

NERS 312

Elements of Nuclear Engineering and Radiological Sciences II

aka **Nuclear Physics for Nuclear Engineers**

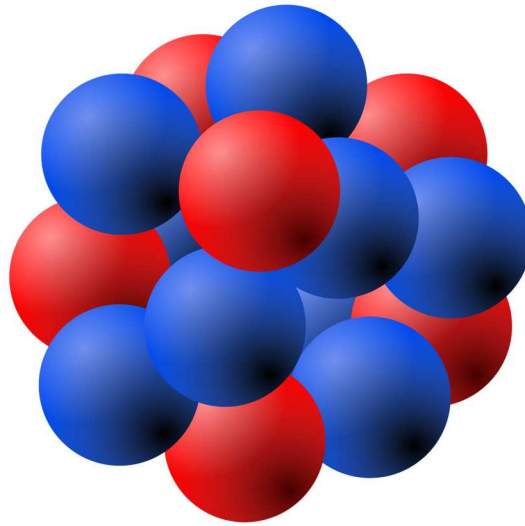
Lecture Notes for Chapter 10: Nuclear Properties

Supplement to (Krane II: Chapters 1 & 3)

*Note: The lecture number corresponds directly to the chapter number in the online book.
The section numbers, and equation numbers correspond directly to those in the online book.*

10.0: Introduction: The nucleus

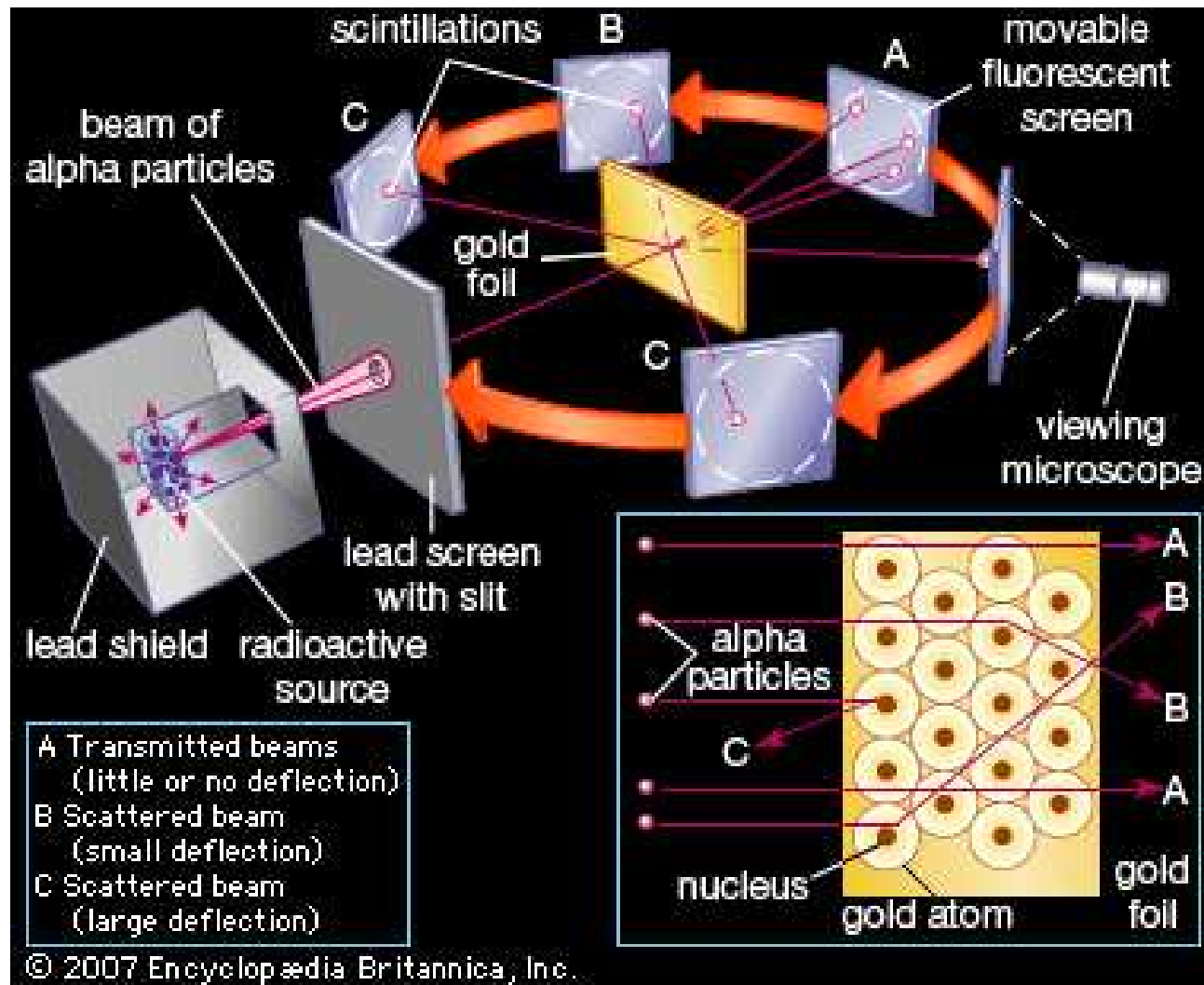
The nucleus
was discovered by
Ernest Rutherford
in 1911



through interpretation of
the classic “ 4π ” scattering
experiment conducted by
PostDoc Hans Geiger (left)
and undergraduate (!)
Ernest Marsden (right)



10.0: Introduction: The “Classic” Rutherford experiment

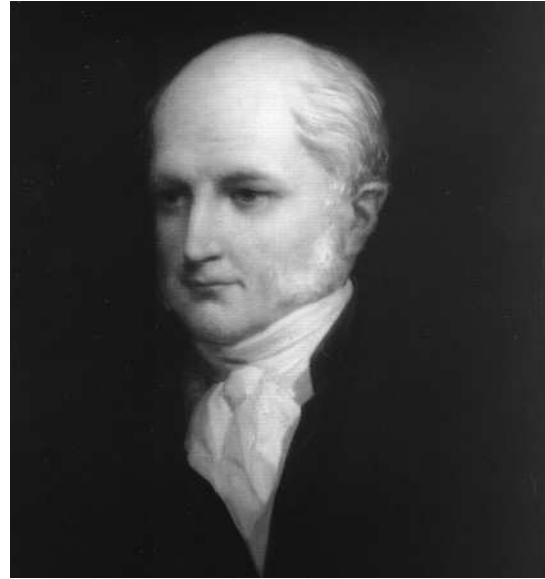


Rutherford concluded that the only cause for the rare observation of a back-scattered α , was the existence of a dense concentration of +ve charge, at the center of atom.

10.0: Introduction: The Nucleons

The nucleus is made up of protons and neutrons *a.k.a.* the “nucleons”.

Protons were proposed by William Prout in 1815
and discovered by Rutherford in 1920



In 1920, Rutherford proposed existence of the neutron
later discovered by James Chadwick in 1932

10.0: Introduction: Characteristics of nucleons

Name (symbol)	mass (MeV/c ²)	charge e	lifetime (s)	magnetic moment $\mu_N = e\hbar/(2m_p)$
neutron (n)	939.565378(21)	0	881.5(1.5)	-1.91304272(45)
proton (p)	938.272046(21)	1	stable	2.792847356(23)

Charge Distribution:

p all +ve charge peaks at about 0.45 fm, with an exponential die-off to ≈ 2 fm

n has a +ve charge that peaks at 0.24 fm, followed by a -ve shell that peaks at 0.95 fm
dies off exponentially, to about 2 fm **Graphs on the next 2 pages**

Properties Common to both nucleons

Structure	Composite (quarks)
Radius	≈ 0.8 fm
Statistics	Fermi-Dirac (fermions)
Family	Baryons (3 quarks)
Intrinsic Spin	$\frac{1}{2}\hbar$
Active forces	Strong, electromagnetic, weak, gravity

10.0: Introduction: Proton Charge Distribution

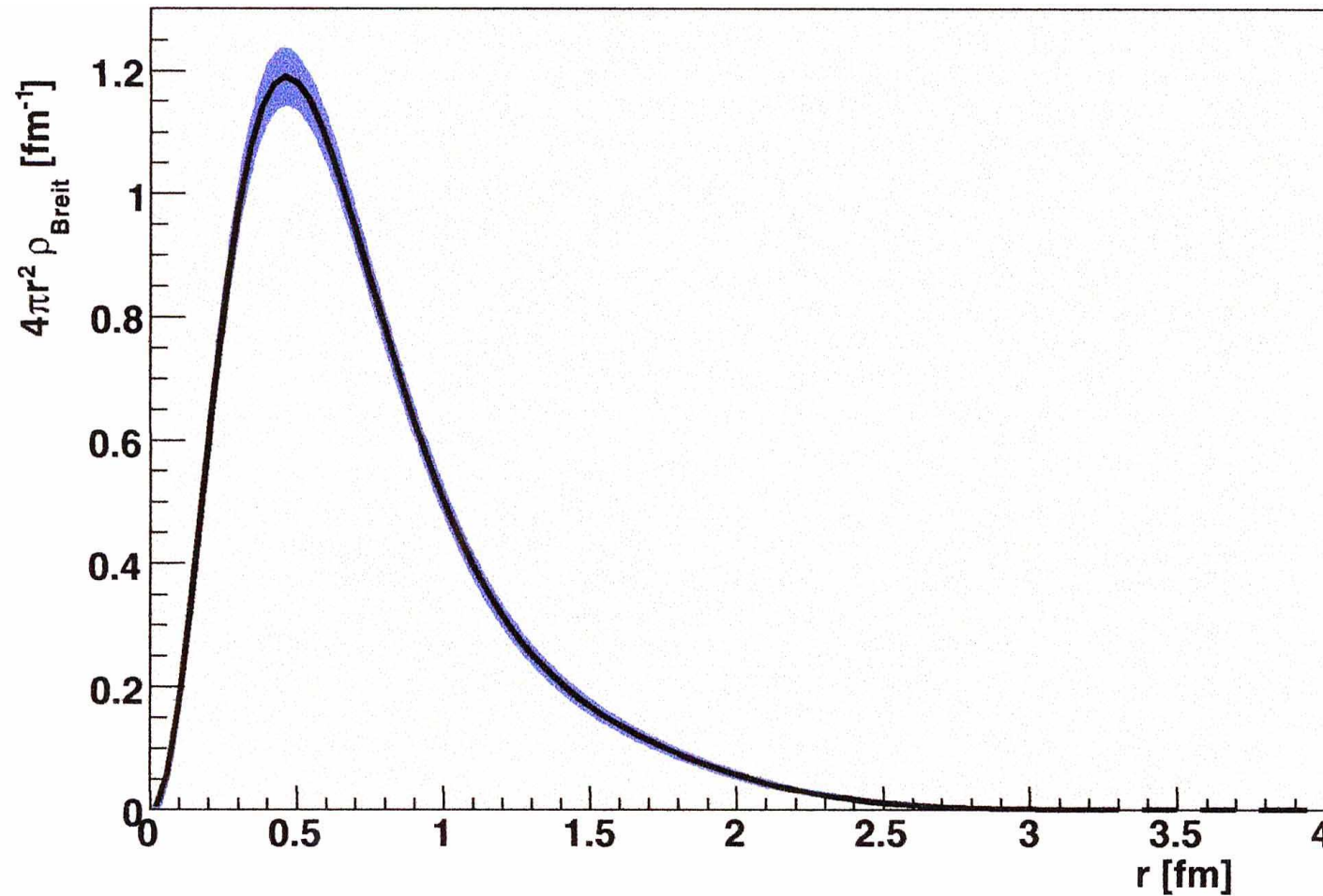


Figure 10.1: From “The Frontiers of Nuclear Science — A Long Range Plan, December 2007”, by the Nuclear Science Advisory Committee. The width of the color band represents the uncertainty.

10.0: Introduction: Neutron Charge Distribution

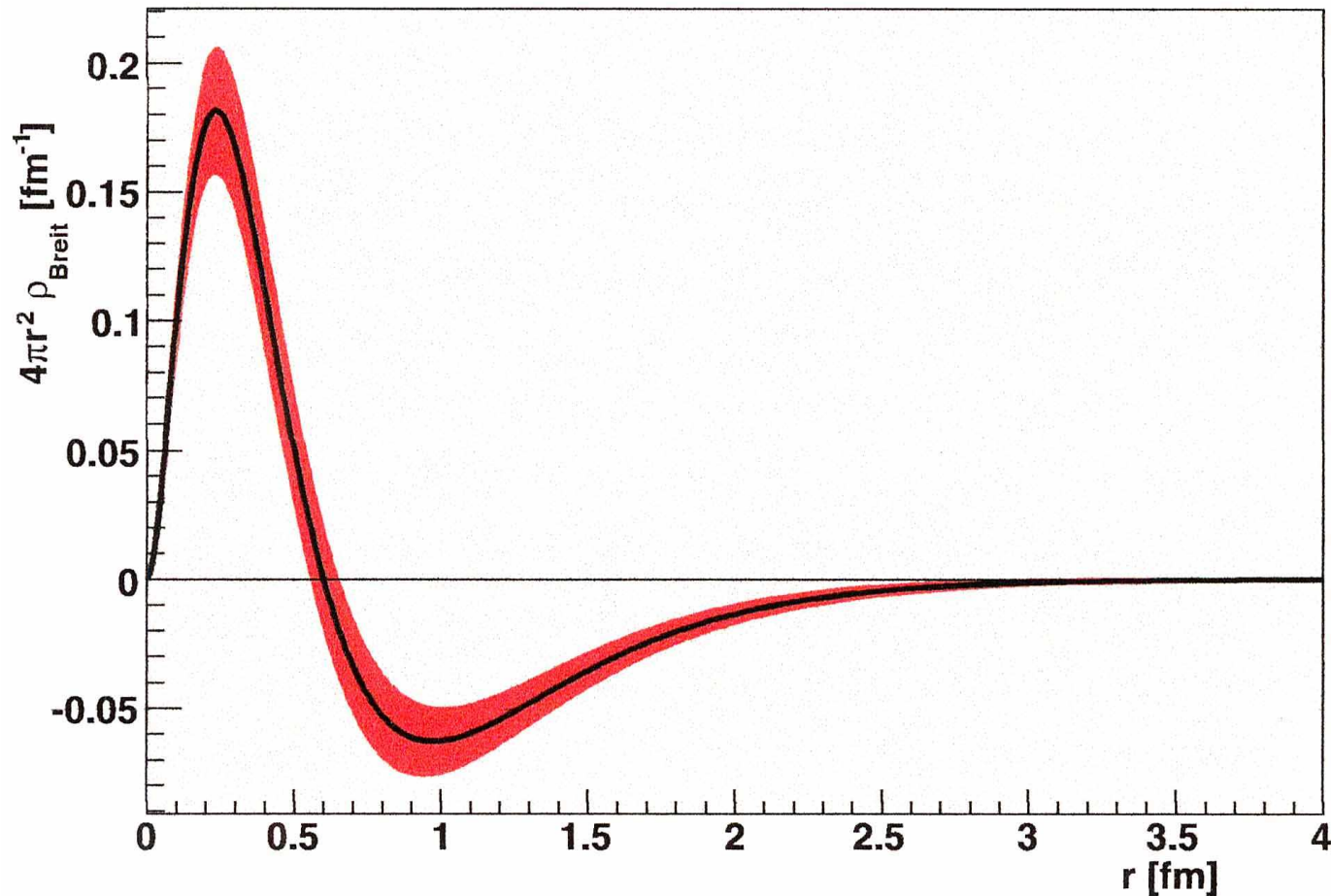


Figure 10.2: From “The Frontiers of Nuclear Science — A Long Range Plan, December 2007”, by the Nuclear Science Advisory Committee. The width of the color band represents the uncertainty.

10.0: Introduction: The nucleon-nucleon force

What binds two nucleons together?

- The nucleon force is a “derivative force”, generated by the underlying constituent particles, much like an electrostatic dipole field is created by two nearby, equal charges, but of opposite sign.
- The n - n , n - p , and p - p nuclear forces are all almost identical.
- There is a strong short-range repulsive force, of the form:

$$V_{nn}^{\text{rep}}(r) \equiv \frac{A_{nn}^{\text{rep}}}{r} \exp(-r/r_{nn}^{\text{rep}}); \quad \text{where } r_{nn}^{\text{rep}} \approx 0.25 \text{ fm}$$

- There is a strong medium-range attractive force, of the form:

$$V_{nn}^{\text{att}}(r) \equiv -\frac{A_{nn}^{\text{att}}}{r} \exp(-r/r_{nn}^{\text{att}}); \quad \text{where } r_{nn}^{\text{att}} \approx 0.36 \text{ fm} \quad \& \quad \frac{A_{nn}^{\text{att}}}{A_{nn}^{\text{rep}}} \approx 0.67$$

- The combined force is:

$$V_{nn}(r) \equiv \frac{A_{nn}^{\text{rep}}}{r} \exp(-r/r_{nn}^{\text{rep}}) - \frac{A_{nn}^{\text{att}}}{r} \exp(-r/r_{nn}^{\text{att}})$$

10.0: Introduction: The nucleon-nucleon force

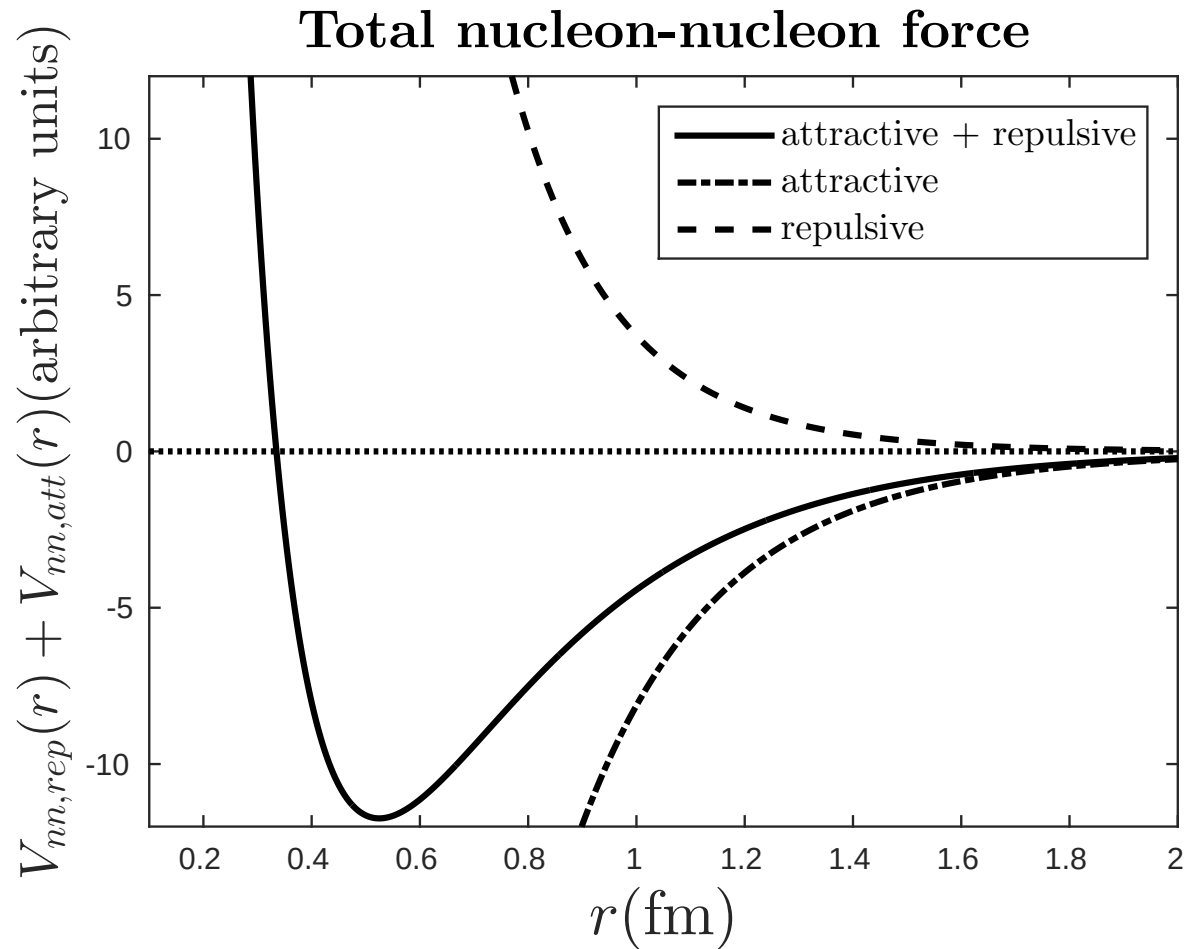


Figure 10.3: The nucleon-nucleon potential, the sum of the repulsive (upper dashed line) and attractive (lower dash-dot line) nuclear potentials.

