Solid Angle Units
(steradians)

Geometric definition of solid angle

For physical processes which are naturally described with a polar coordinate system, it is often necessary to identify the fraction of a unit sphere interior to a surface formed by moving the radial vector to form a conic structure. By convention, the entire unit sphere is defined to have 4π steradians and the steradian is the unit used to describe the "solid angle" associated with any portion of the unit sphere.

If ϕ is the azimuthal angle from a reference pole of the polar coordinate system ("north pole") and θ is the longitudinal angle from some reference plane passing through the polar axis, then a differential quantity of solid angle can be written as;

$$d\Omega = \sin\phi d\phi d\theta$$

The $\sin\phi$ term is required because of the shorter arc traced by $d\theta$ for angles of ϕ near the poles. The total solid angle of the unit shere can then be computed by integration of $d\Omega$:

$$\Omega = \int_0^{2\pi}\int_0^{\pi} \sin\phi d\phi d\theta$$

$$= 2\pi \int_0^{\pi} \sin\phi d\phi = 4\pi$$

Thus the definition of $d\Omega$ leads to the unit sphere having 4π steradians.

Solid Angle Units
(steradians)

Radiation fluence in photons/steradians

For sources which emit radiation from a region small enough to be considered a point source, the radiation will travel radially into space. Typical radionuclide sources will emit radiation with no bias as to the direction and are said to have isotropic emission. A source which emits N photons will thus produce a fluence of N photons per 4π steradians or $N/4\pi$ photons/steradian. If one considers a sphere with a radius of r mm, this source will produce a fluence of photons traveling through the surface of the sphere equal to $N/4\pi r^2$ photons/mm^2.

Radiation fluence can either be expressed in terms of photons/steradian or photons/mm^2. To convert from photons/steradian to or photons/mm^2, simply divide by r^2, as can be seen in the above example for a radionuclide source. It is often more convenient to describe the fluence from a source in photons/steradian since it is independent of the distance (i.e. radius) from the emission point.

For x-ray sources emitting radiation from a small focal spot, the intensity of emitted radiation can be different depending on the angle of emission relative to the target surface. In this case, the emitted fluence can still be expressed as the quantity of radiation emitted in a small solid angle in the direction ϕ, θ. The differential energy spectrum, $N(E, \phi, \theta)$, will thus have units of photons/kev/steradian but may vary for different emission angles.

Radionuclide Decay
Curie/Becquerel

For a sample with N nuclei of a radioactive nuclide, the rate of decay in disintegrations per second is,

$$Q = -\lambda N = dN/dt$$

where Q is the amount of radioactive activity and λ (1/sec) is the decay constant.

The SI unit for activity (Q) is the Becquerel (Bq) which is equivalent to 1.0 disintegrations per second. The traditional unit of activity in the Curie (Ci) where 1.0 Curie is equal to $3.7x10^{10}$ disintegrations/second or 37 GBq. For radionuclides used in nuclear imaging, activities are often in the range from 1 to 20 mCi (40 to 800 MBq).

This simple differential equation has a solution of

$$N = N_o e^{-\lambda t}$$

and therefore;

$$Q = Q_o e^{-\lambda t}$$

The time for a radioactive sample of Q_o activity to decay to an activity equal to 1/2 of Q_o is easily shown to be;

$$T_{\frac{1}{2}} = \frac{ln(2)}{\lambda} = \frac{.693}{\lambda}$$

X-ray Energy Spectrum
Kramers equation

In 1923, Hendrik Antonie Kramers (1894-1952) published a significant theoretical paper which included a derivation of the continuum energy spectrum (ref 1). Kramers began with the quantum theory of Bohr to provide the theoretical basis for his relationship. He showed via Bohr's correspondence principle that this formula is an approximate solution of the processes which occur when a free electron approaches a positive nucleus. This paper is one of the first applications of the then new quantum theory to a practical physics problem.

Kramers equation can be written as:

$$\phi(E_\gamma) = \frac{\psi(E_\gamma)}{E_\gamma} = KZ\frac{(T - E_\gamma)}{E_\gamma}$$

where $\phi(E_\gamma) \equiv d\phi/dE_\gamma$ is the differential number of x-rays produced in dE_γ, and has units of photons/keV/mA-s/sr. T is the electron energy and E_γ is the energy of the emitted xray. Alternatively the equation can be used to describe the differential amount of x-ray energy produced in dE_γ, $\psi(E_\gamma) \equiv d\psi/dE_\gamma$, in units of energy/keV/mA-s/sr where $\psi(E_\gamma) = E \times \phi(E_\gamma)$. K is a slowly varying function of energy in the range of 5 to 7 phots/keV/mA-s/sr:

$K = 6.64 \times 10^8$ phots/keV/mA-s/sr for 30 keV electrons,
$K = 6.31 \times 10^8$ phots/keV/mA-s/sr for 40 keV electrons,
$K = 4.99 \times 10^8$ phots/keV/mA-s/sr for 180 keV electrons.

ref 1. Kramers HA, On the theory of X-ray absorption and of the continuous X-ray spectrum, Philos. Mag., 46(275):836-871, Nov. 1923. (Communicated by Prof N. Bohr, Copenhagen)

X-ray Energy Spectrum
Storm equation

A notable work on the modeling of the continuous spectrum was published by Storm in 1971 (ref 1). Storm formally evaluated several cross sections detailed by Koch and Motz (ref 2). These cross sections have more validity than the Compton and Allison cross section used by most other investigators. He shows that for spectral estimation the best fit to experimental data is obtained with a differential (in energy) cross section derived using the Born approximation with no screening (3BN). He then presents a mathematical fit for the bremsstrahlung intensity which specifically accounts for electron backscatter.

$$\psi(E_\gamma) = (\frac{11}{4\pi} Z \frac{(T - E_\gamma)(1 - e^{-3E_\gamma/E_K})}{(E_\gamma/T)^{1/3}(1 - e^{-T/E_K})}) f_a$$

where $\psi(E_\gamma) = d\psi/dE_\gamma$ is the differential quantity of x-ray energy produced in dE_γ (ergs/keV/mA-s/sr), E_γ is the x-ray energy, T is the electron energy (high voltage), E_K is the K binding energy, and f_a is a self absorption correction term. While this is the earliest reference to use the full integral model, the work employed cross sections which are more valid than those used by subsequent authors.

ref 1. Storm E., Calculated bremsstrahlung spectra from thick tungsten targets, Phys. Rev. A, 5(6):2328-2338, June 1971.

ref 2. Koch HW and Motz JW, Bremsstrahlung cross section formulas and related data, Rev. Mod. Phys., 31(4):920-955, 1959.