

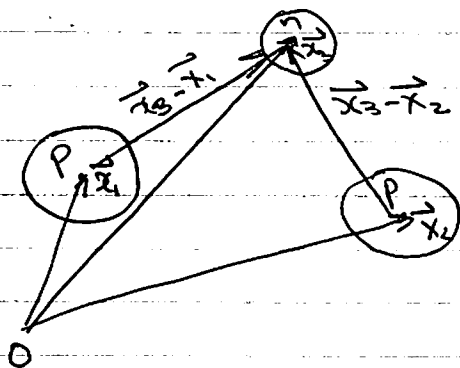
## Nuclear models

Many of the ideas and methods we learned in studying atoms & their quantum behavior, carry over to nuclear physics. But, in some important ways, it is quite different.

Firstly, we don't really know what the nucleon-nucleon potential is. But we do know that it has a central,  $V(r)$ , and non-central part. That's the first complication.

Secondly, the force on one nucleon not only depends on the position of the other nucleons, but also the distances between the other nucleons! These are called 'many-body' forces.

Let's illustrate this by building a  ${}^3\text{T}$  nucleus



The potential on the p at  $\vec{x}_1$  is given by

These would appear in atomic physics

$$\begin{aligned}
 & \left\{ \underbrace{V_{nn}(\vec{x}_3 - \vec{x}_1)}_{\text{influence of } n \text{ at } \vec{x}_3} + \underbrace{V_{nn}(\vec{x}_2 - \vec{x}_1)}_{\text{influence of } p \text{ at } \vec{x}_2} + \underbrace{V_c(|\vec{x}_2 - \vec{x}_1|)}_{\text{Coulomb repulsion of } p \text{ at } \vec{x}_2} \right. \\
 & \left. + \underbrace{V_3(\vec{x}_1 - \vec{x}_2, \vec{x}_1 - \vec{x}_3, \vec{x}_2 - \vec{x}_3)}_{3 \text{ body term}} \right\}
 \end{aligned}$$

The 3-body term is a fundamentally different thing.

You can think of  $V_3$  as a polarization term. The presence of 3 influences how 2 acts on 1, and so on.

People act this way!

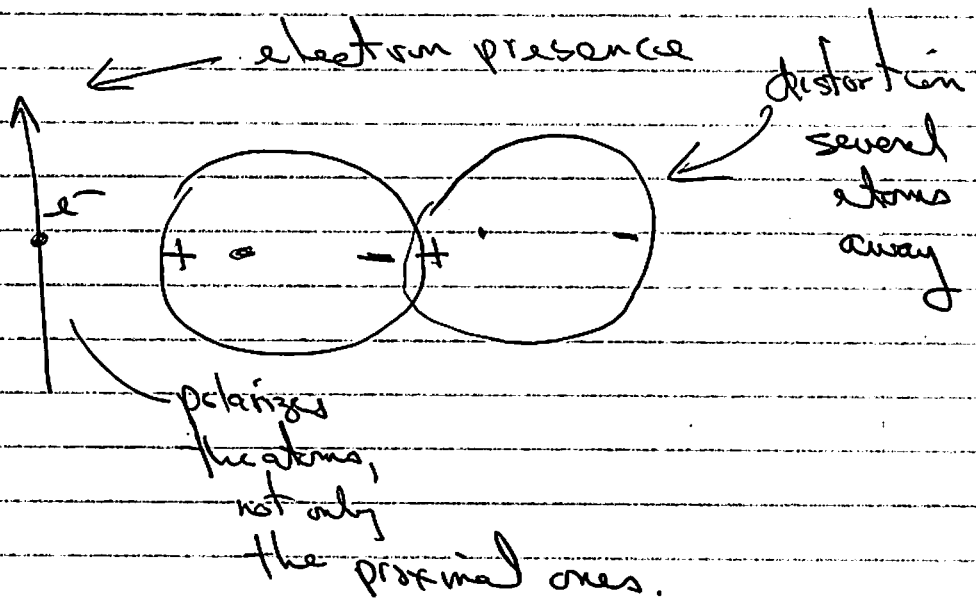
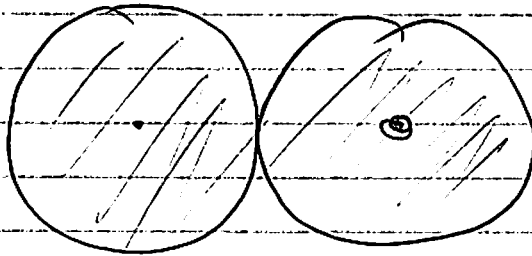
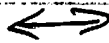
Person 1 may interact with person 2 in a different way if person 3 is present!