10.0: Introduction: The nucleon-nucleon force

The nucleon-nucleon force, key observations and conclusions

- The nuclear force bounds nucleons tightly, it is short-ranged. We can almost think of it as a “contact” force. (Ping-pong balls covered in VaselineTM.)
- Nucleons are fermions, hence n 's and p 's tend to avoid each other.
- The nuclear part of the nucleon-nucleon force is central. (Y_{lm_l})!
- Protons are subject to a repulsive Coulomb force (also central).
- Since n 's and p 's have magnetic moments, they are both subject to magnet and motion ($\vec{v} \times \vec{B}$) forces, *i.e.* , spin-spin, and spin-orbit effects. These have enormous impact because the “magnets” are in close proximity.

10.0: Introduction: Some nomenclature

${}^A_Z\text{X}_N$	Isotope notation
X	Chemical symbol, e.g. Ca, Pb
A	Atomic mass number (sum of n 's and p 's in the nucleus)
Z	Atomic number (or, proton number), the number of p 's in the nucleus
N	Neutron number, the number of n 's in the nucleus
Examples	${}^3_2\text{He}_1$, ${}^{40}_{20}\text{Ca}_{20}$, ${}^{208}_{82}\text{Pb}_{126}$
Variants	${}^{40}\text{Ca}$, Calcium-40, Ca-40

Note that, once X (which encodes Z) and A are given, the rest of the information is redundant, since $A = Z + N$. The full form is usually used only for emphasis.

isotope	Same Z , different N e.g. ${}^{40}\text{Ca}$ and ${}^{41}\text{Ca}$ Mnemonic: From Greek <i>isos</i> (same) <i>topos</i> (place) (coined by F. Soddy 1913) <i>i.e.</i> same place in the periodic table
isotone	Different Z , same N , e.g. ${}^{13}\text{C}$ and ${}^{12}\text{B}$ Mnemonic: isoto P e and isoto N e (coined by K. Guggenheimer 1934)
isobar	Different Z , <u>and</u> N , but same A , e.g. ${}^{12}\text{C}$ and ${}^{12}\text{B}$ Mnemonic: From Greek <i>isos</i> (same) <i>baros</i> (weight)

10.0: Introduction: Nucleus formation

How are nuclei formed?

- Because of the short-ranged nature of the nucleon-nucleon force, and its near extinction at approximately 2 fm, nuclei are tightly bound, and each nucleon in the nuclear core (away from the surface), is bound only by its surrounding neighbors.
- Therefore, the nucleons in the core have their nuclear forces saturated, and it seems to them, that they are in a constant, central (isotropic), nuclear potential. This is an established, experimentally-verified, fact.
- The nucleons near the surface, sometimes called the valence nucleons, are less bound, having fewer neighbors, than their “core” counterparts.
- Nucleons are quantum particles. Quantum particles in bound potentials have quantized energy levels.
- n 's and p 's are fermions, and hence subject to the Pauli Exclusion Principle, within their own group. Therefore, we may build up the nucleus, much as we did for the electrons in the atomic shell (as in NERS 311), except that we have two distinct nucleons to account for.

10.0: Introduction: Nucleus formation

How are nuclei formed? (con't)

- As a nucleus gains nucleons, Coulomb repulsion, a long-range force, is felt by **all** the protons in the nucleus, that is, each proton feels the repulsion of all of the other protons in the nucleus. At some point, Coulomb repulsion overwhelms the nuclear binding potential, and the nucleus can not be stable.
- Nuclei can offset this instability by increasing the number of n 's, compared to the number of p 's, thereby pushing the p 's to greater average radii. This strategy eventually fails for $A > 208$.
- The heaviest isotope with an equal number of n 's and p 's is calcium. The heaviest stable isotope is lead, with $A = 208$.
- The strength of the nuclear force suggests that all nuclei are spherical in shape. (This is very nearly true. Exceptions will be discussed later in the course.) The radius of the nucleus will be discussed in the next section.

10.1: The nuclear radius

We have argued that the nucleus ought to be a spherical object.

Q: How to measure that? A: *Bang things together and interpret!*

Q: How?

A: *Bombard the nucleus with e^- 's (electrons).*

Measure their deflection, from the p 's in the nucleus.

Q: Can you get information about the radius of the nucleus?

A: *Yes! And as it turns out, that's the best way to do it!*

But, you should account for a few things ... a few very important things.

10.1: e^- scattering to measure the radius of the nucleus

What projectile energy should I use?

Here we appeal to Quantum Mechanics:

The effective size of the wave associated with a relativistic electron is:

$$\frac{\lambda}{2\pi} = \frac{\hbar}{p_e} = \frac{\hbar c}{p_e c} \approx \frac{\hbar c}{E_e} = \frac{197 [\text{MeV}\cdot\text{fm}]}{E_e [\text{MeV}]} \lesssim R_N \quad ; \quad R_N \equiv \text{radius of the nucleus [fm]}$$

We now know that nuclear radii fall in the range 2–8 fm.

$\therefore E_e$ should be 10's or even 100's of MeV.

At much greater energy, about 2 GeV, one is able to “see” the quark structure inside a nucleon!

How do I analyze the results of the scattering experiments?

Now, this is a long story ...

10.1: e^- scattering analysis

It starts with consideration of a factor we'll call the “scattering amplitude”, F , where

$$F(\vec{k}_i, \vec{k}_f) = N_F \langle e^{i\vec{k}_f \cdot \vec{x}} | V(\vec{x}) | e^{i\vec{k}_i \cdot \vec{x}} \rangle, \quad (10.1)$$

where ...

$e^{i\vec{k}_i \cdot \vec{x}} \equiv$ the initial unscattered e^- wavefunction

$e^{i\vec{k}_f \cdot \vec{x}} \equiv$ the final scattered e^- wavefunction

$\vec{k}_i \equiv$ the initial e^- wavenumber

$\vec{k}_f \equiv$ the final e^- wavenumber

$V(\vec{x}) \equiv$ the electrostatic Coulomb potential arising from the +ve charge distribution in the nucleus

$N_F \equiv$ a proportionality constant to be determined later

10.1: e^- scattering analysis (con't)

Evaluating ...

$$\begin{aligned} F(\vec{k}_i, \vec{k}_f) &= N_F \int d\vec{x} e^{-i\vec{k}_f \cdot \vec{x}} V(\vec{x}) e^{i\vec{k}_i \cdot \vec{x}} \\ &= N_F \int d\vec{x} V(\vec{x}) e^{i(\vec{k}_i - \vec{k}_f) \cdot \vec{x}} \\ &= N_F \int d\vec{x} V(\vec{x}) e^{i\vec{q} \cdot \vec{x}}, \end{aligned}$$

where $\vec{q} \equiv \vec{k}_i - \vec{k}_f$ is called the *momentum transfer*.

It is the momentum taken (transferred) from $\vec{k}_i$ to produce $\vec{k}_f$, since $\vec{k}_f = \vec{k}_i - \vec{q}$.

We see that F is a function of the momentum transform alone:

$$F(\vec{q}) = N_F \int d\vec{x} V(\vec{x}) e^{i\vec{q} \cdot \vec{x}}. \quad (10.2)$$

We also see that scattering amplitude is proportional to the 3D Fourier Transform of the potential. We are now, hopefully, in familiar mathematical territory!

10.1: e^- scattering analysis (con't)

We note, that the expression we started with, viz. $F(\vec{k}_i, \vec{k}_f) = N_F \langle e^{i\vec{k}_f \cdot \vec{x}} | V(\vec{x}) | e^{i\vec{k}_i \cdot \vec{x}} \rangle$ is **completely** general.

The $\langle \rangle$ term is to be interpreted as follows:

$V(\vec{x})$ (whatever that happens to be) is an “action” that operates on the initial wavefunction $\exp(i\vec{k}_i \cdot \vec{x})$.

The calculation implied by the $\langle \rangle$, an integration over all space, measures the overlap of the transformed (i.e. acted upon by $V(\vec{x})$) initial wavefunction $\exp(i\vec{k}_i \cdot \vec{x})$, with the final wavefunction $\exp(i\vec{k}_f \cdot \vec{x})$.

How accurate is this “scattering factor”?

There are several approximations implied by our formulation, that we have not spoken of.

Can you suggest what they are?

(It turns out that the approximations have little effect on our intended application.)

10.1: e^- scattering analysis (con't) (an aside on accuracy)

In this application...

1. $V(\vec{x})$ is treated as a classical, continuous charge distribution. In fact, the operator is made up of the quantum mechanical E&M operator over the composite proton wavefunction.
2. The proton charge density is not continuous, nor is it static.
3. The initial and final wavefunctions are not distorted by the potential. This is called the First Born Approximation. It happens to work for this application because the E&M forces are relatively weak.

Returning to our analysis ...

10.1: e^- scattering analysis (con't)

The scattering potential takes the form:

$$V(\vec{x}) = -\frac{Ze^2}{4\pi\epsilon_0} \int d\vec{x}' \frac{\rho_p(\vec{x}')}{|\vec{x} - \vec{x}'|} , \quad (10.3)$$

$\rho_p(\vec{x}') \equiv$ the “number” density of protons in the nucleus, normalized so that:

$$\int d\vec{x}' \rho_p(\vec{x}') \equiv 1 . \quad (10.4)$$

The potential at $\vec{x}$ arises from the electrostatic attraction of the elemental charges in $d\vec{x}$, integrated over the nucleus.

Putting it all together:

$$F(\vec{q}) = N_F \left(-\frac{Ze^2}{4\pi\epsilon_0} \right) \int d\vec{x} \int d\vec{x}' \frac{\rho_p(\vec{x}')}{|\vec{x} - \vec{x}'|} e^{i\vec{q} \cdot \vec{x}} . \quad (10.5)$$

We choose the constant of proportionality in $F(\vec{q})$, to require that $F(0) \equiv 1$. The motivation for this choice is that, when $\vec{q} = 0$, the charge distribution is known to have no effect on the projectile. If a potential has no effect on the projectile, then we can rewrite (10.5) as

$$F(0) \equiv 1 = N_F \left(-\frac{Ze^2}{4\pi\epsilon_0} \right) \int d\vec{x} \int d\vec{x}' \frac{\rho_p(\vec{x}')}{|\vec{x} - \vec{x}'|} , \quad \text{or...} \quad (10.6)$$

10.1: e^- scattering analysis (con't)

$$N_F = \left[\left(-\frac{Ze^2}{4\pi\epsilon_0} \right) \int d\vec{x} \int d\vec{x}' \frac{\rho_p(\vec{x}')}{|\vec{x} - \vec{x}'|} \right]^{-1},$$

and

$$F(\vec{q}) = \left[\int d\vec{x} \int d\vec{x}' \frac{\rho_p(\vec{x}')}{|\vec{x} - \vec{x}'|} e^{i\vec{q}\cdot\vec{x}} \right] / \left[\int d\vec{x} \int d\vec{x}' \frac{\rho_p(\vec{x}')}{|\vec{x} - \vec{x}'|} \right]$$

Remarkably, we can collapse the expression above significantly.

The details of this calculation will be left to enthusiastic students to discover for themselves. The final result is:

$$F(\vec{q}) = \int d\vec{x} \rho_p(\vec{x}) e^{i\vec{q}\cdot\vec{x}}, \quad (10.7)$$

and the “scattering amplitude” has been shown to be the 3D Fourier transform of the charge distribution.

We shall soon see, that $\sqrt{|F(\vec{q})|^2}$ is a function that can be explicitly measured in experiments.

First, let's work out some simple examples of $F(\vec{q})$.

10.1.1: $F(\vec{q})$ for spherical charge distributions

For spherical charge distributions (the usual case), $\rho_p(\vec{x}) = \rho_p(r)$, (10.7) can be further reduced. We start with:

$$F(\vec{q}) = \int d\vec{x} \rho_p(\vec{x}) e^{i\vec{q} \cdot \vec{x}}$$

Now, we convert $(x, y, z) \longrightarrow (r, \theta, \phi)$ and align $\vec{q} \longrightarrow q\hat{z}$.

Since the charge distribution is spherically symmetric, we can choose any axis to align $\vec{q}$ with, and the $\hat{z}$ -axis is the most convenient.

$$F(q) = \int_0^{2\pi} d\phi \int_0^\infty r^2 dr \rho_p(r) \int_0^\pi \sin \theta d\theta e^{iqr \cos \theta} \quad [\text{note } F(\vec{q}) \longrightarrow F(q)] \quad (10.8)$$

$$= 2\pi \int_0^\infty r^2 dr \rho_p(r) \int_0^\pi \sin \theta d\theta e^{iqr \cos \theta} \quad [\text{did the integral over } \phi]$$

$$= 2\pi \int_0^\infty r^2 dr \rho_p(r) \int_{-1}^1 d\mu e^{iqr\mu} \quad [\text{change of variable } \mu = \cos \theta]$$

$$= 2\pi \int_0^\infty r dr \rho_p(r) \left(\frac{2}{q}\right) \sin qr \quad [\text{did the integral over } \mu]$$

$$= \frac{4\pi}{q} \int_0^\infty r dr \rho_p(r) \sin qr \quad [\text{in final form}] \quad (10.9)$$

10.1.1: Measurements of nuclear radius

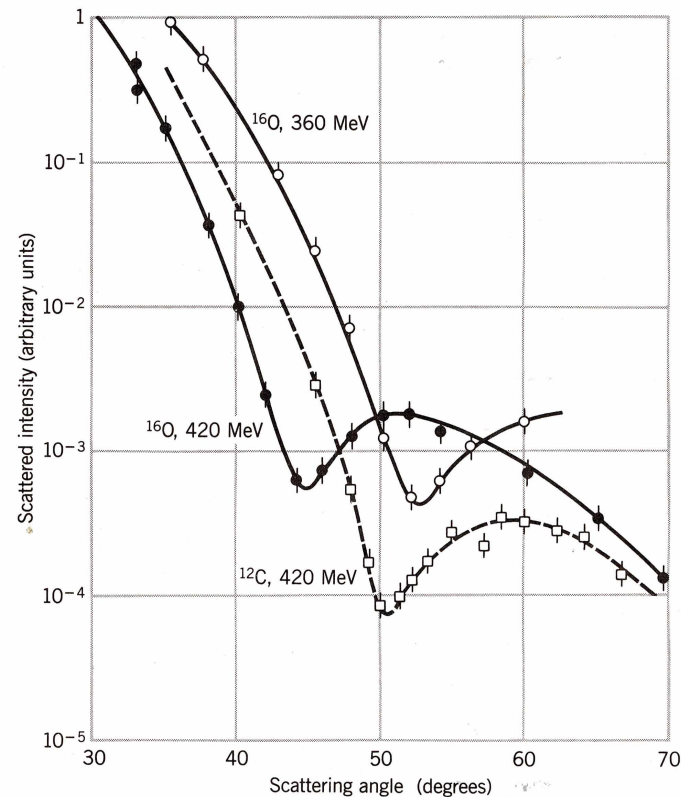


Figure 3.1 Electron scattering from ^{16}O and ^{12}C . The shape of the cross section is somewhat similar to that of diffraction patterns obtained with light waves. The data come from early experiments at the Stanford Linear Accelerator Center (H. F. Ehrenberg et al., *Phys. Rev.* **113**, 666 (1959)).

Figure 10.4: From "Introductory Nuclear Physics", by Kenneth Krane

10.1.1: Measurements of nuclear radius

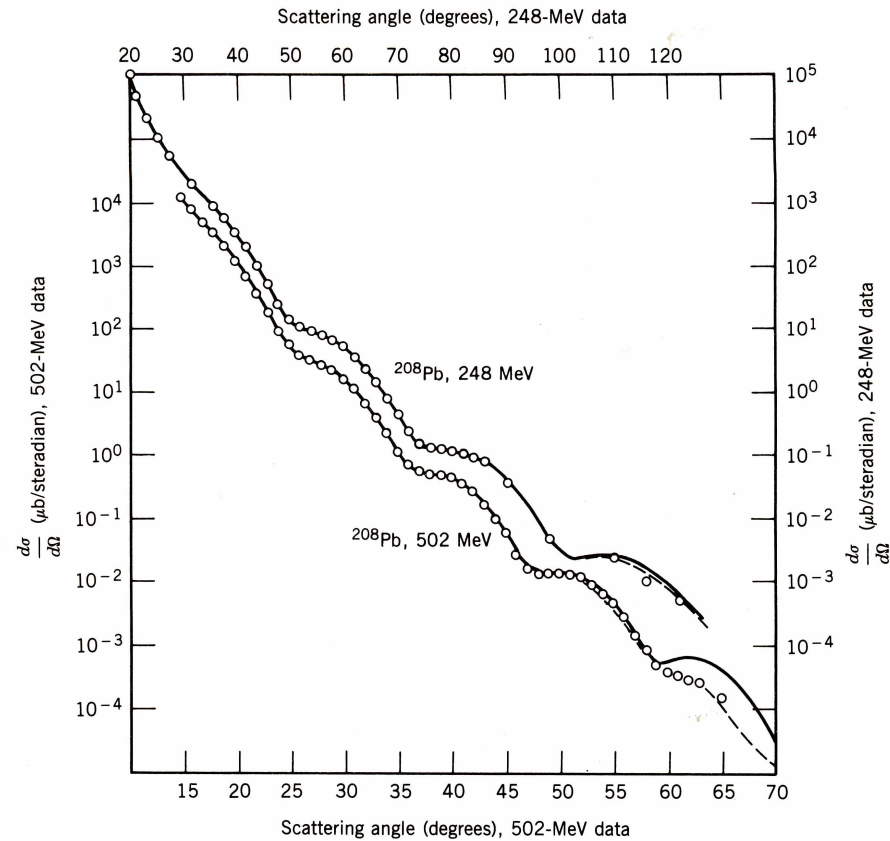


Figure 3.2 Elastic scattering of electrons from ^{208}Pb . Note the different vertical and horizontal scales for the two energies. This also shows diffractionlike behavior, but lacks sharp minima. (J. Heisenberg et al., *Phys. Rev. Lett.* **23**, 1402 (1969).)

Figure 10.5: From “Introductory Nuclear Physics”, by Kenneth Krane

10.1.1: Measurements of nuclear radius

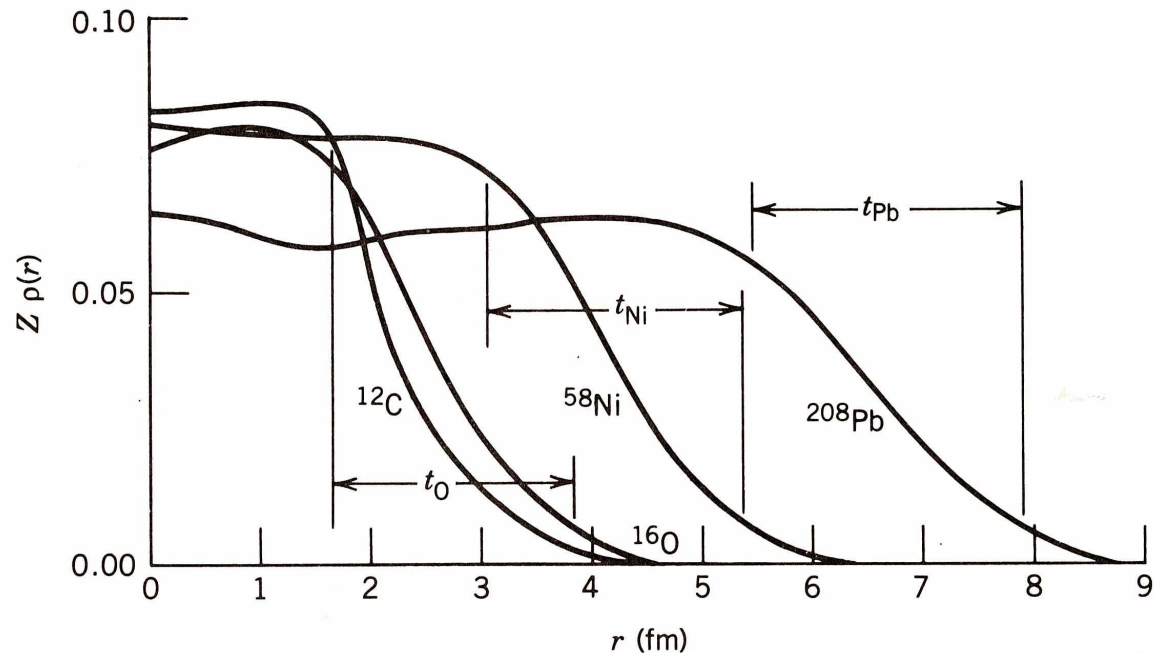


Figure 3.4 The radial charge distribution of several nuclei determined from electron scattering. The skin thickness t is shown for O, Ni, and Pb; its value is roughly constant at 2.3 fm. The central density changes very little from the lightest nuclei to the heaviest. These distributions were adapted from R. C. Barrett and D. F. Jackson, *Nuclear Sizes and Structure* (Oxford: Clarendon, 1977), which gives more detail on methods of determining $\rho(r)$.

