radiation (i.e., attenuation of the latter is assumed to be negligible). Secondary charged particles are thus produced uniformly throughout $\mathcal{V}$, not necessarily isotropically, but with the same directional and energy distribution everywhere. If the minimum distance separating the boundaries of $\mathcal{V}$ and the smaller internal volume ν is greater than the maximum range of charged particles present, CPE exists in ν (see text).**

the boundary of ν. Particle e_1 crosses ν and carries out of that volume a kinetic energy of, say $2/3$ of its original energy. A second identical interaction occurring at point P_2 generates charged particle e_2, which enters ν with $2/3$ of its original energy, and leaves with $1/3$ of that energy. Likewise a third identical interaction at P_3 generates charged particle e_3, which enters ν with $1/3$ of its original energy, and expends all of that energy in ν. Thus CPE exists in the non-stochastic limit, and the total kinetic energy expended in ν by the three particles is equal to that which e_1 alone would have expended if its entire track had remained inside ν.

Substituting Eq. (4.84) into Eq. (4.55) assuming the non-stochastic limit for the latter, we see that for CPE conditions

$$\bar{\varepsilon}^{\mathrm{n}}_{\mathrm{tr}} = \bar{\varepsilon} + (\bar{R}_{\mathrm{out}})_{\mathrm{u}} - (\bar{R}_{\mathrm{out}})^{\mathrm{non-r}}_{\mathrm{u}} - \bar{R}^{\mathrm{r}}_{\mathrm{u}} \tag{4.85}$$

However, under those same conditions we may also assume that any radiative interaction by a charged particle after it leaves ν will be replaced by an identical interaction inside ν, as shown in Fig. 4.11. Thus

$$(\bar{R}_{\mathrm{out}})_{\mathrm{u}} = (\bar{R}_{\mathrm{out}})^{\mathrm{non-r}}_{\mathrm{u}} + \bar{R}^{\mathrm{r}}_{\mathrm{u}} \tag{4.86}$$

provided that the volume ν is small enough to allow radiative-loss photons to escape, as shown in the same figure. For that case Eqs. (4.85) and (4.86) can be simplified to the equality

$$\bar{\varepsilon} = \bar{\varepsilon}^{\mathrm{n}}_{\mathrm{tr}} \tag{4.87}$$

Reducing ν to the infinitesimal volume $\mathrm{d}\nu$, containing mass $\mathrm{d}m$ about a point of interest P, we can write

$$\frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}m} = \frac{\mathrm{d}\bar{\varepsilon}^{n}_{\mathrm{tr}}}{\mathrm{d}m} \tag{4.88}$$

and hence

$$D_{\mathrm{med}} \overset{\mathrm{CPE}}{=} [K_{\mathrm{el}}]_{\mathrm{med}} \tag{4.89}$$

where the CPE above the equals sign emphasizes the dependence of the equality on that condition. Notice that since Eqs. (4.88) and (4.89) apply to an infinitesimal volume, the required equality of Eq. (4.66) is assured, since radiative-loss photons will certainly escape in that case.

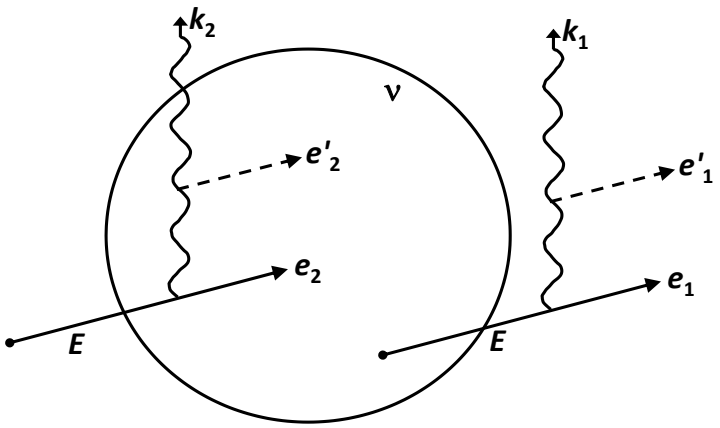


Figure 4.12: **Clarification of Eqs. (4.86) and (4.87). CPE exists (in the non-stochastic limit) because electron e_2 enters the volume ν with a kinetic energy E equal to that carried out by electron e_1. If e_1 then emits a photon of energy k_1, e_2 will also emit an identical photon with energy k_2 (on the average). If k_2 escapes from ν, then $(\bar{R}_{\rm out})_{\rm u} = k_2 = k_1 = \bar{R}^r_{\rm u}$, and since $(\bar{R}_{\rm out})^{\rm non-}_{\rm u}{}^r = 0$, Eq. (4.86) is satisfied. However, if k_2 is absorbed within ν, producing a secondary electron e'_2, then $(\bar{R}_{\rm out})_{\rm u} = 0$ but $\bar{R}^r_{\rm u}$ still equals k_1, and $(\bar{R}_{\rm out})^{\rm non-}_{\rm u}{}^r = 0$ as before, so Eq. (4.86) is no longer satisfied. Therefore Eq. (4.87) also is only valid for small enough volumes to allow the escape of radiative losses. Equations (4.88) and (4.89) will always be valid because the relevant volume is infinitesimal.**

Equation (4.89) demonstrates that under CPE conditions at a point in a medium, the absorbed dose is equal to the *electronic* kerma there. This is true irrespective of radiative losses[11]. Eq. (4.89) is a very important relationship, as it equates the measurable quantity dose D with the calculable quantity electronic kerma $K_{\rm el}$.