These many-body forces are hard to set a grip on. Nuclear theory is basically a phenomenological one based on measurement, and 3 and higher body forces are hard to measure.

However, ...

(1) Just for polarization

vibration

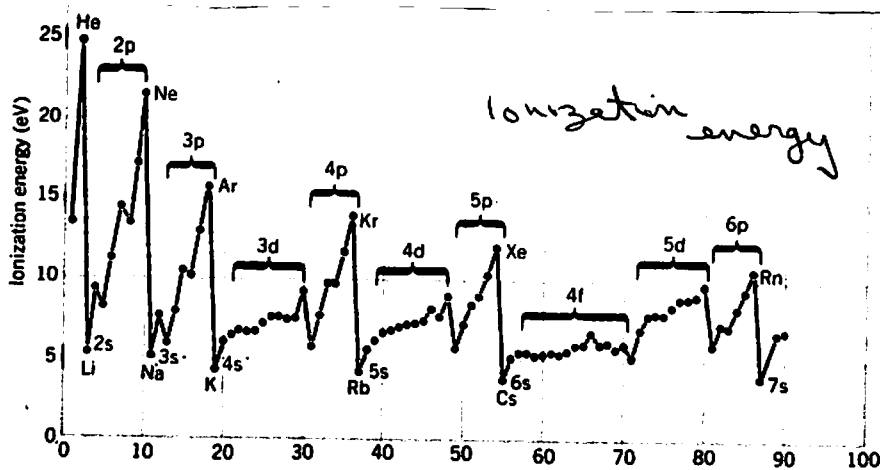
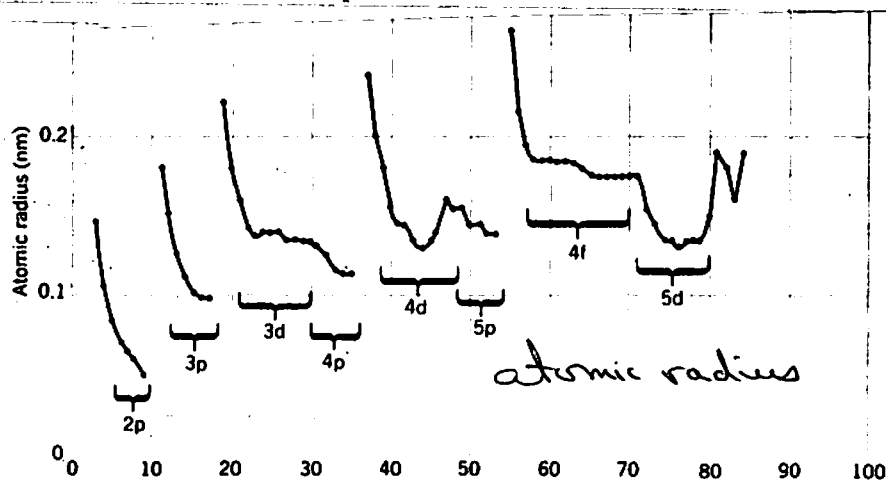