Figure 10.6: From "Introductory Nuclear Physics", by Kenneth Krane

10.1.1: Conclusions of nuclear radius measurements

1. The central density, is (roughly) constant, almost independent of atomic number, and has a value about $0.13/\text{fm}^3$. This is very close to the density of nuclear matter in the infinite radius approximation,

$$\rho_0 = 3/(4\pi R_0^3) .$$

2. The “skin depth”, s , is (roughly) constant as well, almost independent of atomic number, with a value of about 2.9 fm, typically. The skin depth is usually defined as the difference in radii of the nuclear densities at 90% and 10% of maximum value.
3. Scattering measurements suggest a best fit to the radius of nuclei:

$$R_N = R_0 A^{1/3} \quad ; \quad R_0 \approx 1.22 \text{ [fm]} \quad ; \quad 1.20 \longrightarrow 1.25 \text{ is also common.} \quad (10.10)$$

A convenient parametric form of the nuclear density was proposed by Woods and Saxon (*ca.* 1954).

$$\rho_N(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R_N}{t}\right)}$$

where t is a surface thickness parameter, related to s , by $s = 4t \log(3)$.

10.1.1: Conclusions of nuclear radius measurements (con't)

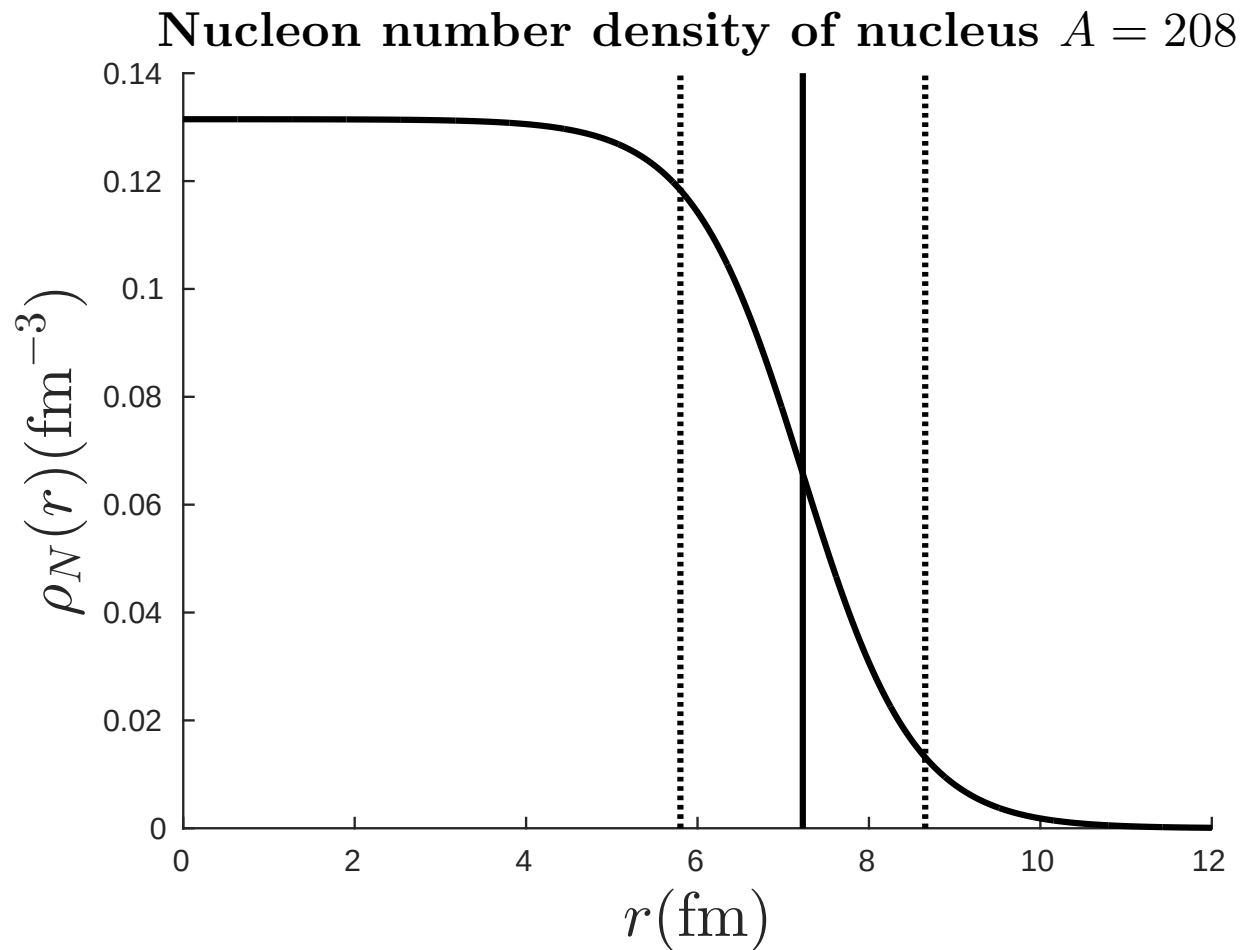
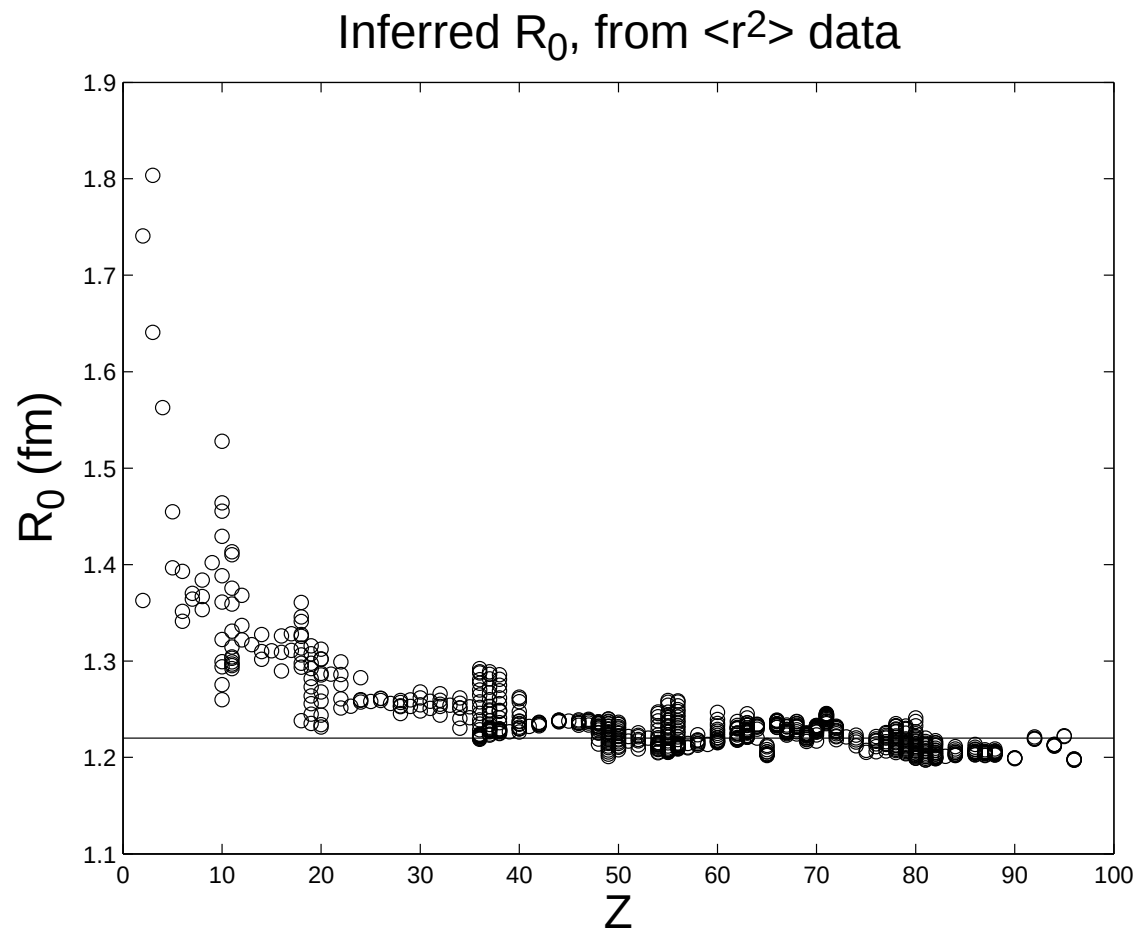


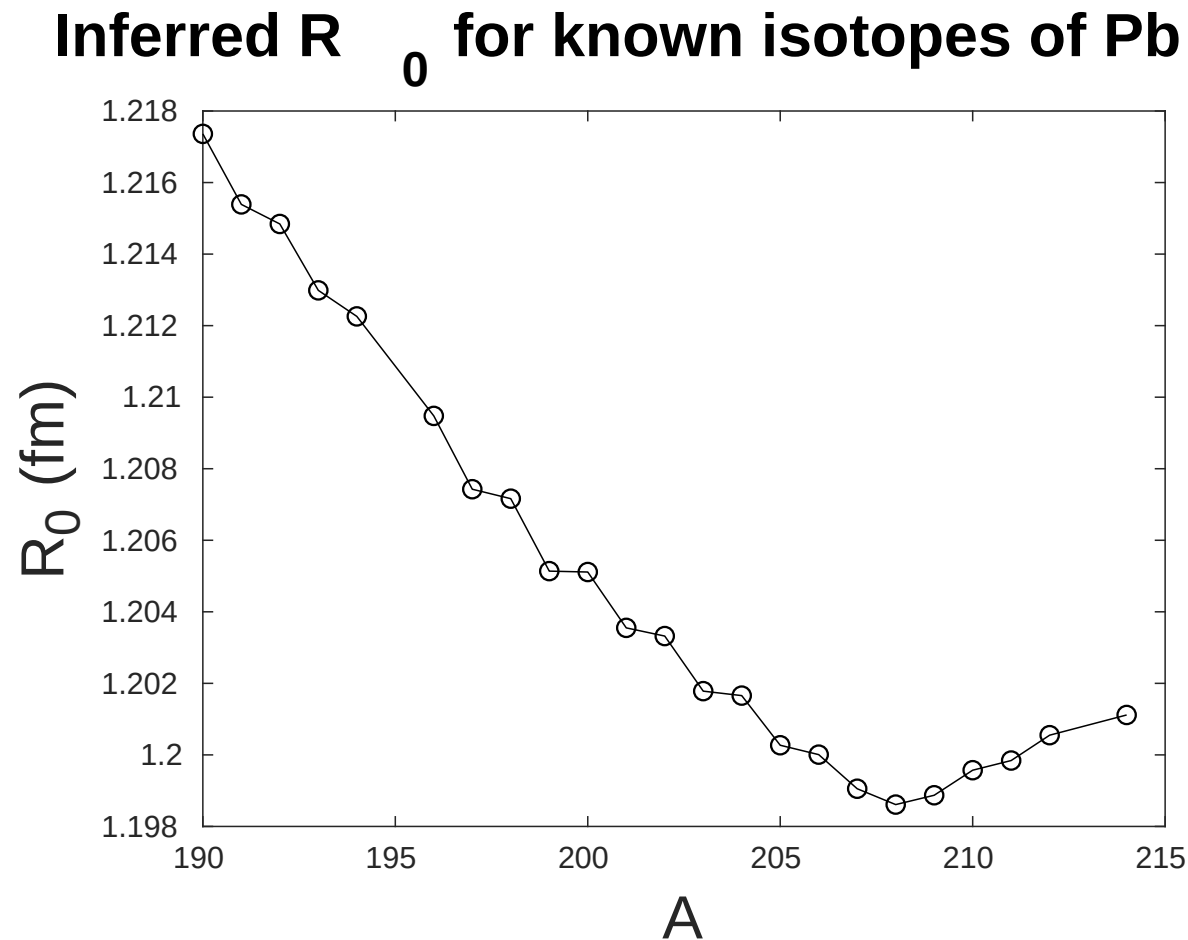
Figure 10.7: The Woods-Saxon model of the nucleon number density. In this figure, $A = 208$, $R_0 = 1.22$ (fm), and $t = 0.65$ (fm). The skin depth is shown, delimited by vertical dotted lines.

10.1.1: Conclusions of nuclear radius measurements (con't)

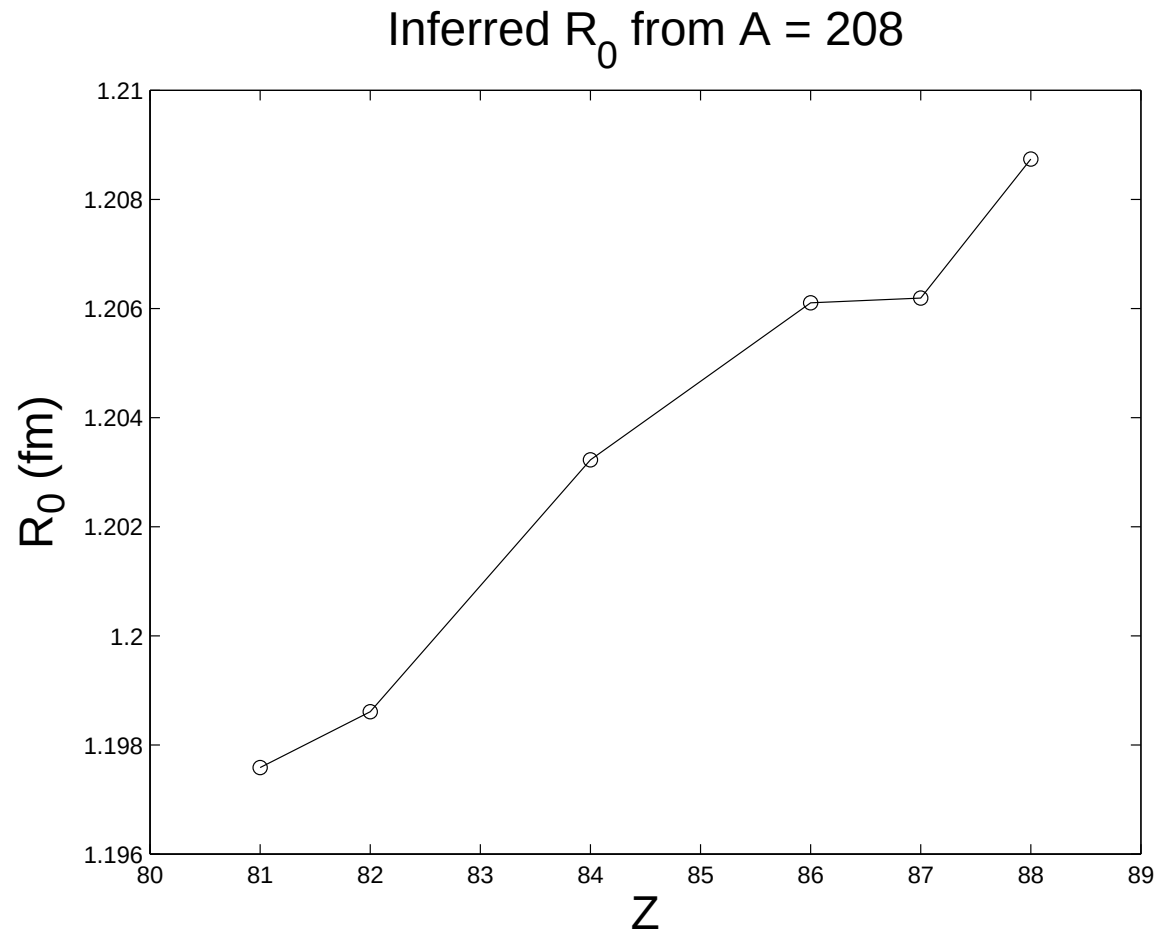
Comparison of known charge radii, the R_0 inferred from that data and the accepted value of $R_0 = 1.22$. The data is from: *A consistent set of nuclear rms charge radii: properties of the radius surface $R(N, Z)$* by I. Angeli, published in Atomic Data and Nuclear Data Tables 87 (2004) 185—206.



10.1.1: Conclusions of nuclear radius measurements (con't)



10.1.1: Conclusions of nuclear radius measurements (con't)



10.1.1: Example $|F(\vec{q})|^2$ from a uniform charge distribution

In this case, the normalized proton density takes the form:

$$\rho_p(r) = \frac{3}{4\pi R_N^3} \Theta(R_N - r) . \quad (10.11)$$

Thus, combining (10.9) and (10.11), gives, after some reorganization:

$$F(q) = \frac{3}{(qR_N)^3} \int_0^{(qR_N)} dz \, z \sin z , \quad (10.12)$$

which is easily evaluated to be,

$$F(q) = \frac{3[\sin(qR_N) - qR_N \cos(qR_N)]}{(qR_N)^3} , \quad (10.13)$$

for which $F(0) = 1$, as expected.

Technical side note:

The following Mathematica code was useful in deriving the above relations.

```
(* Here Z == q*R_N: *)
```

```
(3/Z^3)*Integrate[z Sin[z], {z,0,Z}]
```

```
Series[3*(Sin[Z] - Z*Cos[Z])/Z^3,{Z,0,2}]
```

10.1.1: ...Example $|F(\vec{q})|^2$ from a uniform charge distribution...

Note the minima when $\tan(qR_N) = qR_N$. Measurements do not have such deep minima, because: 1) the nuclear edge is blurred, not sharp, 2) the projectiles are polyenergetic, 3) the detectors have imperfect resolution.

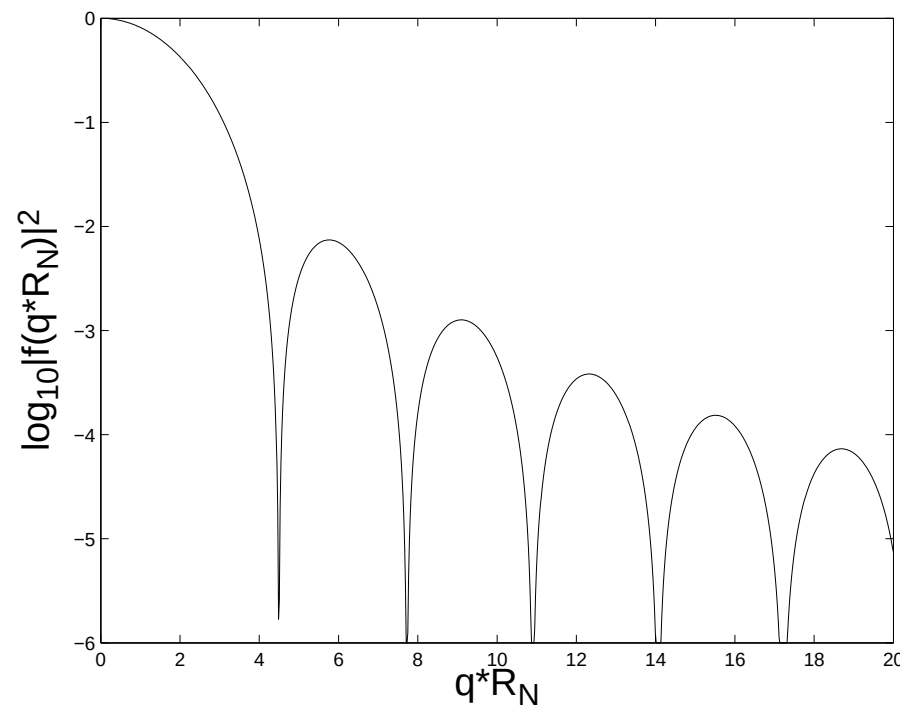


Figure 10.8: Graphical output corresponding to (10.13).

10.1.1: ...Example $|F(\vec{q})|^2$ from a uniform charge distribution...