More explicitly, following Eqs.(4.88) and (4.89), for monoenergetic photons we have

$$D_{\rm med} \overset{\rm CPE}{=} [K_{\rm el}]_{\rm med} = \Phi_{\rm med}\, k \left[\frac{\mu_{\rm en}(k)}{\rho}\right]_{\rm med} \tag{4.90}$$

and for a photon fluence spectrum, described by Φ_k

$$D_{\rm med} \overset{\rm CPE}{=} [K_{\rm el}]_{\rm med} = \int_0^{k_{\rm max}} [\Phi_k]_{\rm med}\, k\, [\mu_{\rm en}(k)/\rho]_{\rm med}\, {\rm d}k \tag{4.91}$$

The above expressions are cornerstones of photon dosimetry theory; for example they play a central role in 'large detector' cavity theory (see Chapter 9).

When CPE conditions exist, and the electron fluence spectrum Φ_E produced by photons (or uncharged particles in general) is known, the absorbed dose can instead be determined using the concept of cema, e.g.

$$D_{\rm med} \overset{\rm CPE}{=} C_{\rm med} = \int_0^{E_{\rm max}} \left[\Phi_E^{\rm prim}\right]_{\rm med} [S_{\rm el}(E)/\rho]_{\rm med}\, {\rm d}E \tag{4.92}$$

where $\Phi_E^{\rm prim}$ corresponds to the first generation of electrons (or charged particles in general) produced by uncharged particles. A similar expression can be written in terms of the restricted cema if the total electron fluence spectrum (of all electron generations), $\Phi_E^{\rm tot}$, would be known; this requires the introduction of a cut-off energy Δ related to the volume within which energy deposition can be considered to be local or 'on the spot' (see Chapter 9)). Obviously, expressions (4.91) and (4.92) yield the same absorbed

[11] For a real detector that is not infinitesimal, the re-absorption of radiative losses might require correction, as for example evaluated by Burns (2012) for the measurement of air kerma in ^{60}Co γ radiation.

dose to the medium, a property sometimes used to verify the consistency of Φ_k and $\Phi_E^{\rm prim}$ calculations under CPE conditions (see Section 9.7.2).

4.9.3 Restricted CPE for external sources of charged particles (RCPE)

In cases where the radiation source consists of charged particles the only situation where CPE exists is for a medium containing a uniformly distributed source emitting charged particles e.g. beta-emitting radioactive nuclei. This case has been dealt with in Section 4.9.1(a).

If the medium is irradiated by an external charged-particle source, such as an electron or proton beam, the situation differs from that discussed in the preceding section for uncharged particles. In this case there can never be CPE for the obvious reason that the primary particles lose energy continuously as they travel within the volume; in other words, there is no approximation to a uniform field of charged particles within the volume under the conditions described previously. There is, however, a less demanding form of charge particle equilibrium that can be said to exist if one considers only charged particles below a certain cut-off energy Δ. For these short-range particles, and for an appropriate small volume ν, each particle of a given energy leaving ν is replaced by an identical particle of the same energy entering, in terms of expectation values. This equilibrium is referred to here as *restricted charged-particle equilibrium*, RCPE. Evidently, if there is CPE then there must also be RCPE. However, in a medium irradiated by a charged-particle beam, even if CPE does not exist there can be a good approximation to RCPE provided that the depth is sufficient for the build-up of secondary particles. It follows that this is *not* the case very close to the surface when a charged-particle beam irradiates a medium – see Chapter 6. The small volume corresponds to what is termed a 'Bragg-Gray detector' and its size can be related to the range of the particles at the cut-off energy Δ, a procedure that will be discussed further under cavity theory in Chapter 9.

If RCPE is present then by analogy with Eqs. (4.89)-(4.92) there will be equality between cema and absorbed dose

$$D_{\rm med} \overset{\rm RCPE}{=} C_{\rm med} \tag{4.93}$$

and hence, we can write, for monoenergetic charged particles (c.f. Eq. (4.90))

$$D_{\rm med} \overset{\rm RCPE}{=} \Phi_{\rm med} \left[\frac{S_{\rm el}(E)}{\rho}\right]_{\rm med} \tag{4.94}$$

and for a charged-particle fluence spectrum (c.f. Eq. (4.92))

$$D_{\rm med} \overset{\rm RCPE}{=} \int_0^{E_{\rm max}} \left[\Phi_E^{\rm prim}\right]_{\rm med} \left[S_{\rm el}(E)/\rho\right]_{\rm med} {\rm d}E \tag{4.95}$$

The above expression (4.93), which can also be written as $dose = cema$, is a further key theoretical result; for example, it is the basis of 'Bragg-Gray cavity integrals' (see Chapter 9).

4.10 Partial charged-particle equilibrium (PCPE)

As the energy of uncharged particles increases, the penetrating power of the generated secondary charged particles increases more rapidly than the penetrating power of the primary radiation. Table 4.2 expresses this for both photons and neutrons; for example, $\sim 10\%$ photon attenuation occurs in a water layer equal in thickness ($\sim 4.8\,{\rm cm}$) to the maximum range of secondary electrons produced by 10 MeV photons. The neutron effect is much smaller ($\sim 0.5\%$) at that energy, assuming hydrogen-recoil proton secondaries.

As a result of this phenomenon charged-particle equilibrium will not be well-approximated at high energies. Referring to Fig. 4.11, the number of charged particles generated at point P_3 will be greater than

Table 4.2: **Approximate attenuation of photons and neutrons in a layer of water equal to the maximum range of secondary charged particles. The photon values have been computed using the CSDA range of the maximum secondary electron energy and the photon attenuation coefficient (see Data Tables); the neutron values are taken from Greening (1981).**

Primary radiation energy (MeV)	Photon attenuation (%) over maximum secondary electron range	Neutron attenuation (%) over maximum secondary proton range
0.1	0.026	0.05
1.0	12.3	0.04
10	10.1	0.5
30	19.9	1.5

at P_1, because of the attenuation of the uncharged particles in penetrating from the depth of P_3 to that of P_1 in the medium. The degree of CPE failure becomes progressively greater for higher energies, as the table indicates. This relatively minor CPE failure is termed here *partial charged-particle equilibrium* (PCPE). Attix (1979, 1986) introduced the concept using the term *transient* charged-particle equilibrium (TCPE), and it is commonly used in this form; however, because the concept of 'transient' is related to a time variation and in this case the variation is spatial, PCPE is preferred throughout this book.