we will persist with developing simple models for nuclei

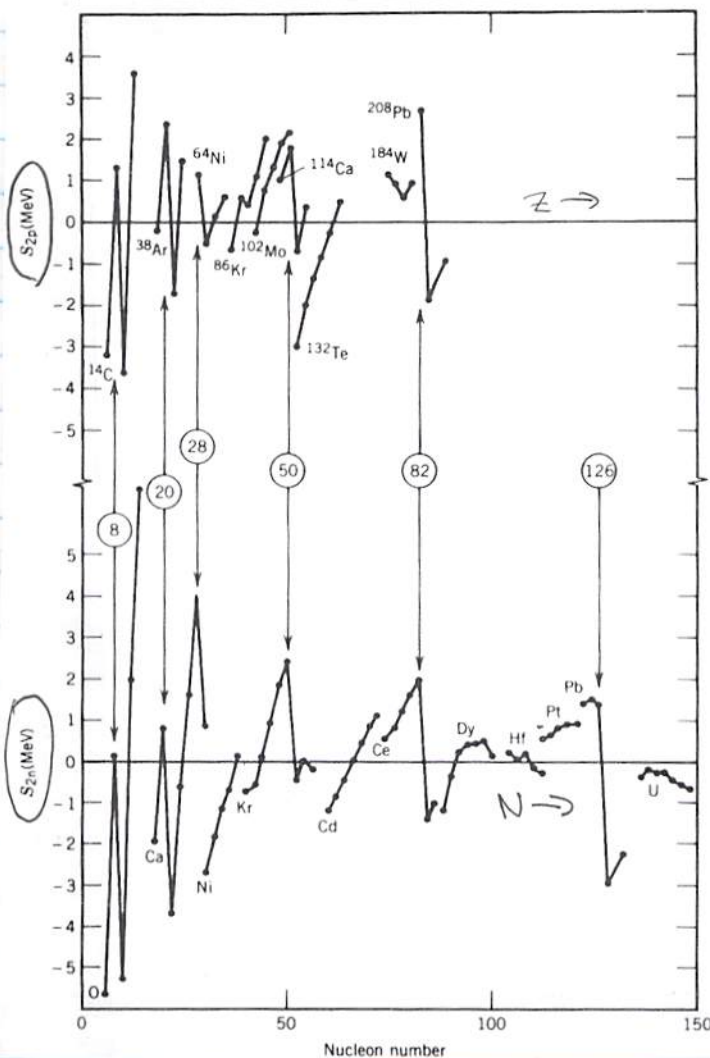
These models organize the way we think about nuclei based upon some intuitive guesses. Should the guess have predictive power, it predicts some behavior we can measure, then we've learned something. Not the whole picture, but at least some aspect of it.

## 5.1 The Shell Model

Atomic system show a very pronounced shell structure.



Nuclei, as well, show a "shell-like" structure



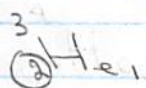
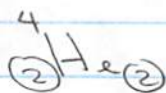
←  $2p$  separation energy

←  $2n$  separation energy

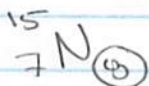
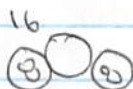
The magic numbers represent closed shells

2, 8, 20, 28, 50, 82, 126

# The stable magic nuclei


 $1.38 \times 10^{-4} \%$ 


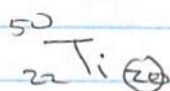
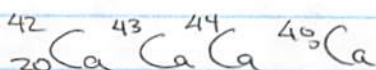
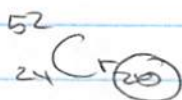
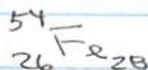
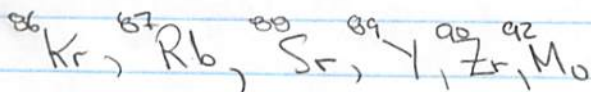
doubly magic