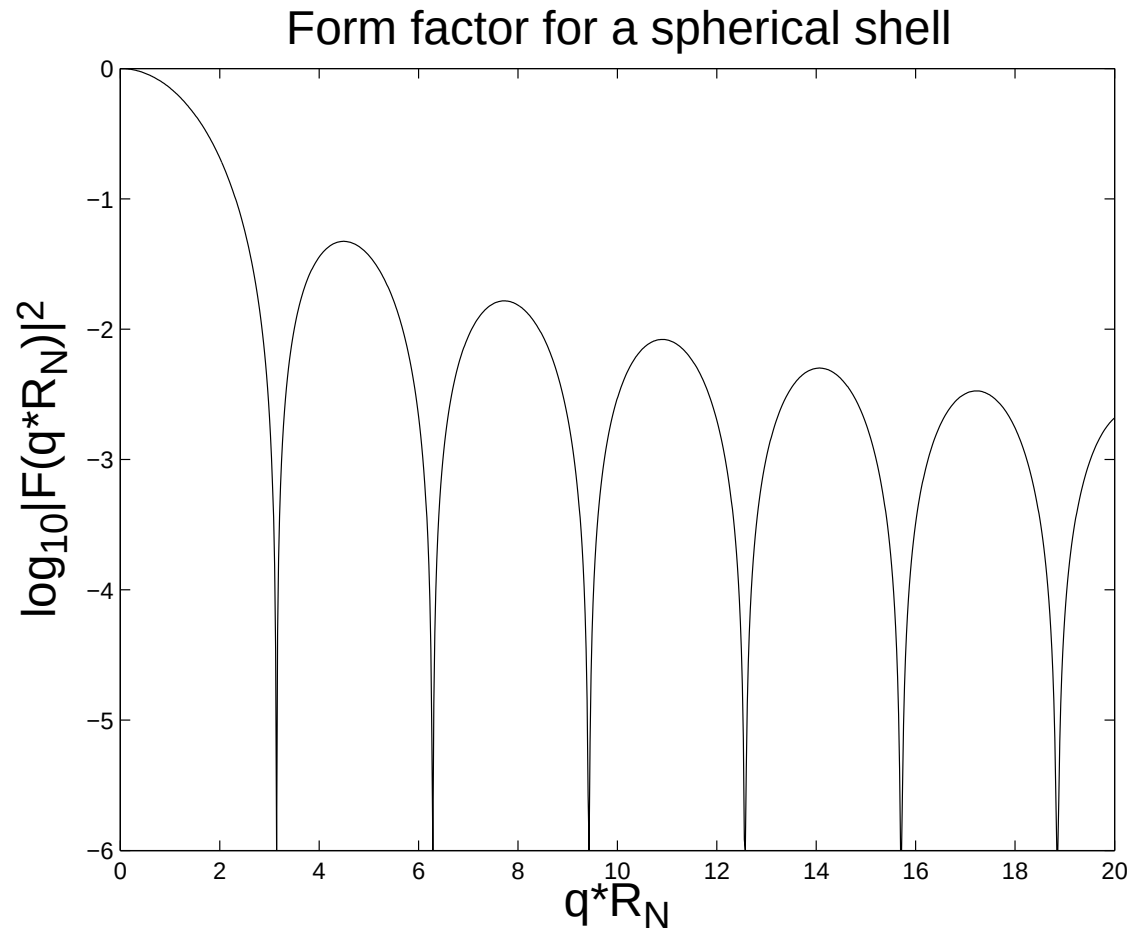
Technical side note:

The following Matlab code was useful in producing the above graph.

```
N = 1000; fMin = 1e-6; zMax = 20; % Graph data
z = linspace(0,zMax,N); f = 3*(sin(z) - z.*cos(z))./z.^3;
f(1) = 1; % Overcome the singularity at 0
f2 = f.*f;
for i = 1:N
    f2(i) = max(fMin,f2(i));
end
plot(z,log10(f2),'-k')
xlabel('\fontsize{20}q*R_N')
ylabel('\fontsize{20}log_{10}|f(q*R_N)|^2')
```

10.1.1: Another example (this one is not in the book)

Just for practice, we will try another example $\rho_p(r) = \delta(R_N - r)/4\pi R_N^2$
Following the same procedure as the previous example $F(q) = \sin(qR_N)/qR_N$



10.1.2: Finally ... how to get $F(q)$ from measurements

Technical side note: The mathematical details can be found in the supplemental notes.

The supplementary notes obtain the following, very significant result, that a full quantum mechanical derivation of the cross section of e^-N scattering obtains the following result:

$$\frac{d\sigma_{eN}}{d\Omega} = \frac{d\sigma_{eN}^{\text{Ruth}}}{d\Omega} |F(q)|^2, \quad (10.14)$$

where $d\sigma_{eN}^{\text{Ruth}}/d\Omega$ is the classical Rutherford cross section discussed in NERS311, and $|F(q)|^2$ is the scattering amplitude that we have been discussing.

Hence, we have a direct experimental determination of the form factor, as a ratio of the measurement data (the measured cross section), to the theoretical function (the Rutherford cross section):

$$|F(q)|^2 = \left(\frac{d\sigma_{eN}^{\text{meas}}}{d\Omega} \right) / \left(\frac{d\sigma_{eN}^{\text{Ruth}}}{d\Omega} \right). \quad (10.15)$$

10.1.2: The quantum mechanical Rutherford cross section

All that remains is to take the square root, and invert the Fourier Transform, to get $\rho(r)$. This is always done via a relatively simple numerical process.

Although the form factor $|F(q)|^2$ is given in terms of q , we may cast it into more recognizable kinematic quantities as follows. Recall,

$$q = \sqrt{q^2} = \sqrt{|\vec{k}_i - \vec{k}_f|^2} = \sqrt{2k^2(1 - \cos \theta)} , \quad (10.16)$$

the final step above being obtained since this is an elastic scattering process, where

$$k = |\vec{k}_i| = |\vec{k}_f| \text{ and } \vec{k}_i \cdot \vec{k}_f = k^2 \cos \theta.$$

10.1.3: Nuclear size from spectroscopy measurements

An ideal probe of the effect of a finite-sized nucleus, would be a $1s$ atomic state.

The $1s$ state has the most probability density in the vicinity of the nucleus.

The shift of energy (from 1st-order perturbation theory), of the $1s$ can be estimated as follows:

$$\Delta E_{1s} = \langle \psi_{1s} | V_o(r) - V_-(r) | \psi_{1s} \rangle , \quad (10.17)$$

where the ψ_{1s} is the $1s$ wavefunction for the point-like nucleus, $V_o(r)$ is the Coulomb potential for the finite nucleus, and $V_-(r)$ is the point-like Coulomb potential.

10.1.3: Nuclear size from spectroscopy measurements (con't)

For a uniform sphere of charge, we know from Classical Electrostatics, that $V_o(r) = V.(r)$ for $r \geq R_N$.

$$\begin{aligned} V.(0 \leq r < \infty) &= -\frac{Ze^2}{4\pi\epsilon_0 r} \\ V_o(r \leq R_N) &= -\frac{Ze^2}{4\pi\epsilon_0 R_N} \left[\frac{3}{2} - \frac{1}{2} \left(\frac{r}{R_N} \right)^2 \right] \\ V_o(r \geq R_N) &= V.(r \geq R_N) \end{aligned} \tag{10.18}$$

or

$$V_o(r) - V.(r) = \frac{Ze^2}{4\pi\epsilon_0 R_N} \left[\frac{R_N}{r} - \frac{3}{2} + \frac{1}{2} \left(\frac{r}{R_N} \right)^2 \right] \theta(R_N - r)$$

We evaluate this by combining the above with $\Delta E_{1s} = \langle \psi_{1s} | V_o(r) - V.(r) | \psi_{1s} \rangle$ and using the hydrogenic wavefunctions given in NERS311 and also in Krane II (Tables 2.2 and 2.5)

$$\psi_{1s, (n,l,m_l)=(1,0,0)} = \left\{ R_{n=1,l=0}(r) \right\} \left[Y_{l=0,m_l=0}(\theta, \phi) \right] = \left\{ 2 \left(\frac{Z}{a_0} \right)^{3/2} e^{-Zr/a_0} \right\} \left[\frac{1}{4\pi} \right]^{1/2}$$

10.1.3: Nuclear size from spectroscopy measurements (con't)

and obtain:

$$\begin{aligned} \Delta E_{1s} = & \langle \psi_{1s} | V_o(r) - V_-(r) | \psi_{1s} \rangle \\ & \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta \int_0^\infty dr r^2 |\psi_{1s}(\vec{x})|^2 [V_o(r) - V_-(r)] \\ & \frac{e^2}{4\pi\epsilon_0} \frac{4Z^4}{a_0^3} \frac{1}{R_N} \int_0^{R_N} dr r^2 e^{-2Zr/a_0} \left[\frac{R_N}{r} - \frac{3}{2} + \frac{1}{2} \left(\frac{r}{R_N} \right)^2 \right], \end{aligned} \quad (10.19)$$

where $a_0 = 5.2917721092(17) \times 10^{-11}$ m, *i.e.*, the Bohr radius.

10.1.3: Nuclear size from spectroscopy measurements (con't)

In unitless quantities, we may rewrite the above as:

$$\Delta E_{1s} = Z^2 \alpha^2 (m_e c^2) \left(\frac{2ZR_N}{a_0} \right)^2 \int_0^1 dz e^{-(2ZR_N/a_0)z} \left[z - \frac{3}{2}z^2 + \frac{z^4}{2} \right]. \quad (10.20)$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{1}{137.035999074(44)} \text{ (fine-structure constant, unitless).}$$

Across all the elements, the dimensionless parameter $(2ZR_N/a_0)$ spans the range $2 \times 10^{-5} \rightarrow \approx 10^{-2}$. Hence, the contribution to the exponential, in the integral, is inconsequential. (You should not make this approximation for muonic atoms [next section].) The remaining integral is a pure number and evaluates to $1/10$. Thus, we may write:

$$\Delta E_{1s} \approx \frac{1}{10} Z^2 \alpha^2 (m_e c^2) \left(\frac{2ZR_N}{a_0} \right)^2. \quad (10.21)$$

This correction is about 1 eV for $Z = 100$ and much smaller for lighter nuclei.

This is a very small difference!

10.1.3: Nuclear size from spectroscopy measurements (con't)

Not only is this shift in energy small, it measures the shift in energy by introducing a realistic charge distribution, and comparing the bound state energy with the bound-state energy arising from a point nucleus.

Point nuclei, do not exist in nature, so, we still have work to do!

We don't have anything to compare with experiment yet.

So ...

10.1.3: Nuclear size from spectroscopy measurements (con't)

Nuclear size determination from an isotope shift measurement

Let us imagine how we are to determine the nuclear size, by measuring the energy of the photon that is given off, from a $2p \rightarrow 1s$ transition.

The Schrödinger equation predicts that the energy of the photon will be given by:

$$(E_{2p \rightarrow 1s})_o = (E_{2p \rightarrow 1s})_i + \langle \psi_{2p} | V_o(r) - V_i(r) | \psi_{2p} \rangle - \langle \psi_{1s} | V_o(r) - V_i(r) | \psi_{1s} \rangle , \quad (10.22)$$

or,

$$(\Delta E_{2p \rightarrow 1s})_o = \langle \psi_{2p} | V_o(r) - V_i(r) | \psi_{2p} \rangle - \langle \psi_{1s} | V_o(r) - V_i(r) | \psi_{1s} \rangle , \quad (10.23)$$

expressing the change in the energy of the photon, due to the effect of finite nuclear size.

The latter term, $\langle \psi_{1s} | V_o(r) - V_i(r) | \psi_{1s} \rangle$, has been calculated in (10.21). We now consider the former term, $\langle \psi_{2p} | V_o(r) - V_i(r) | \psi_{2p} \rangle \dots$

10.1.3: Nuclear size from spectroscopy measurements (con't)

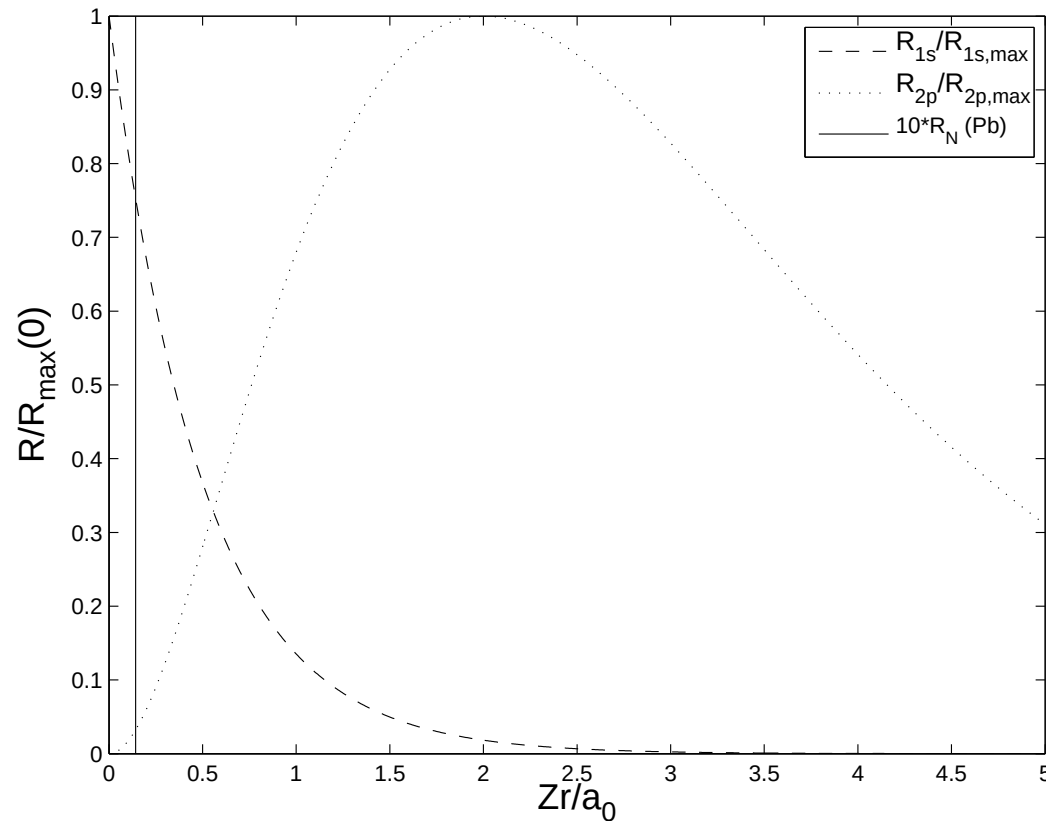


Figure 10.9: Overlap of $1s$ and $2p$ electronic orbitals with the nuclear radius. The nuclear radius depicted is for $A = 208$ and has been scaled upward by 10 for display purposes.

Figure 10.9 shows the $1s$ and $2p$ hydrogenic radial probabilities for the $1s$ and $2p$ states, each divided by their respective maxima. The vertical line near the origin is the $10\times$ the radius of an $A = 208$, $Z = 82$ nucleus.

10.1.3: Nuclear size from spectroscopy measurements (con't)

The overlap of the $2p$ state with the nucleus is $\ll$ than that of the $1s$ state. Hence, $\langle \psi_{2p} | V_o(r) - V(r) | \psi_{2p} \rangle \approx 0$. Therefore, that the transition photon's energy is:

$$\Delta E_{2p \rightarrow 1s} \approx -\langle \psi_{1s} | V_o(r) - V(r) | \psi_{1s} \rangle \approx -\frac{1}{10} Z^2 \alpha^2 (m_e c^2) A^{2/3} \left(\frac{2ZR_0}{a_0} \right)^2. \quad (10.24)$$

Still do not have anything we can measure!

We can not measure a photon's energy from a point nucleus!

Instead, measure the transition energy for two isotopes of the same element, A and A' . The point nucleus drops out of the equation (and the measurement):

$$\begin{aligned} \Delta E_{2p \rightarrow 1s}(A) - \Delta E_{2p \rightarrow 1s}(A') &= \\ &= -\langle 1s | V_o(r) - V(r) | 1s \rangle_A + \langle 1s | V_o(r) - V(r) | 1s \rangle_{A'} \\ &= \langle 1s | V_o(r) | 1s \rangle_{A'} - \langle 1s | V_o(r) | 1s \rangle_A - \langle 1s | V(r) | 1s \rangle_{A'} + \langle 1s | V(r) | 1s \rangle_A \\ &= \langle 1s | V_o(r) | 1s \rangle_{A'} - \langle 1s | V_o(r) | 1s \rangle_A - [\langle 1s | V(r) | 1s \rangle_{A'} - \langle 1s | V(r) | 1s \rangle_A] = 0 \\ &= \frac{1}{10} Z^2 \alpha^2 (m_e c^2) \left(\frac{2ZR_0}{a_0} \right)^2 (A'^{2/3} - A^{2/3}). \end{aligned} \quad (10.25)$$

10.1.3: Nuclear size from spectroscopy measurements (con't)

The measured quantity is called the K X-ray *isotope shift*. The following few pages show measurements of isotope shifts, for K X-Rays and optical photon isotope shifts.

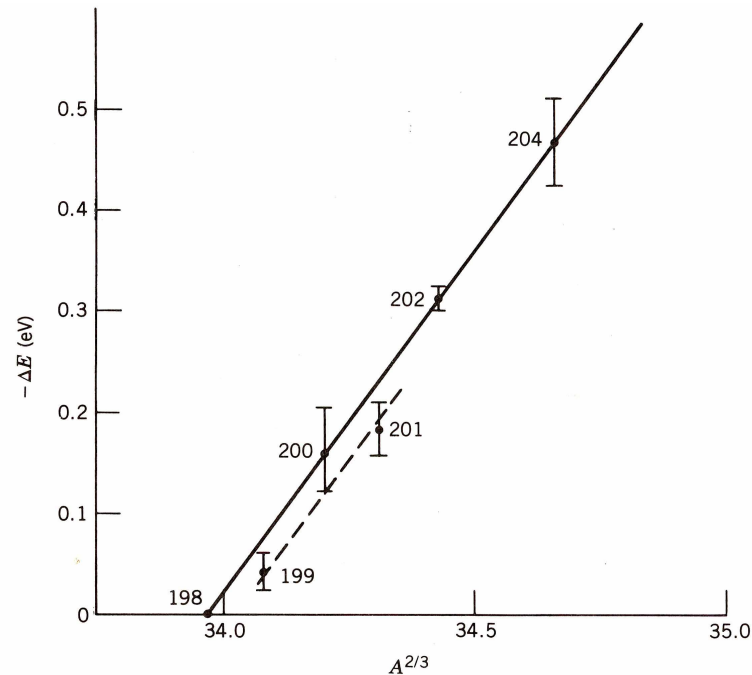


Figure 3.6 K X-ray isotope shifts in Hg. The energy of the K X ray in Hg is about 100 keV, so the relative isotope shift is of the order of 10^{-6} . The data show the predicted dependence on $A^{2/3}$. There is an “odd-even” shift in radius of odd-mass nuclei relative to their even- A neighbors, brought about by the orbit of the odd particle. For this reason, odd- A isotopes must be plotted separately from even- A isotopes. Both groups, however, show the $A^{2/3}$ dependence. The data are taken from P. L. Lee et al., *Phys. Rev. C* **17**, 1859 (1978).

Figure 10.10: Fig 3.6 from Krane, K X-ray shifts for Hg.

10.1.3: Nuclear size from spectroscopy measurements (con't)

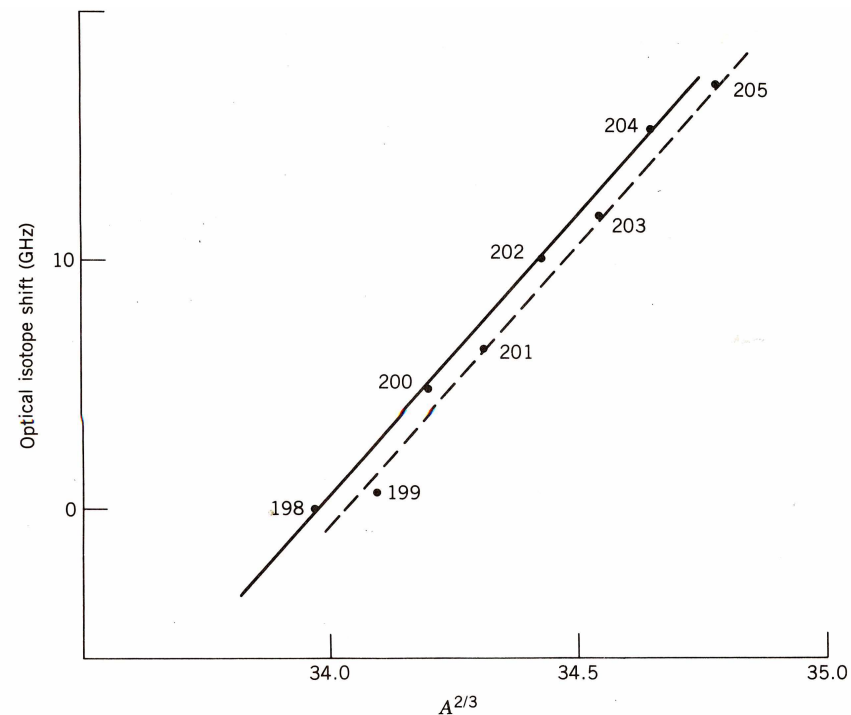


Figure 3.7 Optical isotope shifts in Hg isotopes from 198 to 205, measured relative to 198. These data were obtained through laser spectroscopy; the experimental uncertainties are about $\pm 1\%$. The optical transition used for these measurements has a wavelength of 253.7 nm, and the isotope shift is therefore about one part in 10^7 . Compare these results with Figure 3.6. Data taken from J. Bonn et al., *Z. Phys. A* **276**, 203 (1976).