4.10.1 PCPE and relationships between dose, kerma and electronic kerma

The relationships between absorbed dose D, kerma K and electronic kerma $K_{\rm el}$ as a function of depth in a medium irradiated by a photon beam are of fundamental interest. The data in Table 4.2 are the key to understanding how these relationships change with photon energy.

Fig. 4.13 is for a monoenergetic 250 keV photon beam incident on water; the depth scale only extends to 0.04 cm in order to show the extremely narrow region over which the absorbed dose 'builds up' to a maximum (see Chapter 6). In this energy range, radiative energy losses by the secondary charged particles (electrons in this case) are negligible ($g_{\rm w} \approx 0.0007$) and hence $K_{\rm el}$ is practically identical to K; furthermore the photon attenuation over a distance equal to the maximum secondary electron range $R_{\rm max}$[12], approximately 0.02 cm, is negligible (c.f. Table 4.2), giving the appearance of an unattenuated beam at this 'magnification'. Consequently at depths greater than 0.02 cm there is almost perfect charged-particle equilibrium.

At higher photon energies the picture changes completely as Fig. 4.14 shows for a 25 MeV monoenergetic beam incident on water. There is now a clear separation between kerma and electronic kerma, because radiative losses are no longer negligible ($g_{\rm w} \approx 0.09$), and between electronic kerma and dose, because photon attenuation over the maximum secondary electron range $R_{\rm max}$ is appreciable (around 15%). As a consequence of this attenuation, charged-particle equilibrium can never be complete; it is often called *partial*.

It is interesting to note that between these two extremes there can be a particular range of photon energies for which the effects of radiative losses and photon attenuation are similar in magnitude, that is, the separation of kerma and electronic kerma is similar to that of electronic kerma and dose, causing the kerma and dose curves to essentially overlap. This situation is shown in Fig. 4.15 for a 2.5 MeV monoenergetic photon beam incident on water.

At the depth of $R_{\rm max}$, equal to the maximum distance secondary charged particles starting at the surface

[12]Note that $R_{\rm max}$ is the maximum value of the mean forward range $R_{\rm mf}$ (see Chapter 2), i.e. it is the maximum penetration depth of the electrons produced by the incident photons.

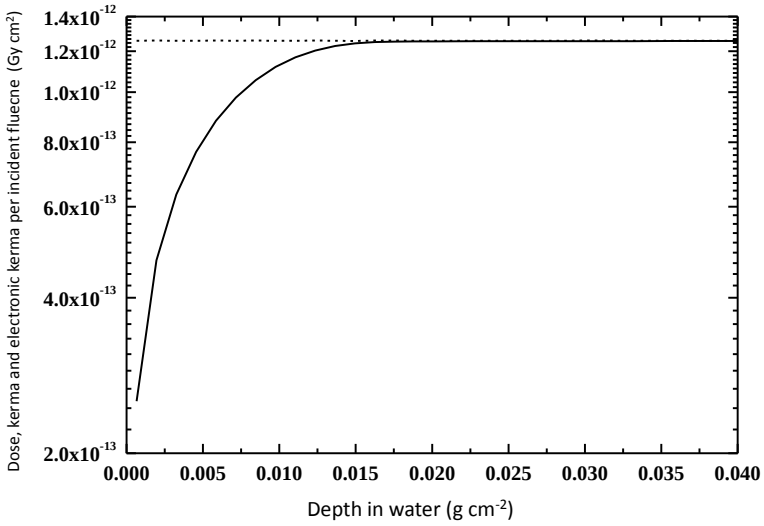


Figure 4.13: **Monte Carlo calculations of the depth-dependence of dose (full curve) and kerma (dotted curve) in a 250 keV photon broad beam incident on a water phantom. Beyond the 'build-up' depth, kerma and dose are practically identical; kerma and electronic kerma are indistinguishable at all depths. Data from Kumar *et al.* (2015).**

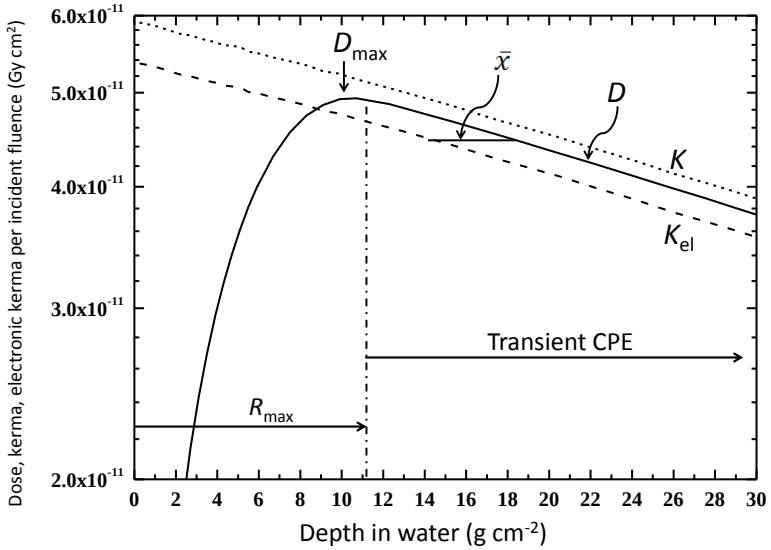


Figure 4.14: **Monte-Carlo calculations of the variation with depth of dose D, kerma K and electronic kerma $K_{\rm el}$ along the central axis for a 25 MeV photon broad beam incident on water. The *partial* CPE region beyond the depth of maximum secondary electron range, $R_{\rm max} = 11.2$ cm, is shown; the displacement $\bar{x}$ between the electronic kerma and dose curves is also indicated. Note that the abscissa (Gy cm^2) is logarithmic and therefore zero depth, where $D \approx 0$, cannot be shown. Data from Kumar *et al.* (2015).**

