

Lecture Mar. 20th

- ppt upto Slide 15 incl.
- Kinematics (on whiteboard)
- ppt starting at Slide #18

Kinematics notes starting on
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Short hand $M = M_X c^2$ rest mass energy of parent
 $m = M_{X'} c^2$ " " " daughter
 $k = m(\gamma - 1)$ $\gamma = \gamma_{X'}$ $T_{X'} = m_X c^2 k$
 $E = E_\gamma$

CoE $M = m\gamma + E$ (1)

or $M = m(\gamma - 1) + m + E$

or $Q = k + E$ (2) since $M - m = Q$

CoM: The X' & E trajectories are collinear
 Thus

$m\beta\gamma = E$ (3)

$\beta = \beta_{X'}$

(3)² $\Rightarrow$

$m^2 \beta^2 \gamma^2 = E^2$

$m^2 (\gamma - 1)(\gamma + 1) = E^2$

$\left\{ \begin{array}{l} \gamma^2 = \frac{1}{1 - \beta^2} \\ \gamma^2 (1 - \beta^2) = 1 \\ \gamma^2 \beta^2 = \gamma^2 - 1 \\ = (\gamma - 1)(\gamma + 1) \end{array} \right.$

$k(k + 2m) = E^2$ (4)

$k = m(\gamma - 1)$

$m(\gamma + 1) = m(\gamma - 1 + 2)$

$= k + 2m$

(2) $\Rightarrow$ (4) $\Rightarrow$

$k(k + 2m) = (Q - k)^2$

$k^2 + 2mk = Q^2 - 2Qk + k^2$

$2k(m + Q) = Q^2$

QED

$k = \frac{Q^2}{2(m + Q)} = \frac{Q^2/m}{2(1 + \frac{Q}{m})}$ (5) $\left| T_{X'} = \frac{Q}{2} \left[\frac{Q/m_X c^2}{1 + \frac{Q}{m_X c^2}} \right] \right.$

(3)

$$E = Q - k \quad \text{from (2)}$$

$$= Q - \frac{Q^2}{2m(1 + \frac{Q}{m})} \quad \text{from (5)}$$

$$= Q \left[\frac{2m(1 + \frac{Q}{m}) - Q}{2m(1 + \frac{Q}{m})} \right]$$

$$= Q \left[\frac{2m + 2Q - Q}{2m(1 + \frac{Q}{m})} \right]$$

$$= Q \left[\frac{1 + \frac{Q}{2m}}{1 + \frac{Q}{m}} \right]$$

$$E = Q \left[\frac{1 + \frac{Q}{2m}}{1 + \frac{Q}{m}} \right]$$

$$E = Q \left[\frac{1 + \frac{Q}{2mc^2}}{1 + \frac{Q}{mc^2}} \right] \quad (6)$$

Taylor expand (5) & (6) to $O(1)$ in $\frac{Q}{mc^2}$

$$(7) \left\{ \begin{aligned} E &\approx Q \left[1 - \frac{1}{2} \frac{Q}{mc^2} \right] + O\left(\left[\frac{Q}{mc^2}\right]^2\right) \\ T_{x'} &= \frac{Q^2}{2mc^2} + O\left(\left[\frac{Q}{mc^2}\right]^2\right) \end{aligned} \right.$$

(4)

Relativistic correction?

To do this Taylor expand (5) & (6) to $O\left[\left(\frac{Q}{m c^2}\right)^2\right]$

$$K = \frac{Q^2}{2m} \left(1 - \frac{Q}{m c^2}\right)$$

$$\frac{1}{1+x} = \frac{Q^2}{2m c^2} \left(1 - \frac{Q}{m c^2}\right)$$

$$\frac{1}{1+x} =$$

$$1 - x + x^2 - x^3 \dots$$

$$\approx \frac{Q}{A \cdot 1000 \text{ (MeV)}}$$

$$A \cdot 1000 \text{ (MeV)}$$

$$\approx \frac{10^{-4} - 10^{-2}}{A}$$

Can certainly ignore
esp for heavier nuclei

$$E = Q \left[1 - \frac{Q}{2m}\right] \left[1 + \frac{Q}{m}\right]^{-1}$$

$$= Q \left[1 + \frac{Q}{2m}\right] \left[1 - \frac{Q}{m} + \left(\frac{Q}{m}\right)^2 - \left(\frac{Q}{m}\right)^3 \dots\right]$$

$$= Q \left[1 - \frac{Q}{m} + \left(\frac{Q}{m}\right)^2 - \left(\frac{Q}{m}\right)^3 + \frac{Q}{2m} - \frac{Q^2}{2m^2} + \frac{Q^3}{2m^3} \dots\right]$$

$$= Q \left[1 - \frac{Q}{2m} + \frac{1}{2} \left(\frac{Q}{m}\right)^2\right] \dots$$

(5)

$$E = Q \left[1 - \frac{Q}{2m} \right] + \frac{1}{2} \frac{Q^3}{m^2}$$

$$\frac{E}{Q} = \underbrace{1 - \frac{Q}{2m}}_{\text{non relativistic}} + \underbrace{\frac{1}{2} \frac{Q^2}{m^2}}_{\frac{1}{2} \frac{[0.1 \rightarrow 10]^2 [\text{MeV}]^2}{A 10^6 \text{ MeV}}}$$

$$\frac{1}{2} \frac{10^{-8} \rightarrow 10^{-4}}{A}$$

can also safely ignore