Figure 10.11: Fig 3.7 from Krane, optical shifts for Hg.

10.1.3: Nuclear size from spectroscopy measurements (con't)

A better probe of nuclear shape can be done by forming muonic atoms, formed from muons (usually from cosmic rays), that replace an inner K-shell electron, and has significant overlap of its wavefunction with the nucleus.

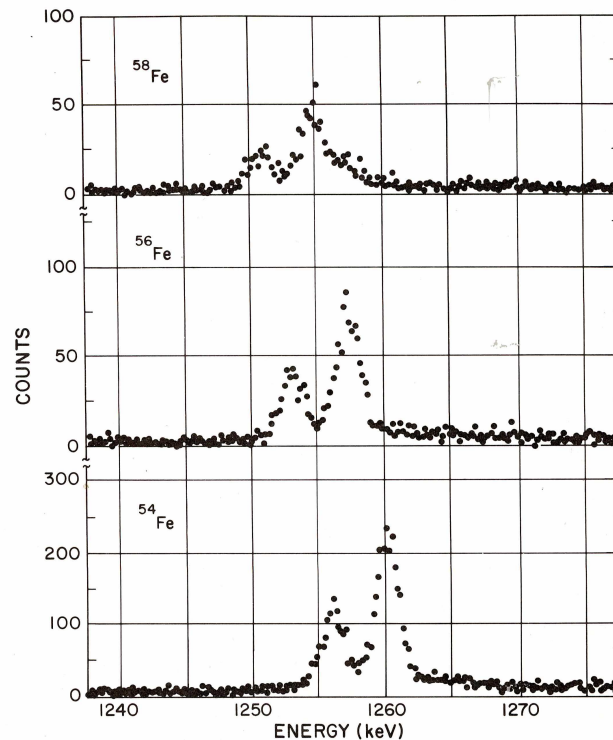


Figure 3.8 The muonic K X rays in some Fe isotopes. The two peaks show the $2p_{3/2}$ to $1s_{1/2}$ and $2p_{1/2}$ to $1s_{1/2}$ transitions, which have relative intensities in the ratio 2:1 determined by the statistical weight $(2j + 1)$ of the initial state. The isotope shift can clearly be seen as the change in energy of the transitions. The effect is about 0.4%, which should be compared with the 10^{-6} effect obtained with electronic K X rays (Figure 3.6). From E. B. Shera et al., *Phys. Rev. C* **14**, 731 (1976).

Figure 10.12: Fig 3.8 from Krane, K X-ray shifts for muonic Fe.

10.1.3: Nuclear size from spectroscopy measurements (con't)

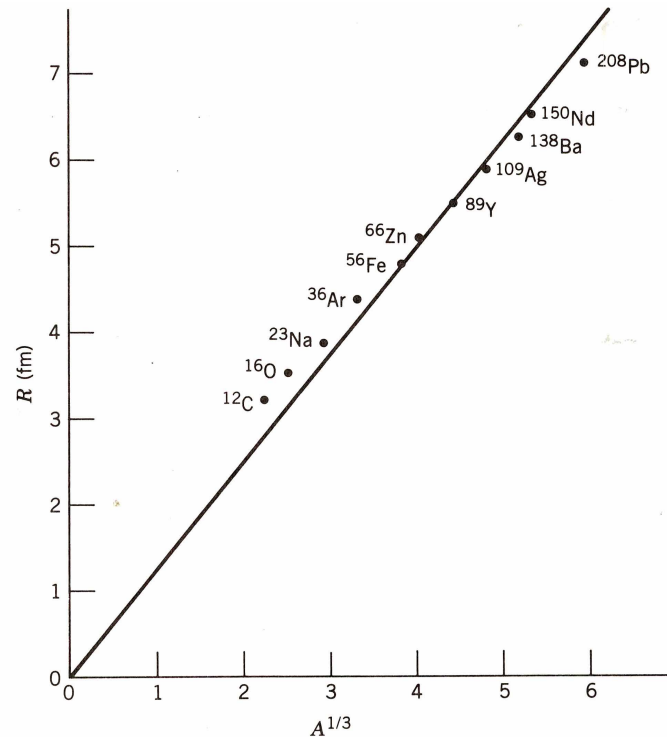


Figure 3.9 The mean nuclear radius determined from muonic X-ray measurements. As in Figure 3.5, the data depend roughly linearly on $A^{1/3}$ (again forcing the line to go through the origin). The slope of the line gives $R_0 = 1.25$ fm. The data are taken from a review of muonic X-ray determinations of nuclear charge distributions by R. Engfer et al., *Atomic Data and Nuclear Data Tables* **14**, 509 (1974).

Figure 10.13: Fig 3.9 from Krane, composite K X-ray shift data.

All these data are consistent with a nuclear size with a radius, $R_N = R_0 A^{1/3}$, and a value for $R_0 \approx 1.2$ fm.

10.1.3: Charge radius from Coulomb energy in mirror nuclei...

Charge radius from Coulomb energy in mirror nuclei

A mirror-pair of nuclei are two nuclei that have the same atomic mass, but the number of protons in one, is the number of neutrons in the other, and the number of protons and neutrons in one of the nuclei differs by only 1. So, if Z is the atomic number of the higher atomic number mirror nucleus, it has $Z - 1$ neutrons. Its mirror pair has $Z - 1$ protons and Z neutrons. The atomic mass of both is $2Z - 1$. Examples of mirror pairs are: ${}^3\text{H}/{}^3\text{He}$, and ${}^{39}\text{Ca}/{}^{39}\text{K}$.

These mirror-pairs are excellent laboratories for investigating nuclear radius since the nuclear component of the binding energy of these nuclei ought to be the same, if the strong force does not distinguish between nucleons. The only remaining difference is the Coulomb self-energy, and that is related to the radius.

10.1.3: ...Charge radius from Coulomb energy in mirror nuclei...

For a charge distribution with Z protons, the Coulomb self-energy is:

$$E_C = \frac{1}{2} \frac{Z^2 e^2}{4\pi\epsilon_0} \int d\vec{x}_1 \rho_p(\vec{x}_1) \int d\vec{x}_2 \rho_p(\vec{x}_2) \frac{1}{|\vec{x}_1 - \vec{x}_2|} . \quad (10.26)$$

The factor of $1/2$ in front of (10.26) accounts for the double counting of repulsion that takes place when one integrates over the nucleus twice, as implied in (10.26).

For a uniform, spherical charge distribution of the form,

$$\rho_p(\vec{x}) = \frac{3}{4\pi R_N^3} \Theta(R_N - r) . \quad (10.27)$$

As derived on the next pages:

$$E_C = \frac{3}{5} \frac{Z^2 e^2}{4\pi\epsilon_0 R_N} . \quad (10.28)$$

10.1.3: ...Charge radius from Coulomb energy in mirror nuclei...

For a uniform, spherical charge distribution, given by (10.27):

$$\begin{aligned} E_C &= \frac{1}{2} \frac{Z^2 e^2}{4\pi\epsilon_0} \left(\frac{3}{4\pi R_N^3} \right)^2 \int_{|\vec{x}_1| \leq R_N} d\vec{x}_1 \int_{|\vec{x}_2| \leq R_N} d\vec{x}_2 \frac{1}{|\vec{x}_1 - \vec{x}_2|} \\ &= \frac{1}{2} \frac{Z^2 e^2}{4\pi\epsilon_0 R_N} \left(\frac{3}{4\pi} \right)^2 \int_{|\vec{u}_1| \leq 1} d\vec{u}_1 \int_{|\vec{u}_2| \leq 1} d\vec{u}_2 \frac{1}{|\vec{u}_1 - \vec{u}_2|} \quad [\vec{x} \rightarrow r\vec{u} \text{ for both integrals}] \\ &= \frac{1}{2} \frac{Z^2 e^2}{4\pi\epsilon_0 R_N} I, \end{aligned} \tag{10.29}$$

where

$$I = \left(\frac{3}{4\pi} \right)^2 \int_{|\vec{u}_1| \leq 1} d\vec{u}_1 \int_{|\vec{u}_2| \leq 1} d\vec{u}_2 \frac{1}{|\vec{u}_1 - \vec{u}_2|}. \tag{10.30}$$

From (10.30), one sees that I has the interpretation as a pure number representing the average of $|\vec{u}_1 - \vec{u}_2|^{-1}$, for two vectors, $\vec{u}_1$ and $\vec{u}_2$, integrated uniformly over the interior of a unit sphere. So, now it just remains, to calculate I . We'll work this out explicitly because the calculation is quite delicate.

10.1.3: ...Charge radius from Coulomb energy in mirror nuclei...

Features of this derivation are seen in several areas of Nuclear and Radiological Science. Expanding the 3-dimensional integrals in (10.30) results in:

$$I = \left(\frac{3}{4\pi}\right)^2 \int_0^{2\pi} d\phi_1 \int_0^\pi d\theta_1 \sin \theta_1 \int_0^1 du_1 u_1^2 \int_0^{2\pi} d\phi_2 \int_0^\pi d\theta_2 \sin \theta_2 \int_0^1 du_2 u_2^2 \frac{1}{|\vec{u}_1 - \vec{u}_2|}.$$

The following expression results from having done both azimuthal integrals, once having aligned the z -axis of the coordinate system with $\vec{u}_1$, when performing the 3 inner integrals. Then with the transformation $\cos \theta_1 \rightarrow \mu_1$ and $\cos \theta_2 \rightarrow \mu_2$, we obtain:

$$\begin{aligned} I &= \left(\frac{9}{2}\right) \int_0^1 du_1 u_1^2 \int_0^1 du_2 u_2^2 \int_{-1}^1 d\mu_2 \frac{1}{\sqrt{u_1^2 + u_2^2 - 2u_1 u_2 \mu_2}} \\ &= \left(\frac{9}{2}\right) \int_0^1 du_1 u_1 \int_0^1 du_2 u_2 [(u_1 + u_2) - |u_1 - u_2|] \\ &= 9 \int_0^1 du_1 u_1 \left[\int_0^{u_1} du_2 u_2^2 + u_1 \int_{u_1}^1 du_2 u_2 \right] \\ &= 9 \int_0^1 du_1 \left[\frac{u_1^2}{2} - \frac{u_1^4}{6} \right] = 9 \left[\frac{1}{6} - \frac{1}{30} \right] = \frac{6}{5}. \end{aligned} \tag{10.31}$$

A common error in performing the above integral results from ignoring the absolute value in the 2nd step. Recall that $\sqrt{a^2} = |a|$, not a .

10.1.3: ...Charge radius from Coulomb energy in mirror nuclei...

Finally, combining (10.29) and (10.31) gives us the final result expressed in (10.28). The Coulomb energy differences are measured through β -decay endpoint energies (more on this later in the course), which yield very good information on the nuclear radius. The difference in Coulomb energies is given by:

$$\begin{aligned}\Delta E_C &= \frac{3}{5} \frac{e^2}{4\pi\epsilon_0 R_N} [Z^2 - (Z-1)^2] \\ &= \frac{3}{5} \frac{e^2}{4\pi\epsilon_0 R_N} (2Z-1) \\ &= \frac{3}{5} \frac{e^2}{4\pi\epsilon_0 R_0} A^{2/3},\end{aligned}\tag{10.32}$$

where, in the last step, we let $R_N = R_0 A^{1/3}$. Recall, $A = 2Z - 1$ for mirror nuclei.

10.1.3: ...Charge radius from Coulomb energy in mirror nuclei...

ΔE_C data, for all the mirror nuclei

$Z_< Z_>$	I^π	ΔE_C (kev)
${}^3_1\text{H}_2 {}^3_2\text{He}_1$	1/2+	763.764(0.004)
${}^5_2\text{He}_3 {}^5_3\text{Li}_2$	3/2-	1072(71)
${}^7_3\text{Li}_4 {}^7_4\text{Be}_3$	3/2-	1644(1)
${}^9_4\text{Be}_5 {}^9_5\text{B}_4$	3/2-	1850(1)
${}^{11}_5\text{B}_6 {}^{11}_6\text{C}_5$	3/2-	2765(1)
${}^{13}_6\text{C}_7 {}^{13}_7\text{N}_6$	1/2-	3003(1)
${}^{15}_7\text{N}_8 {}^{15}_8\text{O}_7$	1/2-	3536(1)
${}^{17}_8\text{O}_9 {}^{17}_9\text{F}_8$	5/2+	3543(1)
${}^{19}_9\text{F}_{10} {}^{19}_{10}\text{Ne}_9$	1/2+	4021(1)
${}^{21}_{10}\text{Ne}_{11} {}^{21}_{11}\text{Na}_{10}$	3/2+	4330(1)
${}^{23}_{11}\text{Na}_{12} {}^{23}_{12}\text{Mg}_{11}$	3/2+	4839(2)
${}^{25}_{12}\text{Mg}_{13} {}^{25}_{13}\text{Al}_{12}$	5/2+	5059(1)
${}^{27}_{13}\text{Al}_{14} {}^{27}_{14}\text{Si}_{13}$	5/2+	5595(1)
${}^{29}_{14}\text{Si}_{15} {}^{29}_{15}\text{P}_{14}$	1/2+	5725(1)
${}^{31}_{15}\text{P}_{16} {}^{31}_{16}\text{S}_{15}$	1/2+	6178(2)
${}^{33}_{16}\text{S}_{17} {}^{33}_{17}\text{Cl}_{16}$	3/2+	6365(1)
${}^{35}_{17}\text{Cl}_{18} {}^{35}_{18}\text{Ar}_{17}$	3/2+	6748(1)
${}^{37}_{18}\text{Ar}_{19} {}^{37}_{19}\text{K}_{18}$	3/2+	6931(1)
${}^{39}_{19}\text{K}_{20} {}^{39}_{20}\text{Ca}_{19}$	3/2+	7313(2)
${}^{41}_{20}\text{Ca}_{21} {}^{41}_{21}\text{Sc}_{20}$	7/2-	7278(1)
${}^{43}_{21}\text{Sc}_{22} {}^{43}_{22}\text{Ti}_{21}$	7/2-	7650(8)
${}^{45}_{22}\text{Ti}_{23} {}^{45}_{23}\text{V}_{22}$	7/2-	7915(17)
${}^{47}_{23}\text{V}_{24} {}^{47}_{24}\text{Cr}_{23}$	3/2-	8234(14)
${}^{49}_{24}\text{Cr}_{25} {}^{49}_{25}\text{Mn}_{24}$	5/2-	8497(25)
${}^{51}_{25}\text{Mn}_{26} {}^{51}_{26}\text{Fe}_{25}$	5/2-	8802(15)
${}^{55}_{27}\text{Co}_{28} {}^{55}_{28}\text{Ni}_{26}$	7/2-	9476(11)
${}^{59}_{29}\text{Cu}_{30} {}^{59}_{30}\text{Zn}_{27}$	3/2-	9874(40)

10.1.3: ...Charge radius from Coulomb energy in mirror nuclei

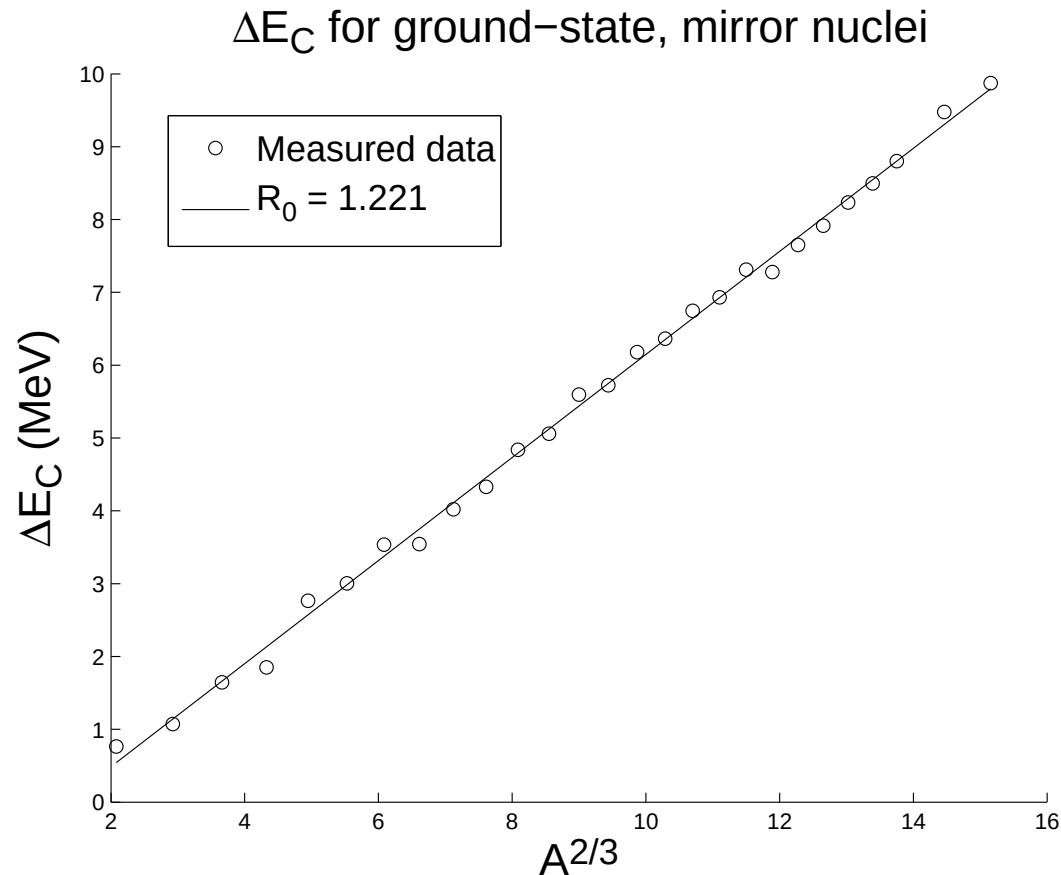
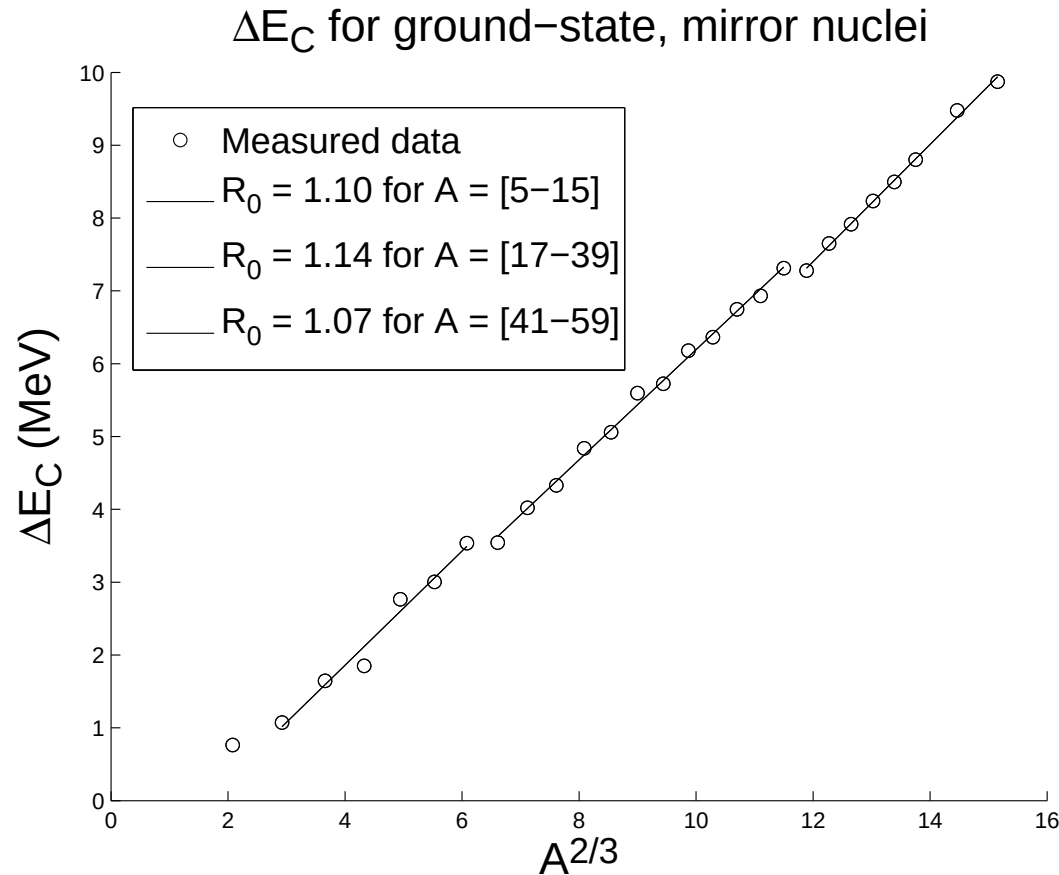