can penetrate in the direction of the incident particles (see Fig. 4.14), which is greater than the depth of $D_{\rm max}$, the D-curve becomes approximately parallel to the $K_{\rm el}$ curve. The dose D becomes proportional to electronic kerma $K_{\rm el}$, and *partial charged-particle equilibrium* (PCPE) is said to exist. Following Roesch (1958) the relationship between electronic kerma and dose can be expressed as

$$K_{\rm el} \overset{\rm PCPE}{=} D\, e^{-\bar{\mu}\,\bar{x}} \tag{4.96}$$

which can be re-written as

$$D \overset{\rm PCPE}{=} K_{\rm el}\, e^{\bar{\mu}\,\bar{x}} \simeq K_{\rm el}\,(1 + \bar{\mu}\,\bar{x}) \tag{4.97}$$

where $\bar{\mu}$ (the *effective photon attenuation coefficient*, see Chapter 5) is the common slope of the D, K

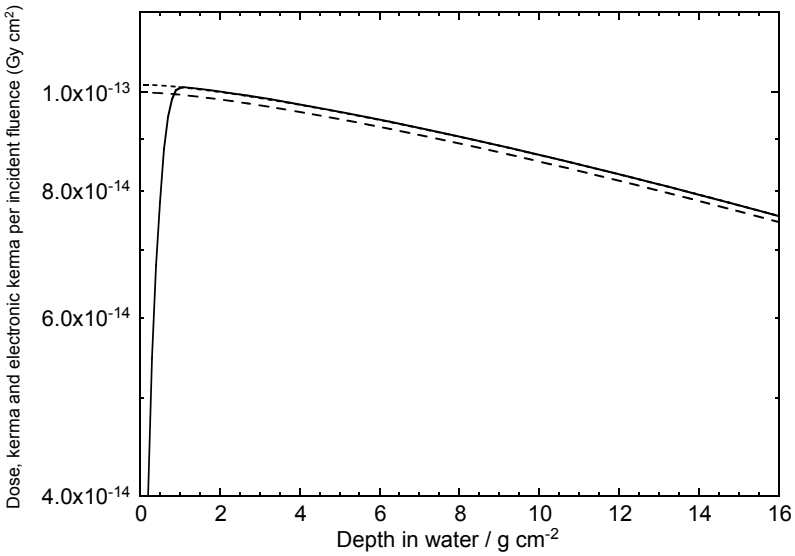


Figure 4.15: **Monte-Carlo calculations of the variation with depth of dose D (full curve), kerma K (short dashes) and electronic kerma $K_{\rm el}$ (long dashes) along the central axis for a 2.5 MeV photon broad beam incident on water.**

and $K_{\rm el}$ curves and $\bar{x}$ is the mean distance from the point of creation of the secondary charged particles to the point at which they deposit dose. In Fig. 4.14 $\bar{x}$ is shown as the distance separating the depths where $K_{\rm el}$ and D have equal values. Note that $\bar{\mu}$ and $\bar{x}$ must be expressed in consistent reciprocal units so that their product is dimensionless. The 'PCPE' above the equal signs in Eqs. (4.96) and (4.97) indicates that these equalities are valid only where partial CPE exists. We can now understand why, beyond the depth $R_{\rm max}$, the dose at a given depth is higher than the electronic kerma – it is because this dose arises from the electronic kerma at a distance $\bar{x}$ upstream, where it is higher by the approximate factor $(1 + \bar{\mu}\,\bar{x})$. This factor is often denoted as $\beta = D/K_{\rm el}$ but, to avoid confusion with the widely used $\beta = v/c$, a subscript indicating the medium where D and $K_{\rm el}$ refer to will be used in this book, see e.g. $\beta_{\rm med}$ and $\beta_{\rm w}$ in Eqs. (9.28) and (11.12), respectively; it is sometimes referred to as a correction for the *mean centre of electron production* (CEP) or the mean origin of electrons (see e.g. Boutillon and Niatel 1973).

The above discussion applies equally well to Figs. 4.13 – 4.15, whether radiative losses are negligible or not. The D-curve continues to bear the same relationship to the $K_{\rm el}$-curve, but where $K_{\rm rad} \neq 0$ the K-curve is higher than the $K_{\rm el}$-curve by the amount $K_{\rm rad} = [(\mu_{\rm tr} - \mu_{\rm en})/\mu_{\rm tr}]\,K$.

It is instructive to do a '*Gedankenexperiment*' (thought experiment) with respect to the figures above. Imagine a strong constant magnetic field being applied to the phantom with lines of force parallel to its surface plane. All charged particles would then be forced to remain near the depth plane of their origin, following helical paths with the helix axis lying in the plane of origin of the particle. For a sufficiently strong magnetic field the particles hypothetically could be forced to remain arbitrarily close to their depth of origin. What would happen to the D-curve under these conditions? *It would align itself with the $K_{\rm el}$-curve, coinciding with it at all depths, and CPE would exist at all depths.*[13] Since the magnetic field will not affect the bremsstrahlung losses in e.g. Fig. 4.14, the $K_{\rm el}$-curve would remain below the K-curve by the same amount as before. It seems clear from this that the integral of the D-curve from depth 0 to ∞, with or without the magnetic field and regardless of radiative losses, should equal the corresponding integrated value of $K_{\rm el}$ (neglecting losses of charged particles scattering out the front surface of the medium when the magnetic field is absent).