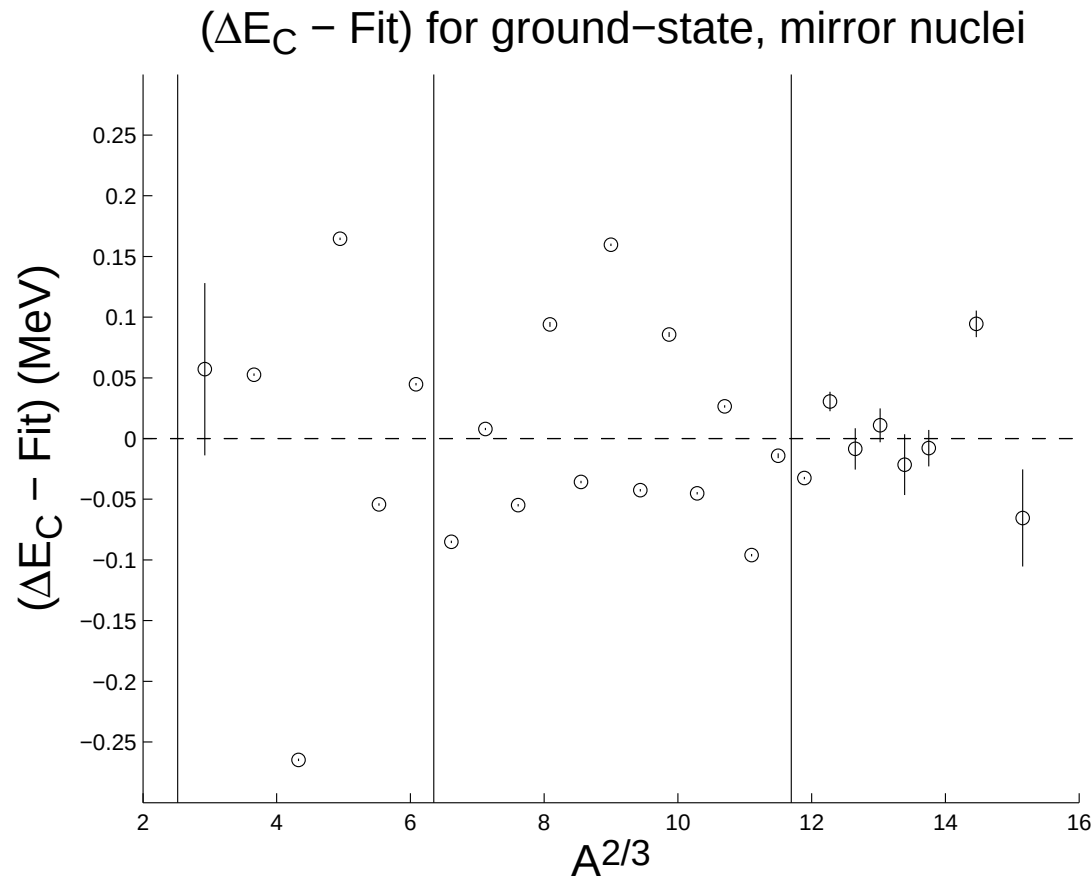
Figure 10.14: The Coulomb energy differences of all the ground-state mirror nuclei. The expected $A^{2/3}$ dependence is shown, and the slope of the line, from a fit to all the data, is $R_0 = 1.221$ fm.

10.1.3: ...Charge radius from Coulomb energy in mirror nuclei



The Coulomb energy differences of all the ground-state mirror nuclei, with separate fits within shell.

10.1.3: ...Charge radius from Coulomb energy in mirror nuclei



The Coulomb energy differences of all the ground-state mirror nuclei, with separate fits within each shell, less the value of the fits.

10.2: Mass and Abundance of Nuclei

Note to students: Read 3.2 in Krane on your own. You are responsible for this material, but it will not be covered in class.

10.3: Nuclear binding energy

Atomic and Nuclear rest mass energies

The rest mass energy of a neutral atom, $m_A c^2$ (often called $m(^A X)c^2$) and the rest mass energy of its nucleus, $m_N c^2$, are related by:

$$m_A c^2 = m_N c^2 + [Z m_e c^2 - B_e(Z, A)] , \quad (10.33)$$

where $B_e(Z, A)$ is the *electronic* binding, the sum of the binding energies of all the electrons in the atomic cloud, *i.e.*

$$B_e(Z, A) = \sum_{i=1}^Z B_i^e ,$$

where B_i^e is the electronic binding energy of the i 'th atomic electron.

$B_e(Z, A)$ can be as large as 1 MeV in the heavier atoms!

However, $B_e(Z, A)$ is swamped by factors of 10^5 – 10^6 by $m_N c^2 \approx A \times 1000$ MeV.

Hence, $m_A c^2 \approx m_N c^2 + Z m_e c^2$, is used for simplicity, particularly when mass *differences* are discussed, as the electronic binding component largely cancels out.

10.3: ...Nuclear binding energy (con't)...

One may estimate the total electronic binding as done in the following example.

Technical aside: Estimating the electronic binding in Pb:

Lead has the following electronic configuration:

$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^2 5p^6 4f^{14} 5d^{10} 6s^2 6p^2$,

or, occupancies of 2, 8, 18, 32, 18, 4 in the $n = 1, 2, 3, 4, 5, 6$ atomic shells. Thus,

$$B_e(82, 208) \approx (82)^2 (13.6 \text{ eV}) \left(2 + \frac{8}{2^2} + \frac{18}{3^2} + \frac{32}{4^2} + \frac{18}{5^2} + \frac{4}{6^2} \right) = 0.8076 \text{ MeV} .$$

This is certainly an overestimate, since electron repulsion in the atomic shells has not been accounted for. However, the above calculation gives us some idea of the magnitude of the total electronic binding. (A more refined calculation gives 0.2074 MeV, indicating that the overestimate is as much as a factor of 4.) So, for the time being, we shall ignore the total electronic binding but keep it in mind, should the need arise.

10.3: ...Nuclear binding energy (con't)...

By analogy, the formula for the *nuclear binding energy*, $B_N(Z, A)$, for atom X , with atomic mass $m(^A X)$ is

$$B_N(Z, A) = \{ Zm_p + Nm_n - [m(^A X) - Zm_e] \} c^2 , \quad (10.34)$$

or,

$$B_N(Z, A) = [Z(m_p + m_e) + Nm_n - m(^A X)] c^2 ,$$

Using $m_p + m_e \approx m(^1 H)$, (10.34) $\longrightarrow$

$$B_N(Z, A) = [Zm(^1 H) + Nm_n - m(^A X)] c^2 . \quad (10.35)$$

Note, with electron binding energy, (10.34) $\longrightarrow$

$$B_N(Z, A) = [Z(m_p + m_e) + Nm_n - m(^A X)] c^2 - B_e(Z, A) .$$

Atomic masses usually quoted in *atomic mass unit*, u , $uc^2 = 931.494028(23) \text{ MeV}$

10.3: ...Nuclear binding energy (con't)...

mass defect/mass excess

are common tabulations that are used to infer the nuclear mass/nuclear binding energy
Caveat emptor! Definitions can vary. Check your literature.

In this course we will use $\Delta = [m(^A X) - A]c^2$

Separation energies

neutron separation energy, S_n , energy required to liberate a neutron from the nucleus

$$\begin{aligned} S_n &= B_N(^A_Z X_N) - B_N(^{A-1}_Z X_{N-1}) \quad [\text{Use (10.35) to get}] \\ &= [Zm(^1H) + Nm_n - m(^A_Z X_N)]c^2 - [Zm(^1H) + (N-1)m_n - m(^{A-1}_Z X_{N-1})]c^2 \\ &= [m(^{A-1}_Z X_{N-1}) - m(^A_Z X_N) + m_n]c^2 \end{aligned} \quad (10.36)$$

proton separation energy

$$\begin{aligned} S_p &= B_N(^A_Z X_N) - B_N(^{A-1}_{Z-1} X_N) \\ &= [m(^{A-1}_{Z-1} X_N) - m(^A_Z X_N) + m(^1H)]c^2. \end{aligned} \quad (10.37)$$

10.3: ...Nuclear binding energy (con't)...

Some examples:

$\frac{A}{Z}X_N$	lifetime (s)	Δ (MeV)	S_n (MeV)		S_p (MeV)
$^{16}_8\text{O}_8$	stable (99.76%)	-4.737	15.66	$\approx$	12.13
$^{17}_8\text{O}_9$	stable (0.38%)	-0.810	4.14	$<$	13.78
$^{17}_9\text{F}_8$	64.5	1.952	16.81	$>$	0.60

10.3: ...Nuclear binding energy (con't)...

There are:

- 82 stable (*i.e.* “no measurable decay rate”)
 $^{209}_{83}\text{Bi}$ has a measured half-life of $(19 \pm 2) \times 10^{18}$ years (α -decay)!
- Those 82 stable elements have 256 stable isotopes.
Sn (tin) has 28 known unstable isotopes, ranging from ^{99}Sn – ^{137}Sn plus 10 stable isotopes ranging from ^{112}Sn – ^{126}Sn .
- These stable isotopes, plus the more than 1000 unstable but usable nuclei (from the standpoint of living long enough to provide a direct measurement of mass), can have their binding energy characterized by a universal fitting function, the semi-empirical formula for $B(Z, A) \equiv B_N(Z, A)$, a five-parameter empirical fit to the 1000+ set of data points. (The subscript N is dropped to distinguish B as the formula derived from data fitting.)

10.3: ...Nuclear binding energy (con't)...

Semi-empirical Mass Formula – Binding Energy per Nucleon

The formula for $B(Z, A)$ is given conventionally as:

$$\begin{aligned} B(Z, A) = & \quad a_V A && \text{("volume" term)} \\ & -a_S A^{2/3} && \text{("surface" term)} \\ & -a_C Z(Z-1)A^{-1/3} && \text{("Coulomb repulsion" term)} \\ & -a_{\text{sym}} \frac{(A-2Z)^2}{A} && \text{("symmetry" term)} \\ & +a_p \frac{(-1)^Z [1+(-1)^A]}{2} A^{-3/4} && \text{("pairing" term)} \end{aligned} \quad (10.38)$$

a_i	[MeV]	Description	Source
a_V	15.5	Volume attraction	Liquid Drop Model
a_S	16.8	Surface repulsion	Liquid Drop Model
a_C	0.72	Coulomb repulsion	Liquid Drop Model + Electrostatics
a_{sym}	23	n/p symmetry	Shell model
a_p	34	$n/n, p/p$ pairing	Shell model

Table 10.1: Fitting parameters for the nuclear binding energy

10.3: ...Nuclear binding energy (con't)...

Contributions from the Liquid Drop Model

Volume attraction represents the attraction of a core nucleon to its surrounding neighbors... attractive, and $\propto A$, since $R_N \propto A^{1/3}$... comes from considering the nucleus to be an incompressible fluid of mutually-attracting nucleons, *i.e.* the Liquid Drop Model of the nucleus

Surface attraction accounts for deficit of nuclear force saturation for surface nucleons repulsive, and $\propto R_N^2 \propto A^{2/3}$

Coulomb repulsion is estimated from (10.28)... $\propto 1/R_N \propto A^{-1/3}$... Z^2 is replaced by $Z(Z - 1)$ since a solitary proton does not repulse itself... the electrostatic charge is considered to be spread uniformly through the drop.

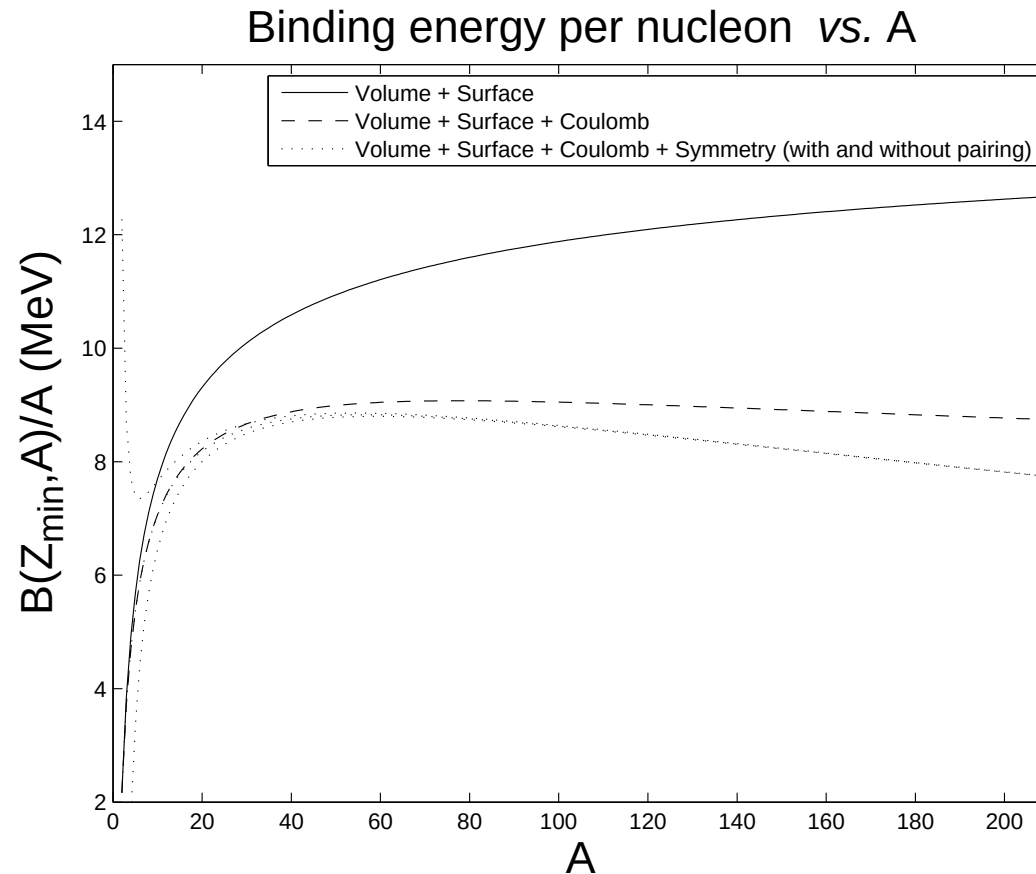
10.3: ...Nuclear binding energy (con't)...

Contributions from the Shell Model

n/p **symmetry** since nuclei like to form with equal numbers of protons and neutrons (Nuclear Shell Model)... $\propto [(A - 2Z)/A]^2$...minimizes (vanishes) when $Z = N$. This is a “Fermi pressure” term, meaning that the nucleus wants to maximize the difference of particle types in the nucleus.

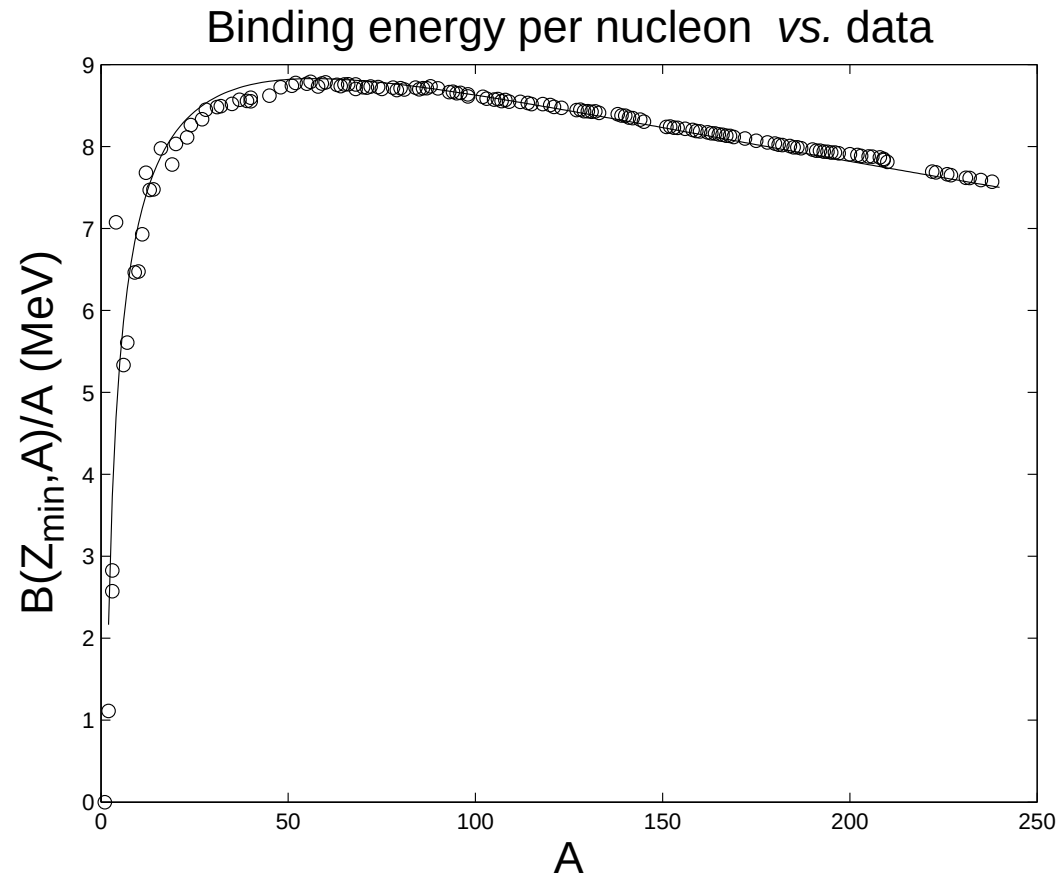
$n/n, p/p$ **pairing** since the Nuclear Shell Model predicts that nuclei prefer when protons or neutrons are paired up in $n - n, p - p$ pairs... attractive for an even-even nucleus (both Z and N are even), repulsive for an odd-odd nucleus, and zero otherwise. The $A^{-3/4}$ comes from Shell Model calculations, and often varies in the literature. This term comes from the spin-spin interaction. Like magnets (which is sort of, what nucleons are), want to anti-align their magnetic poles.

10.3: ...Nuclear binding energy (con't)...



The above plot shows the contributions of the components of $B(Z, A)/A$. The Z employed is the most stable from the standpoint of β decay, discussed later in this chapter.

10.3: ...Nuclear binding energy (con't)...



The above plot gives a comparison of $B(Z, A)/A$ measured data (from Wikipedia “nuclear binding energy”) .

10.3: ...Nuclear binding energy (con't)...

Binding Energy per Nucleon, major results

$B(Z, A)/A$...

- peaks at about $A = 56$ (Fe). Iron and nickel (the iron core of the earth) are natural endpoints of the fusion process.
- is about $8 \text{ MeV} \pm 10 \%$ for $A > 10$.

10.3: ...Nuclear binding energy (con't)...

Using the expression (10.38), *viz.*

$$B(Z, A) = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{\text{sym}} \frac{(A-2Z)^2}{A} \\ + a_p \frac{(-1)^Z [1 + (-1)^A]}{2} A^{-3/4}$$

and adapting (10.35), *viz.*

$$B_N(Z, A) = [Zm(^1H) + Nm_n - m(^AX)]c^2$$

we obtain the *semi-empirical mass formula*:

$$m(^AX) = Zm(^1H) + Nm_n - B(Z, A)/c^2, \quad (10.39)$$

that one may use to estimate $m(^AX)$ from measured values of the binding energy, or vice-versa.

10.3: ...Nuclear binding energy (con't)...

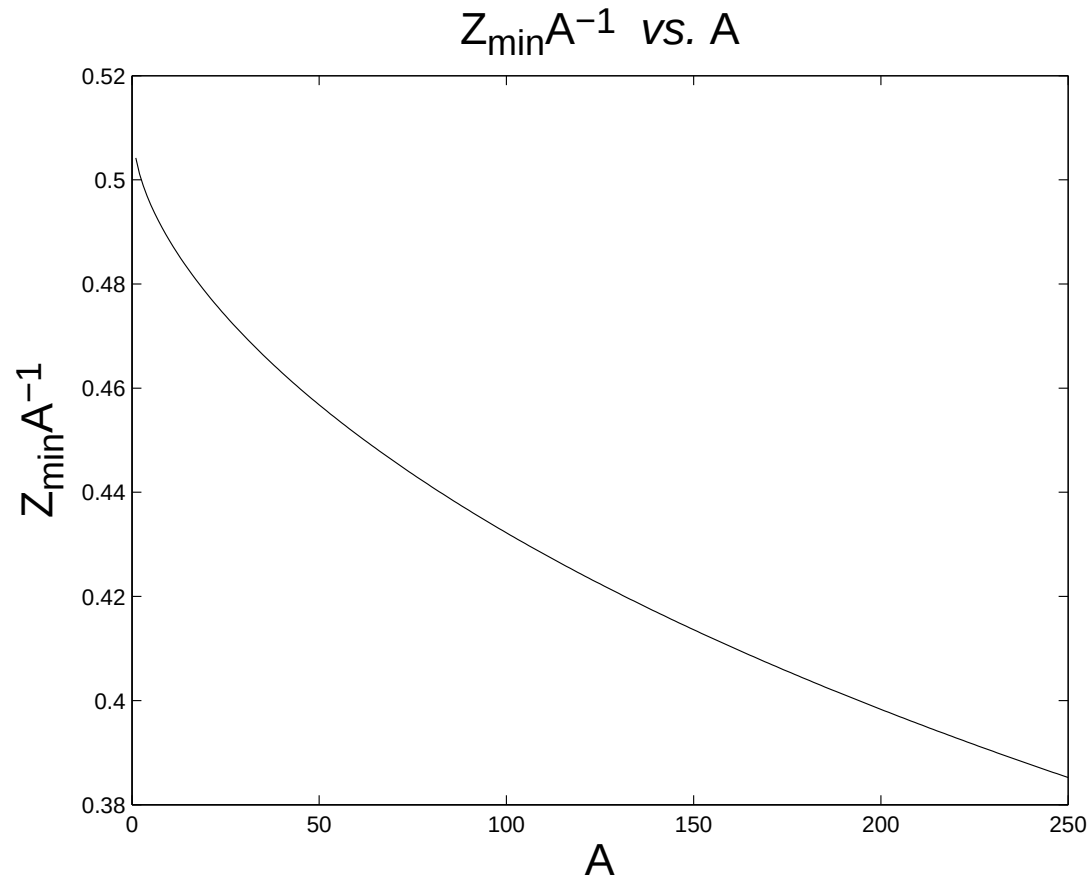
Application to β -decay

β -decay occurs when a proton or a neutron in a nucleus converts to the other form of nucleon, $n \rightarrow p$, or $p \rightarrow n$. (An unbound neutron will also β -decay.) This process preserves A . Therefore, one may characterize β -decay as an isobaric (*i.e.* same A) transition. For fixed A , (10.39) represents a parabola in Z , with the minimum occurring at (Note: There is a small error in Krane's formula below.):

$$Z_{\min} = \frac{[m_n - m(^1\text{H})]c^2 + a_{\text{C}}A^{-1/3} + 4a_{\text{sym}}}{2a_{\text{C}}A^{-1/3} + 8a_{\text{sym}}A^{-1}}. \quad (10.40)$$

We have to use some caution when using this formula. When A is odd, there is no ambiguity. However, when the decaying nucleus is odd-odd, the transition picks up an additional loss in mass of $2a_{\text{p}}A^{-3/4}$, because an odd-odd nucleus becomes an even-even one. Similarly, when an even-even nucleus decays to an odd-odd nucleus, it picks up a gain of $2a_{\text{p}}A^{-3/4}$ in mass, that must be more than compensated for, by the energetics of the β -decay.

10.3: ...Nuclear binding energy (con't)...



10.3: ...Nuclear binding energy (con't)...

Figure 3.18 in Krane illustrates this for two different decays chains.

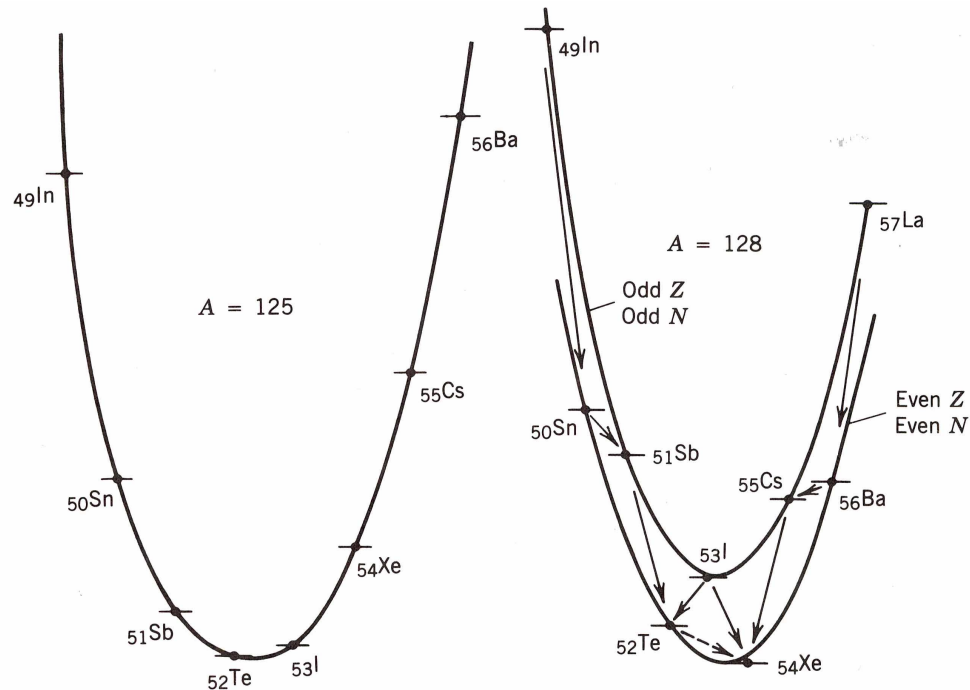


Figure 3.18 Mass chains for $A = 125$ and $A = 128$. For $A = 125$, note how the energy differences between neighboring isotopes increase as we go further from the stable member at the energy minimum. For $A = 128$, note the effect of the pairing term; in particular, ^{128}I can decay in either direction, and it is energetically possible for ^{128}Te to decay directly to ^{128}Xe by the process known as double β decay.

10.3: ...Nuclear binding energy (con't)...

(10.40) can very nearly be approximated by:

$$Z_{\min} \approx \frac{A}{2} \frac{1}{1 + (1/4)(a_C/a_{\text{sym}})A^{2/3}} . \quad (10.41)$$

This shows clearly the tendency for $Z \approx N$ for lighter nuclei. For heavier nuclei, $A \approx 0.41$.

10.4: Angular momentum and parity...

Some notation:

Symbol	meaning
l_i	orbital angular momentum of the i 'th nucleon
s_i	intrinsic spin angular momentum of the i 'th nucleon
j_i	total angular momentum of the i 'th nucleon, <i>i.e.</i> $\vec{j}_i = \vec{l}_i + \vec{s}_i$
$\vec{L}$	sum of all orbital angular momenta in a nucleus <i>i.e.</i> $\vec{L} = \sum_{i=1}^A \vec{l}_i$
$\vec{S}$	sum of all intrinsic spin angular momenta in a nucleus <i>i.e.</i> $\vec{S} = \sum_{i=1}^A \vec{s}_i$

The total angular momentum of a nucleus is formed from the sum of the individual constituents angular momentum, $\vec{l}_i$, and spin, $\vec{s}_i$, angular momentum. The symbol given to the nuclear angular momentum is I .

Thus,

$$\vec{I} = \sum_{i=1}^A (\vec{l}_i + \vec{s}_i) = \vec{L} + \vec{S} . \quad (10.42)$$

10.4: Angular momentum and parity...

These angular momenta add in the Quantum Mechanical sense. That is:

$$\begin{aligned}\langle \vec{I}^2 \rangle &= \hbar^2 I(I+1) \\ I &= 0, \frac{1}{2}, 1, \frac{3}{2}, \dots \\ \langle I_z \rangle &= \hbar m_I \\ m_I &: -I \leq m_I \leq I \\ \Delta m_I &: \text{integral jumps}\end{aligned}\tag{10.43}$$

10.4: ...Angular momentum and parity...

Since neutron and proton spins are half-integral, and orbital angular momentum is integral, it follows that I is half-integral for odd- A nuclei, and integral for even- A nuclei.

Recall that parity is associated with a quantum number of ± 1 , that is associated with the inversion of space. That is, if Π is the parity operator, acting on the composite nuclear wave function, $\Psi(\vec{x}; A, Z)$,

$$\Pi\Psi(\vec{x}; A, Z) = \pm\Psi(-\vec{x}; A, Z) . \quad (10.44)$$

The plus sign is associated with “even parity” and the minus sign with “odd parity”.

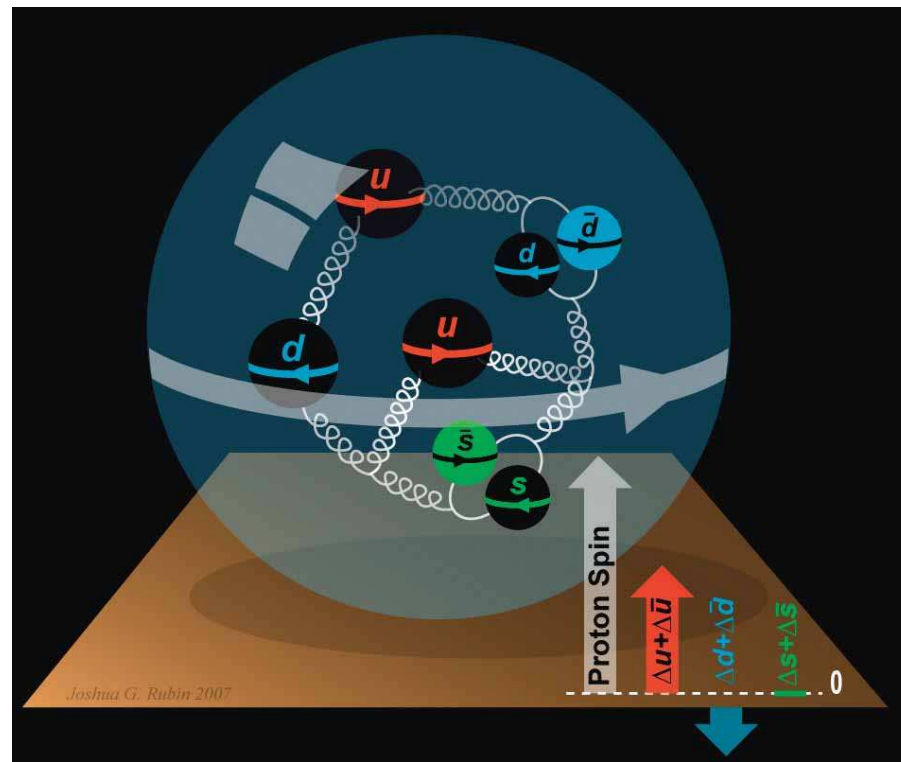
Total spin and parity are measurable, and a nucleus is said to be in an I^π configuration. For example, ^{235}U has $I^\pi = \frac{7}{2}^-$, while ^{238}U has $I^\pi = 0^+$.

10.4: ...Angular momentum and parity...

- If the nuclear strong force were completely central, *i.e.* $V(\vec{x}) = V(r)$, then L , M_L , S , M_S would all be constants of the motion.
- In atomic physics, and nuclear physics, the spin-orbit force has the form $V_{\text{so}}(r)\vec{l} \cdot \vec{s}$. However, in atomic physics, this effect is small, and splits the spectroscopy lines by a small amount (called the “fine” structure).
- In atomic physics, and nuclear physics, the spin-spin force has the form $V_{\text{ss}}(r)\vec{s} \cdot \vec{s}$. However, in atomic physics, this effect is even smaller, and splits the spectroscopy lines by a smaller amount (call the “hyperfine” structure).
- However, in nuclear physics, these forces are very strong, and can not be treated as small perturbations, leading to great mathematical complexity. Moreover, L , M_L , S , M_S are **not** constants of the motion. This has interesting effect, that we shall encounter later. But, total angular momentum, I , is a constant for each nucleus.
- Observational fact: All the even-even nuclei have $I = 0$, because of the strong pairing force.

10.5: Nuclear magnetic and electric moments...

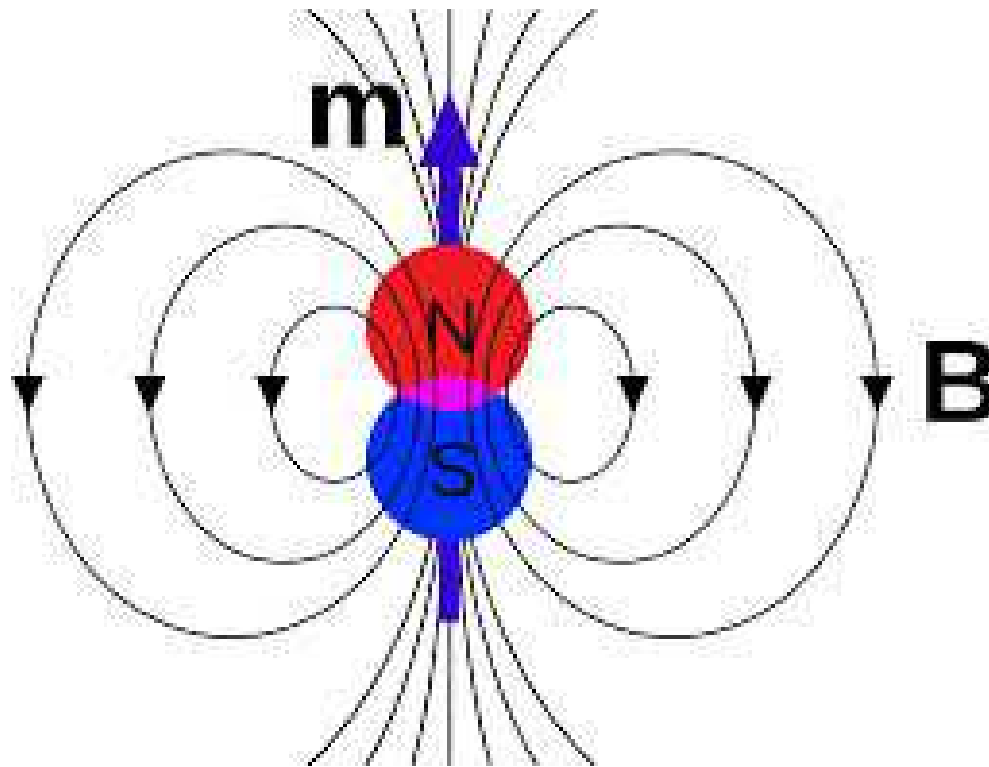
An illustration of the preferred spin orientations of the up, down, and strange quarks and antiquarks within a polarized proton. From “The Frontiers of Nuclear Science — A Long Range Plan, December 2007”, by the Nuclear Science Advisory Committee.



An introduction to moments is given in terms of the gravitational force. The handwritten notes are in Chapter10.pdf, in pdf pages 55-60.

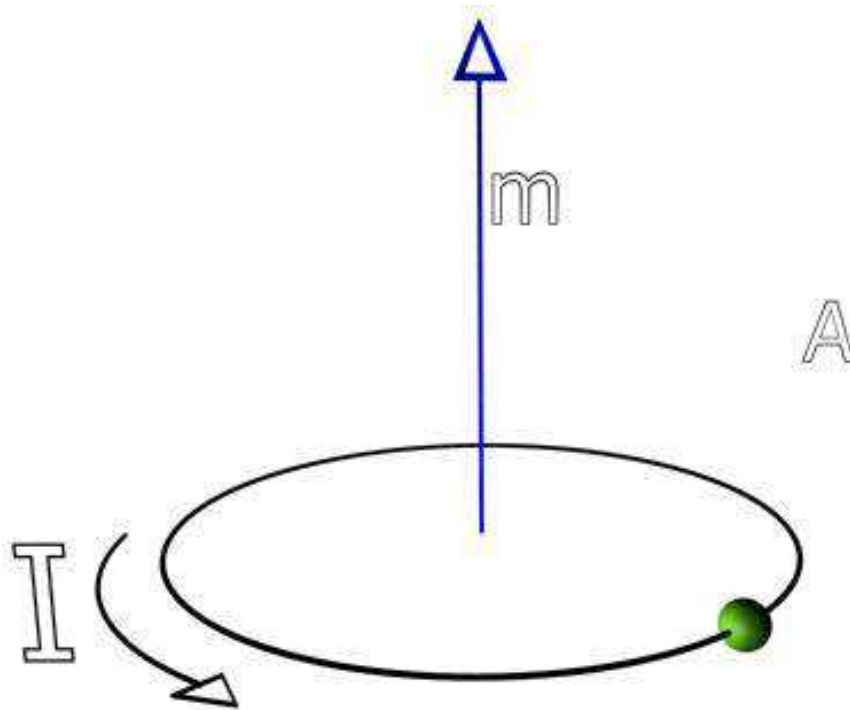
10.6: ...Nuclear magnetic and electric moments...

This is the dipole magnetic field, $\vec{B}$ and the “dipole moment”, $\vec{m}$ or $\vec{\mu}$ of a permanent magnet.



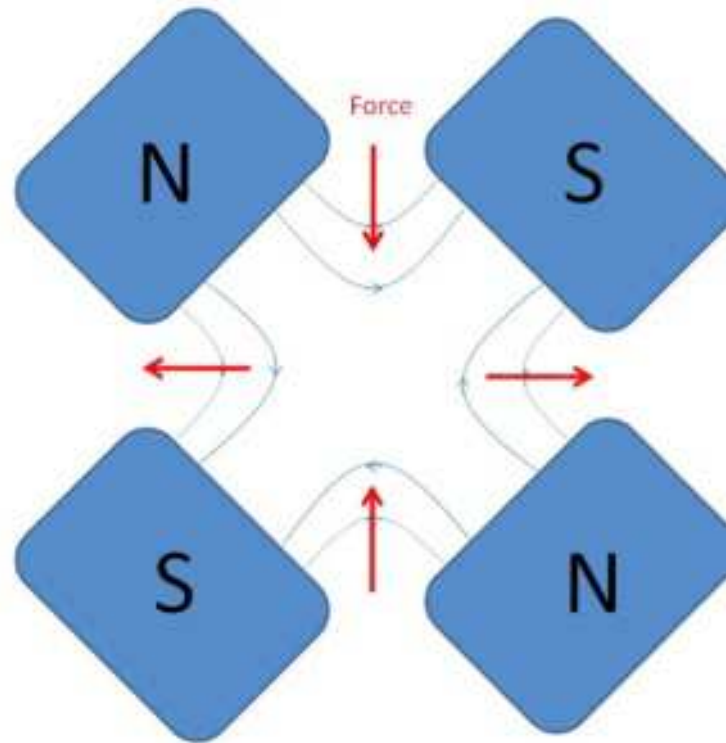
10.7: ...Nuclear magnetic and electric moments...

A moving charge (positive in this case) generates a similar magnetic field and dipole moment. For a negative charge moving in the same direction, the direction of the magnetic moment would be reversed.