[13]Limited of course to the volume within the radiation beam, and remote from the beam edges by a distance at least equal to the range of the charged particles, which are still allowed to move with a velocity component aligned in either direction along the magnetic lines of force.

In conclusion, Eq. (4.97) in principle allows D and $K_{\rm el}$ to be inter-related where partial CPE conditions exist for high-energy indirectly ionizing radiations. However, a knowledge of $\bar{x}$ and the effective attenuation coefficient $\bar{\mu}$ is required for each case, so that Eq. (4.97) is not as readily applicable as the simple equality of D and $K_{\rm el}$ that exists under perfect CPE conditions.

4.11 Summary of the inter-relations between fluence, kerma, cema and dose

In this chapter many of the key relationships between fundamental dosimetric quantities (absorbed dose, kerma, cema) and field quantities (fluence, energy fluence and fluence differential in energy) have been derived, in certain cases requiring specific equilibrium conditions (e.g. CPE) and in others cases not (e.g. the ration between kerma and fluence). Fig. 4.16 summarizes these relationships, which play a key role in Radiation Dosimetry in general and in 'Cavity Theory' (Chapter 9) in particular.

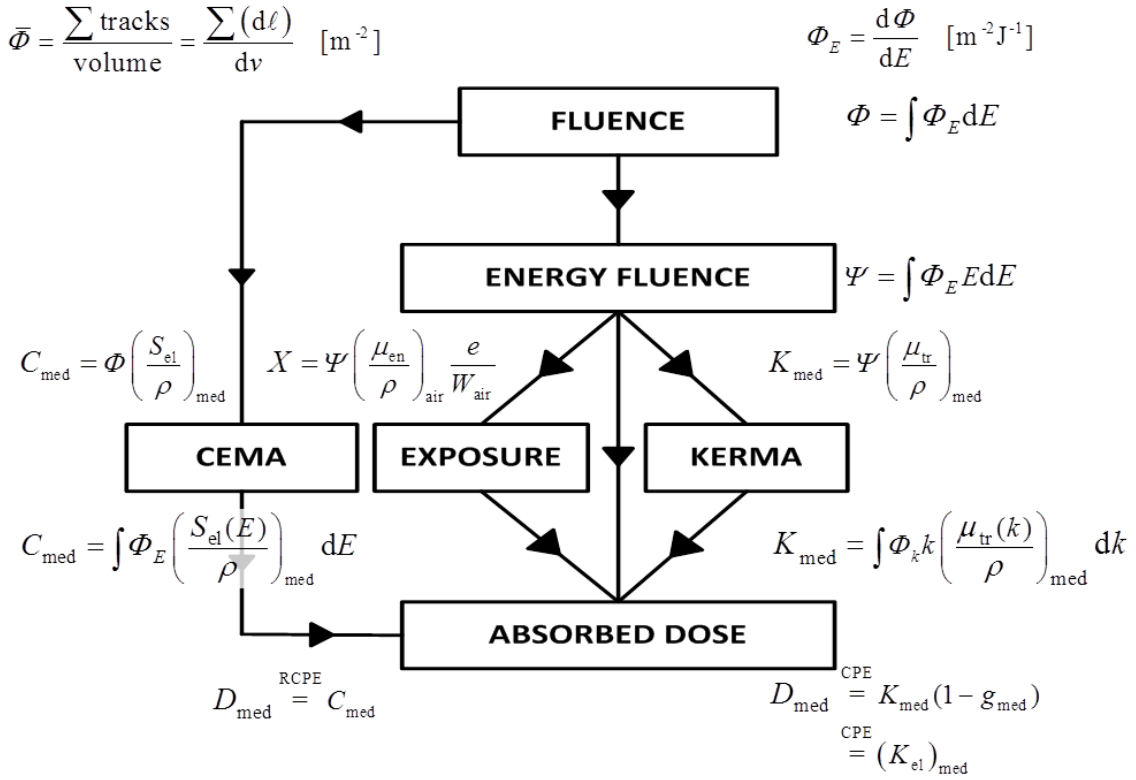


Figure 4.16: **Summary of the relationships between fundamental radiation dosimetry quantities and field quantities.**

4.12 Addendum – Example calculations of (net) energy transferred and imparted

4.12.1 Energy transferred

Figure 4.17 shows a positron entering volume ν and undergoing in-flight annihilation, resulting in two photons with energies k_1 and k_2. The kinetic energy of the positron just before annihilation is E^+. Both photons leave the volume ν.

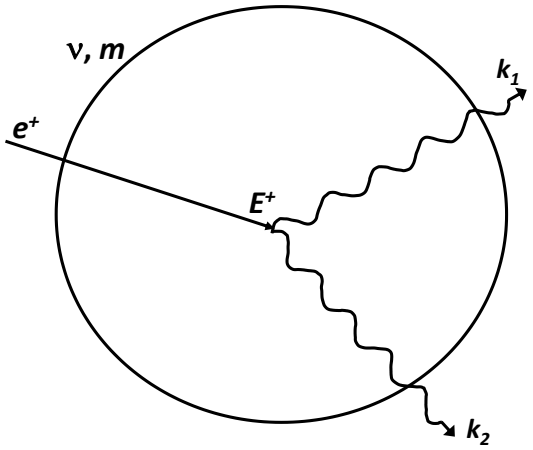


Figure 4.17: **Illustration of the concept of energy transferred, $\varepsilon_{\rm tr}$; a positron with kinetic energy E^+ undergoes in-flight annihilation in volume ν.**

The energy transferred is given by

$$\varepsilon_{\rm tr} = (R_{\rm in})_{\rm u} - (R_{\rm out})_{\rm u}^{\rm non-r} + \sum \mathsf{Q} \tag{4.98}$$

and the different components in this equation are as follows:

(a) $(R_{\rm in})_{\rm u} = 0$ since no uncharged particles enter the volume;

(b) $(R_{\rm out})_{\rm u}^{\rm non-r}$ represents the radiant energy leaving the volume *excluding* radiative losses. Thus, we must take into account the kinetic energy E^+ of the positron before the in-flight annihilation took place.

It is helpful to think of $(R_{\rm out})_{\rm u}^{\rm non-r}$ as the difference between the total radiant energy, $(R_{\rm out})_{\rm u}^{\rm total}$, and the kinetic energy of the charged particles, $(R_{\rm out})_{\rm u}^{\rm KE}$, that went into the creation of the radiant energy, i.e.,

$$(R_{\rm out})_{\rm u}^{\rm non-r} = (R_{\rm out})_{\rm u}^{\rm total} - (R_{\rm out})_{\rm u}^{\rm KE} \tag{4.99}$$

In this case the energy balance of the in-flight annihilation is given by

$$E^+ + 2\,m_{\rm e}c^2 = k_1 + k_2 \tag{4.100}$$

and we have

$$(R_{\rm out})_{\rm u}^{\rm total} = k_1 + k_2 \tag{4.101}$$

and

$$(R_{\rm out})_{\rm u}^{\rm KE} = E^+ \tag{4.102}$$

Thus equation (4.99) becomes

$$(R_{\rm out})_{\rm u}^{\rm non-r} = (R_{\rm out})_{\rm u}^{\rm total} - (R_{\rm out})_{\rm u}^{\rm KE} = k_1 + k_2 - E^+ \tag{4.103}$$

From equation (4.100) we have the energy balance $k_1 + k_2 - E^+ = 2\,m_{\rm e}c^2$, therefore

$$(R_{\rm out})_{\rm u}^{\rm non-r} = 2\,m_{\rm e}c^2 \tag{4.104}$$

(c) $\sum \mathrm{Q}$ represents the energy contribution derived from rest mass, counted as positive for a mass-to-energy conversion; thus for the (in-flight) annihilation

$$\sum \mathrm{Q} = +2\, m_\mathrm{e} c^2 \tag{4.105}$$

The energy transferred is therefore

$$\varepsilon_\mathrm{tr} = 0 - 2m_\mathrm{e}c^2 + 2m_\mathrm{e}c^2 = 0 \tag{4.106}$$

This result makes intuitive sense when one considers energy transferred simply as kinetic energy passed from photons to charged particles.

The *net* energy transferred is the portion of the energy transferred that does **not** contain radiative energy, $(R_\mathrm{u}^\mathrm{r})$, derived from kinetic energy of the charged particles set in motion by uncharged particles in the volume ν. In other words, $\varepsilon_\mathrm{tr}^\mathrm{n} = \varepsilon_\mathrm{tr} - (R_\mathrm{u}^\mathrm{r})$. In the above example, $\varepsilon_\mathrm{tr}^\mathrm{n} = 0$ because there is no energy transferred in the first place.

The case becomes more interesting when one of the annihilation photons undergoes an interaction in the volume. This situation is depicted in Fig. 4.18 where photon k_2 undergoes a Compton interaction.

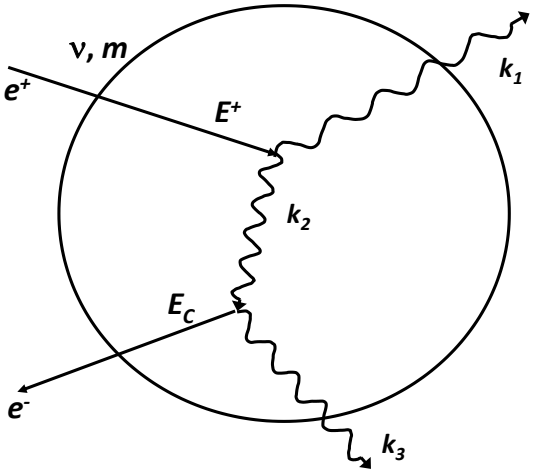


Figure 4.18: **A second illustration of the concept of energy transferred; a positron undergoes in-flight annihilation, and then one of the resulting photons interacts within volume ν.**

The components in Eq. (4.98) are now:

(a) $(R_\mathrm{in})_\mathrm{u} = 0$, since no radiant energy enters the volume

(b) $(R_\mathrm{out})_\mathrm{u}^\mathrm{non-r} = (R_\mathrm{out})_\mathrm{u}^\mathrm{total} - (R_\mathrm{out})_\mathrm{u}^\mathrm{KE} = k_1 + k_3 - E^+$

As above, the energy balance of the in-flight annihilation is given by $E^+ + 2\, m_\mathrm{e}c^2 = k_1 + k_2$. Using this, we can write:

$$(R_\mathrm{out})_\mathrm{u}^\mathrm{non-r} = k_1 + k_3 - \left(k_1 + k_2 - 2\, m_\mathrm{e}c^2\right) = k_3 - k_2 + 2\, m_\mathrm{e}c^2 \tag{4.107}$$

The kinetic energy of the Compton electron is given by $E_\mathrm{C} = k_2 - k_3$. Thus

$$(R_\mathrm{out})_\mathrm{u}^\mathrm{non-r} = 2\, m_\mathrm{e}c^2 - E_\mathrm{C} \tag{4.108}$$

(c) $\sum \mathsf{Q} = +2\,m_{\mathrm{e}}c^2$ for the same reasons as above.

Therefore, from Eq. (4.98) the energy transferred is given by

$$\varepsilon_{\mathrm{tr}} = 0 - 2\,m_{\mathrm{e}}c^2 + E_{\mathrm{C}} + 2\,m_{\mathrm{e}}c^2 = E_{\mathrm{C}} \tag{4.109}$$

The *energy transferred* is thus equal to the energy transferred in the Compton interaction of one of the annihilation photons, which again makes intuitive sense. The *net energy transferred* is $\varepsilon_{\mathrm{tr}}^{\mathrm{n}} = \varepsilon_{\mathrm{tr}} - (R_{\mathrm{u}}^{\mathrm{r}}) = \varepsilon_{\mathrm{tr}}$ since the electron set in motion does not produce bremsstrahlung.