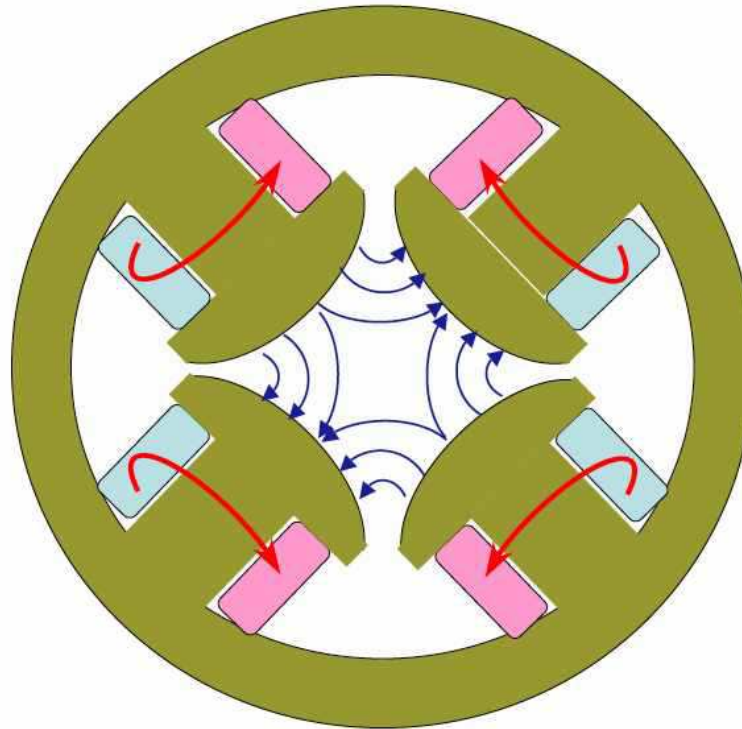
10.8: ...Nuclear magnetic and electric moments...

This is a magnetic quadrupole, formed from pairs of fixed “monopole” magnets. Note the force lines that would deflect moving charged particles.



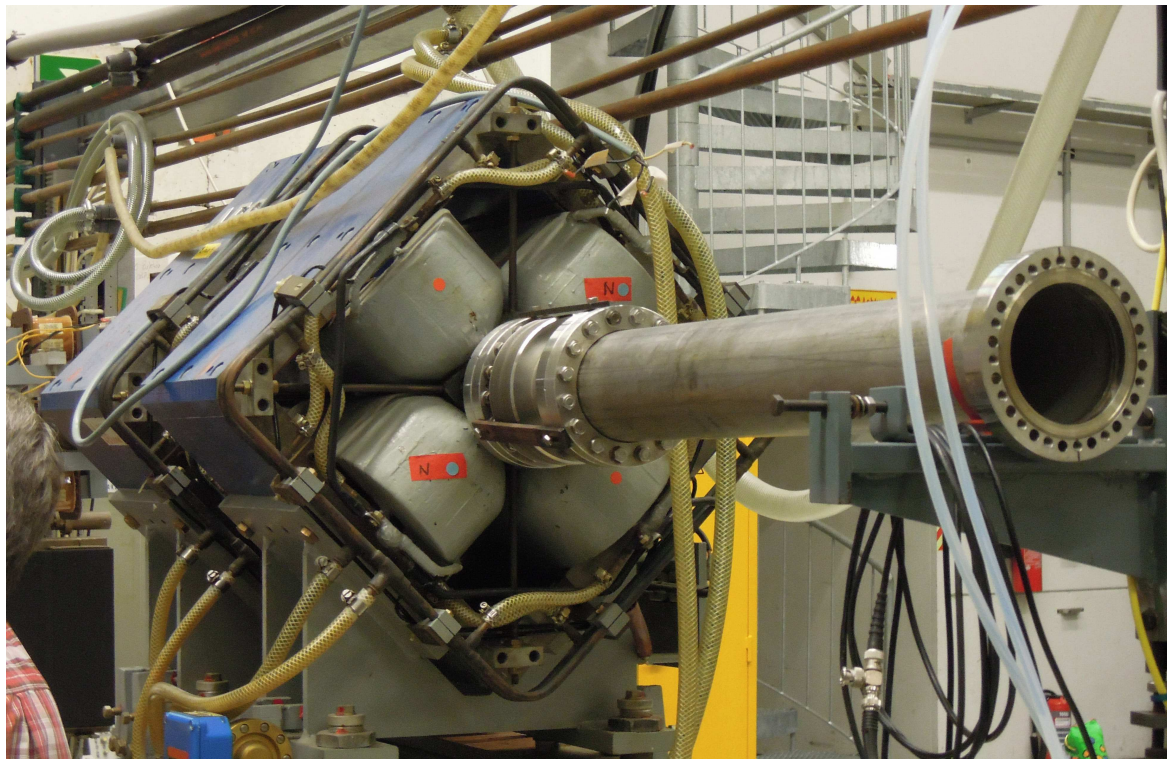
10.9: ...Nuclear magnetic and electric moments...

Magnetic monopoles do not exist in nature, so quadrupole magnets are made up of 4 opposing dipole magnets, in this illustration, electromagnets.



10.10: ...Nuclear magnetic and electric moments...

Pairs of quadrupole magnets, in opposing orientations form a quadrupole lens, used for focusing beams of charged particles. You can see some of these in the MIBL. (This one is from the Maier-Leibnitz lab in Munich.)



10.10.1: Magnetic dipole of nucleons...

Nucleons, in a tightly-bound nucleus, all in close proximity to each other, all moving with velocities of about $0.001 \rightarrow 0.1c$. This is a radical departure from the leisurely orbit of an electron about a nucleus. This is a “mosh pit” of thrashing, slamming nucleons. The forces between them are considerable, and play a vital role in the determination of nuclear structure.

The orbital angular momentum can be characterized in classical electrodynamics in terms of a magnetic moment, $\vec{\mu}$:

$$\vec{\mu} = \frac{1}{2} \int d\vec{x} \, \vec{x} \times \vec{J}(\vec{x}) , \quad (10.45)$$

where $\vec{J}(\vec{x})$ is the *current density*. For the purpose of determining the orbital angular momentum's contribution to the magnetic moment, the nucleons can be considered to be point-like particles.

10.10.1: ...Magnetic dipole of nucleons...

For point-like particles,

$$\mu = |\vec{\mu}| = g_l l \mu_N , \quad (10.46)$$

where l is the orbital angular momentum quantum number, g_l is the *g-factor* or *gyromagnetic ratio* ($g_l = 1$ for protons, $g_l = 0$ for neutrons, since the neutrons are neutral), and the nuclear magnetron, μ_N is:

$$\mu_N = \frac{e\hbar}{2m_p} , \quad (10.47)$$

defined in terms of the single charge of the proton, e , and its mass, m_p . Its current measured value is $\mu_N = 5.05078324(13) \times 10^{-27} \text{ J/T}$.

10.10.1: ...Magnetic dipole of nucleons...

Intrinsic spins of the nucleons also result in magnetic moments. These are given by:

$$\mu = g_s S \mu_N , \quad (10.48)$$

where the *spin g factors* are known to be, for the electron, proton, neutron and muon:

Type	g_s (measured)	g_s (theory)
$e^\pm$	$\pm 2.002319043622(15)$	agree!
p	$+5.585694713(90)$	?
n	$-3.82608545(46)$	?
$\mu^\pm$	$\pm 2.0023318414(12)$	$\pm 2.0023318361(10)$

A simple(!) application of Dirac's *Relativistic Quantum Mechanics* and *Quantum Electrodynamics* (aka QED) leads to the prediction, $g_s = 2$ for the electron. The extra part comes from the *zitterbewegung* of the electron. The fantastic agreement of g_s for the electron, between measurement and theory, 12 decimal places, is considered to be the most remarkable achievement of theoretical physics, and makes QED the most verified theory in existence.

10.10.1: ...Magnetic dipole of nucleons...

- There is no theory for the determination of the nucleon g -factors. However, the measured values allow us to reach an important conclusion: The proton must be something very different from a point charge (else its g_s would be close to 2), and the neutron must be made up of internal charged constituents (else its g_s would be 0).
- These observations laid the groundwork for further investigation that ultimately led to the discovery (albeit indirectly), that neutrons and protons are made up of quarks. (Free quarks have never been observed.) This led to the development of *Quantum Chromodynamics* (aka QCD), that describes the the strong force in fundamental, theoretical terms. The unification of QCD, QED, and the weak force (responsible for β -decay) is called *The Standard Model* of particle physics.
- Measurement and Theory differ, however, for the muon's g_s . It has been suggested that there is physics beyond The Standard Model that accounts for this.
- Table 3.2 in Krane provides some examples. Further exploration awaits our later discussions on nuclear models.

10.10.1: ...Magnetic dipole of nucleons

Table 3.2 Sample Values of Nuclear Magnetic Dipole Moments

Nuclide	$\mu(\mu_N)$
n	-1.9130418
p	+2.7928456
^2H (D)	+0.8574376
^{17}O	-1.89379
^{57}Fe	+0.09062293
^{57}Co	+4.733
^{93}Nb	+6.1705

All values refer to the nuclear ground states; uncertainties are typically a few parts in the last digit. For a complete tabulation, see V. S. Shirley, in *Table of Isotopes* (Wiley: New York, 1978), Appendix VII.

I^π assignments: $p, n = \frac{1}{2}^+$, $^2_1\text{H}_1 = 1^+$, $^{17}_8\text{O}_9 = \frac{5}{2}^+$, $^{57}_{26}\text{Fe}_{31} = \frac{1}{2}^-$, $^{57}_{27}\text{Co}_{30} = \frac{7}{2}^-$, $^{93}_{41}\text{Nb}_{52} = \frac{9}{2}^+$.
Can you draw any conclusions?

10.10.2: Quadrupole moments of nuclei...

The electric quadrupole moment is derived from the following considerations. The electrostatic potential of the nucleus is given by:

$$V(\vec{x}) = \frac{Ze}{4\pi\epsilon_0} \int d\vec{x}' \frac{\rho_p(\vec{x}')}{|\vec{x} - \vec{x}'|} . \quad (10.49)$$

Now, imagine that we are probing the nucleus from a considerable distance, so far away from it, that we can only just discern the merest details of its shape. Given that $\rho_p(\vec{x}')$ is highly localized in the vicinity of the nucleus and our probe is far removed from it, we may expand (10.49) in a Taylor expansion in $|\vec{x}'|/|\vec{x}|$. Thus we obtain:

$$V(\vec{x}) = \frac{Ze}{4\pi\epsilon_0} \left[\frac{1}{|\vec{x}|} \int d\vec{x}' \rho_p(\vec{x}') + \frac{\vec{x}}{|\vec{x}|^3} \cdot \int d\vec{x}' \vec{x}' \rho_p(\vec{x}') + \frac{1}{2|\vec{x}|^5} \int d\vec{x}' (3(\vec{x} \cdot \vec{x}')^2 - |\vec{x}|^2 |\vec{x}'|^2) \rho_p(\vec{x}') \cdots \right] . \quad (10.50)$$

10.10.2: Quadrupole moments of nuclei...

This simplifies to:

$$V(\vec{x}) = \frac{Ze}{4\pi\epsilon_0} \left[\frac{1}{|\vec{x}|} + \frac{Q}{2|\vec{x}|^3} \cdots \right] , \quad (10.51)$$

where

$$Q = \int d\vec{x} \left(3z^2 - r^2 \right) \rho_p(\vec{x}) . \quad (10.52)$$

We have used $\int d\vec{x} \rho_p(\vec{x}) \equiv 1$ for the first integral in (10.50). This is simply a statement of our conventional normalization of $\rho_p(\vec{x})$. We also used $\int d\vec{x} \vec{x} \rho_p(\vec{x}) \equiv 0$ in the second integral in (10.50). This is made possible by choosing the “center of charge” as the origin of the coordinate system for the integral. Finally, the third integral resulting in (10.52), arises from the conventional choice, when there is no preferred direction in a problem, and set the direction of $\vec{x}'$ to align with the z' -axis, for mathematical convenience.

10.10.2: Quadrupole moments of nuclei...

Technical note: *The second integral can be made to vanish through the choice of a center of charge. This definition is made possible because the charge is of one sign. Generally, when charges of both signs are involved in an electrostatic configuration, and their respective centers of charge are different, the result is a non-vanishing term known as the electric dipole moment. In this case, the dipole moment is given by:*

$$\vec{d} = \int d\vec{x}' \vec{x}' \rho(\vec{x}') .$$

Finally, when it is not possible to choose the z -axis to be defined by the direction of $\vec{x}$, but instead, by other considerations, the quadrupole becomes a tensor, with the form:

$$V_q(\vec{x}) = \frac{Ze}{4\pi\epsilon_0} \frac{1}{|\vec{x}|^3} \frac{1}{2} \sum_{i,j} Q_{ij} n_i n_j ,$$

where

$$Q_{ij} = \int d\vec{x} \left(3x'_i x'_j - \delta_{ij} |\vec{x}'|^2 \right) \rho_p(\vec{x}') .$$

Here the quadrupole part of the potential is $V_q(\vec{x})$ and the n_i are the 3 direction components from the center of mass of the charged object to the point where $V_q(\vec{x})$ is measured. Note that $|\hat{n}| = 1$.

10.10.2: Quadrupole moments of nuclei...

The quantum mechanics analog to (10.52) is:

$$Q = \int d\vec{x} \, \psi_N^*(\vec{x})(3z^2 - r^2)\psi_N(\vec{x}) , \quad (10.53)$$

where $\psi_N(\vec{x})$ is the composite nuclear wave function. The electric quadrupole moment of the nucleus is also a physical quantity that can be measured, and predicted by nuclear model theories. See Krane's Table 3.3.

10.10.2: Quadrupole moments of nuclei...

Table 3.3 Some Values of Nuclear Electric Quadrupole Moments

Nuclide	Q (b)
^2H (D)	+ 0.00288
^{17}O	− 0.02578
^{59}Co	+ 0.40
^{63}Cu	− 0.209
^{133}Cs	− 0.003
^{161}Dy	+ 2.4
^{176}Lu	+ 8.0
^{209}Bi	− 0.37

All values refer to nuclear ground states; uncertainties are typically a few parts in the last digit. For a complete tabulation, see V. S. Shirley, in *Table of Isotopes* (Wiley: New York, 1978), Appendix VII.

10.11:Nuclear excited states

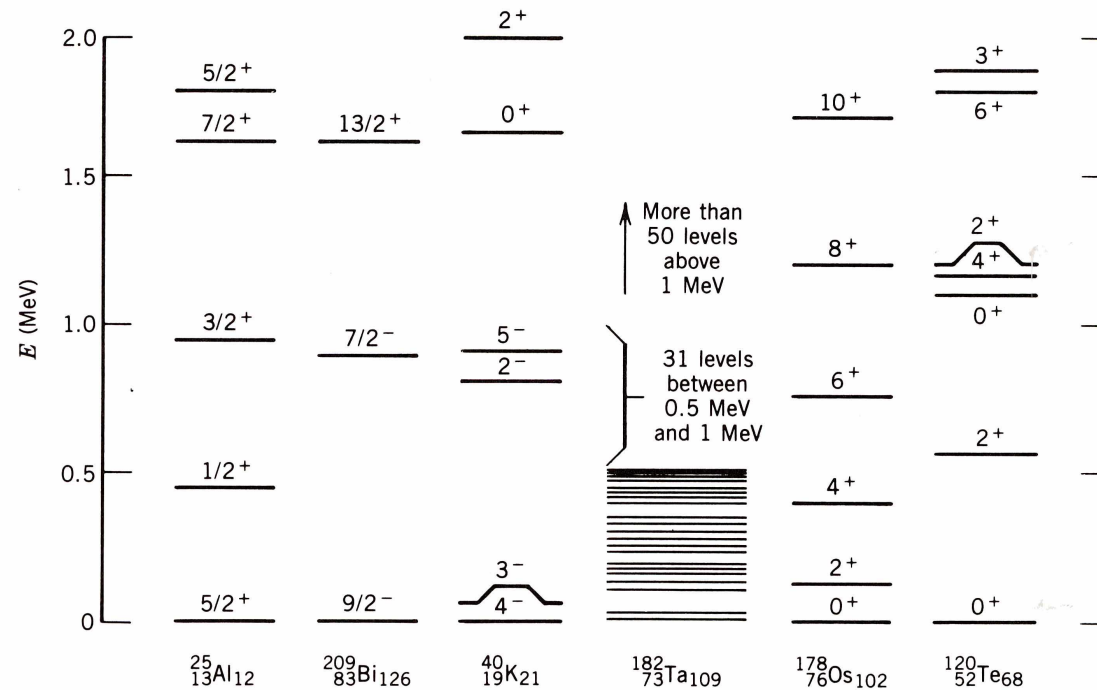


Figure 3.19 Some sample level schemes showing the excited states below 2 MeV. Some nuclei, such as ^{209}Bi , show great simplicity, while others, such as ^{182}Ta , show great complexity. There is a regularity associated with the levels of ^{178}Os that is duplicated in all even- Z , even- N nuclei in the range $150 \leq A \leq 190$. Structures similar to ^{120}Te are found in many nuclei in the range $50 \leq A \leq 150$.

Chapter 10: Nuclear Properties...

Things to think about ...

Chapter 10.0: Introduction ...

1. Describe the charge distribution of the p and n . What do those charge distributions imply?
2. Sketch the repulsive and attractive parts of the nucleon-nucleon force. Add them together, and sketch that.
3. Recall the components of the nucleon-nucleon force.
4. How are nuclei formed?

Chapter 10.1: The nuclear radius ...

1. What projectile is ideal to probe the nuclear charge distribution? What should its energy be?
2. What is momentum transfer? Form factor?
3. What is the characteristic shape of a form factor? What does it mean?
4. How are form factors used to measure the nuclear charge distribution?

Chapter 10: Nuclear Properties...

Chapter 10.1: ...The nuclear radius ...

5. Is the nuclear charge distribution and the nuclear density distribution the same? (Apart from a normalization factor.)
6. What is the Woods-Saxon model of the nuclear charge density?
7. What is R_N , R_0 , and their approximate values?
8. Where does the simple model for R_N break down? Why?
9. How can the size of the nucleus be determine from K -shell decays?
10. What is the “isotope” shift?
11. Why are muonic atoms a great tool in these measurements?
12. How can one measure the charge radius of a nucleus from the Coulomb energy in mirror nuclei? How does this experiment work?
13. Show that

$$\left(\frac{3}{4\pi}\right)^2 \int_{|\vec{u}_1| \leq 1} d\vec{u}_1 \int_{|\vec{u}_2| \leq 1} d\vec{u}_2 \frac{1}{|\vec{u}_1 - \vec{u}_2|} = \frac{6}{5}$$

and describe why this integral is so important in nuclear physics. Where does it come from?

Chapter 10: Nuclear Properties...

Chapter 10.3: Nuclear Binding Energy ...

1. Describe in words, what is the binding energy of the nucleus.
2. Why is atomic binding ignored in all discussions of nuclear binding energy?
3. Explain: $B_N(Z, A) = \{Zm_p + Nm_n - [m(^A X) - Zm_e]\} c^2$.
4. “mass defect/mass excess” What is it?
5. What is the “neutron separation energy”?
6. What is the “proton separation energy”?
7. What is and explain $B(Z, A)$, $B(Z, A)/A$ and the “Semiempirical Mass Formula”
8. Explain and justify, all the components of $B(Z, A)$.
9. Sketch $B(Z, A)/A$.
10. How is $B(Z, A)$ applied to the energetics of β -decay?

Chapter 10.4: Angular Momentum and Parity ...

1. What do $\vec{s}$, $\vec{l}$, $\vec{j}$, $\vec{S}$, $\vec{L}$, $\vec{I}$ mean? How do you calculate/determine/measure them?
How do you add them?
2. What does I^π mean? Are these unique for every isotope? What resource can you use to find them?

Chapter 10: Nuclear Properties...

Chapter 10.5: Nuclear Magnetic and Electric Moments ...

1. Why can you see only one face of the moon? What percentage of the moon's surface can you see from the earth? What is "libration"?
2. What is a magnetic dipole?
3. Why doesn't a nucleus have an electric dipole moment?
4. What is a quadrupole moment?
5. What are the orbital and spin gyromagnetic factors? What are the values of the angular (not the spin) ones?
6. Show that

$$\begin{aligned} V(\vec{x}) &= \lim_{|\vec{x}| \gg R_N} \frac{Ze}{4\pi\epsilon_0} \int d\vec{x}' \frac{\rho_p(\vec{x}')}{|\vec{x} - \vec{x}'|} \\ &= \frac{Ze}{4\pi\epsilon_0} \left[\frac{1}{|\vec{x}|} \int d\vec{x}' \rho_p(\vec{x}') + \frac{\vec{x}}{|\vec{x}|^3} \cdot \int d\vec{x}' \vec{x}' \rho_p(\vec{x}') + \right. \\ &\quad \left. \frac{1}{2|\vec{x}|^5} \int d\vec{x}' (3(\vec{x} \cdot \vec{x}')^2 - |\vec{x}|^2 |\vec{x}'|^2) \rho_p(\vec{x}') \dots \right] \end{aligned}$$