In the case where photon k_2 transfers *all* of its energy within the volume, the situation is amended as follows

$$\begin{aligned}(R_{\mathrm{out}})_{\mathrm{u}}^{\mathrm{non-r}} &= (R_{\mathrm{out}})_{\mathrm{u}}^{\mathrm{total}} - (R_{\mathrm{out}})_{\mathrm{u}}^{\mathrm{KE}} \\ &= k_1 - E^{+} \\ &= k_1 - \left(k_1 + k_2 - 2\,m_{\mathrm{e}}c^2\right) \\ &= 2\,m_{\mathrm{e}}c^2 - k_2\end{aligned} \tag{4.110}$$

while the other contributions to $\varepsilon_{\mathrm{tr}}$ remain unchanged, yielding

$$\varepsilon_{\mathrm{tr}} = 0 - 2m_{\mathrm{e}}c^2 + k_2 + 2m_{\mathrm{e}}c^2 = k_2 \tag{4.111}$$

as expected.

4.12.2 Energy imparted

We now turn our attention to how the quantity *energy imparted*, c.f. Eq. (4.43), can be calculated from the definitions given in the previous section.

In Fig. 4.19 a photon with energy k_1 enters the volume ν and undergoes a Compton interaction which produces a scattered photon with energy k_2 and a secondary electron with kinetic energy *E*. This electron produces a bremsstrahlung photon (k_3) before leaving the volume with remaining energy E'. A further bremsstrahlung photon (k_4) is produced outside the volume.

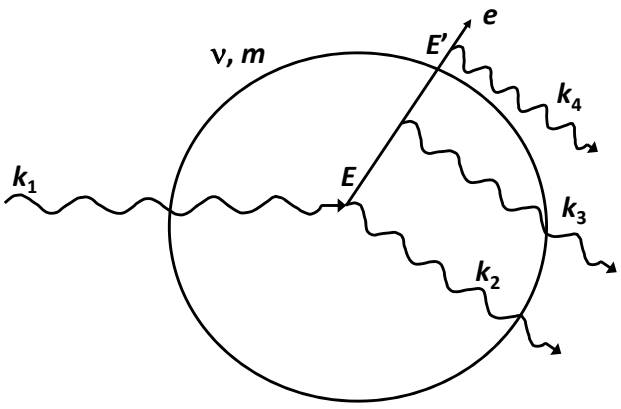


Figure 4.19: **Illustration of the concept of energy imparted for the case of a Compton interaction followed by bremsstrahlung emission. (After Attix 1983).**

In this example the sum of the energies incident on the volume is $\varepsilon_{\rm in} = k_1$, and the sum of the energies leaving the volume is $\varepsilon_{\rm out} = k_2 + k_3 + E'$. Therefore the mean energy imparted in the volume, $\bar{\varepsilon}$, is given by

$$\bar{\varepsilon} = \varepsilon_{\rm in} - \varepsilon_{\rm out} = k_1 - \left(k_2 + k_3 + E'\right) + 0 \tag{4.112}$$

the zero because there are no changes in rest energy.

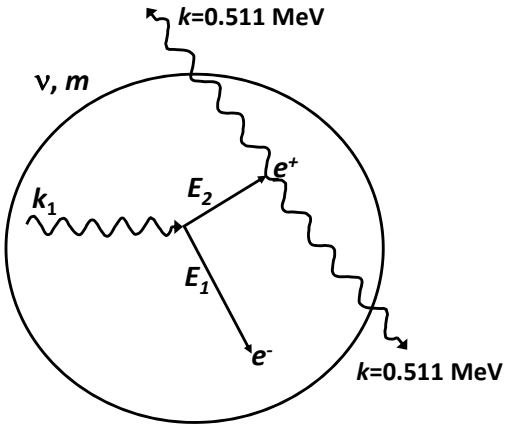


Figure 4.20: **Energy imparted involving γ-ray emission, pair production, and positron annihilation. (After Attix 1983).**

In Fig. 4.20 a photon with energy k_1 is emitted from a radioactive source and undergoes pair production, creating an electron-positron pair with initial kinetic energies E_1 and E_2, respectively. The electron and positron both stop in the volume; the latter undergoes *annihilation* with an atomic electron and their combined rest masses result in two oppositely directed photons, each with energy 0.511 MeV, which both leave the volume.

In this case zero energy is incident on the volume, therefore $\varepsilon_{\rm in} = 0$, and the sum of the energies leaving the volume is $\varepsilon_{\rm out} = 1.022\,{\rm MeV}$. The decrease in rest-mass energy, ΣQ, includes the radioactive disintegration ($+k_1$), the pair production ($-2\,mc^2$) and the annihilation ($+2\,mc^2$), the latter two terms cancelling. Therefore the mean energy imparted in the volume, $\bar{\varepsilon}$, is given by

$$\bar{\varepsilon} = \varepsilon_{\rm in} - \varepsilon_{\rm out} + \sum {\rm Q} = 0 - 1.022\,{\rm MeV} + k_1 \tag{4.113}$$

which can also be written as

$$\bar{\varepsilon} = k_1 - 1.022\,{\rm MeV} = E_1 + E_2 \tag{4.114}$$

Note that there are no radiative losses in this example, as the escaping annihilation photons derive their entire energy from rest mass, not from kinetic energy.

4.13 Exercises

(1) A broad plane-parallel beam of electrons is perpendicularly incident upon a thin foil which scatters the electrons through an average angle of 20°, stopping none of them. (a) What is the ratio of the fluence of primary electrons just behind the foil to that with the foil removed?. (b) What is the ratio of the number of electrons per cm^2 passing through a plane just behind (and parallel to) the foil to that with the foil removed?

Answer: (a) 1.06; (b) Unity

Solution:

(2) The fluence rate decreases with increasing distance from a point source of γ-rays as the inverse square of the distance (see Section 5.7). The strength of the electric field surrounding a point electric charge does likewise. At a point midway between two identical charges the electric field is zero. (a) What is the fluence rate midway between two identical sources?. (b) What is the essential difference between the two cases?

Answer: (a) Twice that due to one of the sources. (b) Fluence rate is a scalar quantity; electric field strength is a vector. Vector addition depends on orientation; scalar addition does not.

(3) A point source of ^{60}Co γ-rays emits equal numbers of photons of 1.17 MeV and 1.33 MeV, giving a fluence rate of 5.7×10^9 photons cm^{-2} s^{-1} at a specified location. What is the energy fluence rate there, expressed (a) in erg cm^{-2} s^{-1} and (b) in J m^{-2} min^{-1} ?

Answer: (a) 1.14 $\times 10^4$ erg cm^{-2} s^{-1}; (b) 685 J m^{-2} min.

(4) In the previous exercise, what is the energy fluence of 1.17 MeV photons during 24 hours, (a) in erg cm^{-2} and (b) in J m^{-2}

Answer: (a) 4.62$\times 10^8$ erg cm^{-2} ; (b)4.62$\times 10^5$ J m^{-2}

(5) As discussed in the text, the unit of energy fluence differential in energy, Ψ_E, coincides with that of fluence, Φ, which sometimes leads to confusion. Show that the integral of the two quantities differential in energy yields the correct units of Φ and Ψ and that absorbed dose calculations with photons (when $D \approx K_{\rm el}$) and electrons have units of Gy.

(6) Certain Monte Carlo code (e.g. Penelope, see Chapter 8) outputs a tally 'FluenceTrackLength' for the spectral fluence per incident particle as tracks integrated over a given region volume, in units of cm/eV. Explain how the user should derive the correct units of Φ_E.

Answer: Dividing by the region volume yields units of Φ_E in $\frac{1}{\rm cm^2 eV}$

(7) A point source isotropically emitting 10^8 fast neutrons per second falls out of its shield onto a railroad platform 3 m horizontally from the track. A train goes by at 60 km/h. Ignoring scattering and attenuation, what is the fluence of neutrons that would strike a passenger at the same height above the track as the source?

Answer: 5$\times 10^5$ n m^{-2}

(8) An x-ray field at a point P contains 7.5×10^8 photons/(m^2 s keV), with energies uniformly distributed from 10 keV to 100 keV. (a) What is the photon fluence rate at *P* ?. (b) What would be the photon fluence in one hour?. (c) What is the corresponding energy fluence, in J/m^2 and erg/cm^2?

Answer: (a) 6.75$\times 10^{10}$photons/(m^{2}s); (b) 2.43$\times 10^{14}$photons/m^2; (c) 2.14 J/m^2 or 2.14$\times 10^3$erg/cm^2

(9) Show that, for a spherical volume, Kellerer's definition of fluence (often cited as Chilton's) gives the same value as the conventional definition in a uniform mono-energetic field. *Hint*: The mean chord length in any convex volume is $\ell = 4\nu/a$, where ν is the volume and a is the surface area.

Answer:

(10) What is $(K_{\rm el})_{\rm air}$ in Gy at a point in air where $X = 47$ roentgens?

Answer: 0.412 Gy

(11) An electron enters a volume ν with a kinetic energy of 4 MeV, and carries 0.5 MeV of that energy out of ν when it leaves. While in the volume it produces a bremsstrahlung x-ray of 1.5 MeV which

escapes from ν. What is the contribution of this event to: (a) The energy transferred?. (b) The net energy transferred?. (c) The energy imparted?

Answer: (a) 0. (b) 0. (c) 2 MeV

(12) A 10 MeV γ-ray enters a volume ν and undergoes pair production, thereby disappearing and giving rise to an electron and positron of equal energies. The electron spends half its kinetic energy in collision interactions before escaping from ν. The positron spends half of its kinetic energy in collisions in ν before being annihilated in flight. The resulting photons escape from ν. Determine (a), (b), and (c) as in the previous exercise.

Answer: (a) 8.98 MeV. (b) 6.73 MeV. (c) 4.49 MeV

(13) The fluence rate of 6 MeV γ-rays is 3.5×10^6 photons cm^{-2} s^{-1} at a point in Pb. What are the values of K and $K_{\rm el}$ there after one week? (Express in units of erg/g and gray.)

Answer: $K = 5.07 \times 10^5\,\mathrm{erg/g} = 50.7\,\mathrm{Gy}$*;* $K_{\rm el} = 3.70 \times 10^5\,\mathrm{erg/g} = 37.0\,\mathrm{Gy}$

(14) A field of 14.5 MeV neutrons deposits a kerma of 1.37 Gy at a point of interest in water. What is the fluence?

Answer: 2.015×10^{10} *neutrons/cm*^{2}.

(15) An x-ray field at a point P in aluminium contains 7.5×10^8 $\mathrm{photons/(m^2\ s\ keV)}$, uniformly distributed from 10 to 100 keV. Calculate the electronic kerma in P for a one hour irradiation, in Gy. (Note: use log-log interpolation from the Data Tables.)

Answer: ~ 0.093 *Gy.*

(16) Consider two flasks containing 5 and 25 cm^3 of water, respectively. They are identically and homogeneously irradiated with γ-rays, making the average kerma equal to 1 Gy in the smaller flask. (a) Neglecting differences in γ-ray attenuation, what is the average kerma in the larger flask? (b) What is the energy transferred in each volume of water?

Answer: (a) 1 Gy. (b) 0.005 J, 0.025 J.