 $99.99986 \%$ 

 $0.366 \%$ 


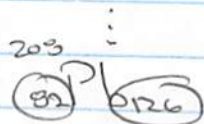
doubly magic

 $99.76 \%$ 

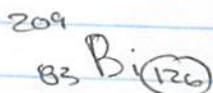

doubly magic

 $96.94 \%$ 

 $5.2 \%$ 

 $83.79 \%$ 

 $5.8 \%$ 


and so on

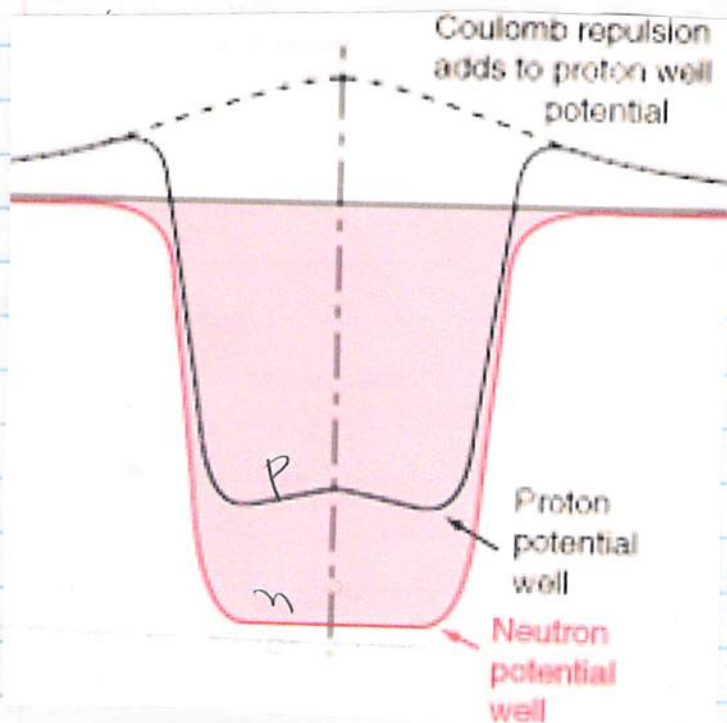


doubly magic

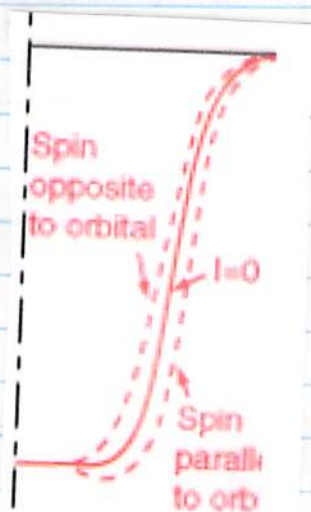
 $52.3 \%$ 

 $100 \%$

# Shell model idea

A nucleus is composed of a "core" that produced a potential that determines the properties of the valence nucleons. These properties determine the behavior of the nucleus much in the same way that the valence electrons in an atom determine its chemical properties.



Spin orbit part



numerical calculation

|     |             |      |
|-----|-------------|------|
| 184 | $3d_{3/2}$  | (4)  |
|     | $4s_{1/2}$  | (2)  |
|     | $2g_{7/2}$  | (8)  |
|     | $3d_{5/2}$  | (6)  |
|     | $1i_{5/2}$  | (16) |
|     | $1i_{1/2}$  | (12) |
|     | $2g_{9/2}$  | (10) |
| 126 | $3p_{1/2}$  | (2)  |
|     | $2f_{5/2}$  | (6)  |
|     | $3p_{3/2}$  | (4)  |
|     | $1i_{3/2}$  | (14) |
|     | $1h_{9/2}$  | (10) |
|     | $2f_{7/2}$  | (8)  |
| 82  | $1h_{11/2}$ | (12) |
|     | $3d_{3/2}$  | (4)  |
|     | $3s_{1/2}$  | (2)  |
|     | $2g_{7/2}$  | (8)  |
|     | $2d_{5/2}$  | (6)  |
| 50  | $2g_{9/2}$  | (10) |
|     | $2p_{1/2}$  | (2)  |
|     | $1f_{5/2}$  | (6)  |
|     | $2p_{3/2}$  | (4)  |
| 28  | $1f_{7/2}$  | (8)  |
| 20  | $3d_{3/2}$  | (4)  |
|     | $2s_{1/2}$  | (2)  |
|     | $1d_{5/2}$  | (6)  |
| 8   |             |      |
|     | $1p_{1/2}$  | (4)  |
|     | $1p_{3/2}$  | (2)  |
| 2   |             |      |
|     | $1s_{1/2}$  | (2)  |

### The neutron central potential

$$V_n(r) = \frac{-V_0}{1 + \exp\left(\frac{r-R}{a}\right)}$$

$$R = 1.25 A^{1/3} \text{ fm}$$

$$a = 0.524 \text{ fm}$$

### The proton central potential

$$V_p(r) = V_n(r) + \frac{(Z-1)e^2}{4\pi\epsilon_0} \underbrace{\int \frac{d\vec{x}' \rho_p(r')}{|\vec{x}-\vec{x}'|}}_{\text{Coulomb part}}$$

becomes a fn of  $r$   
after integration

### Spin orbit potential

The spin-orbit potential has the form

$$V_{so}(\vec{r}) = V_{so}(r) \vec{l} \cdot \vec{s}$$

Recall,  $\vec{j} = \vec{l} + \vec{s}$

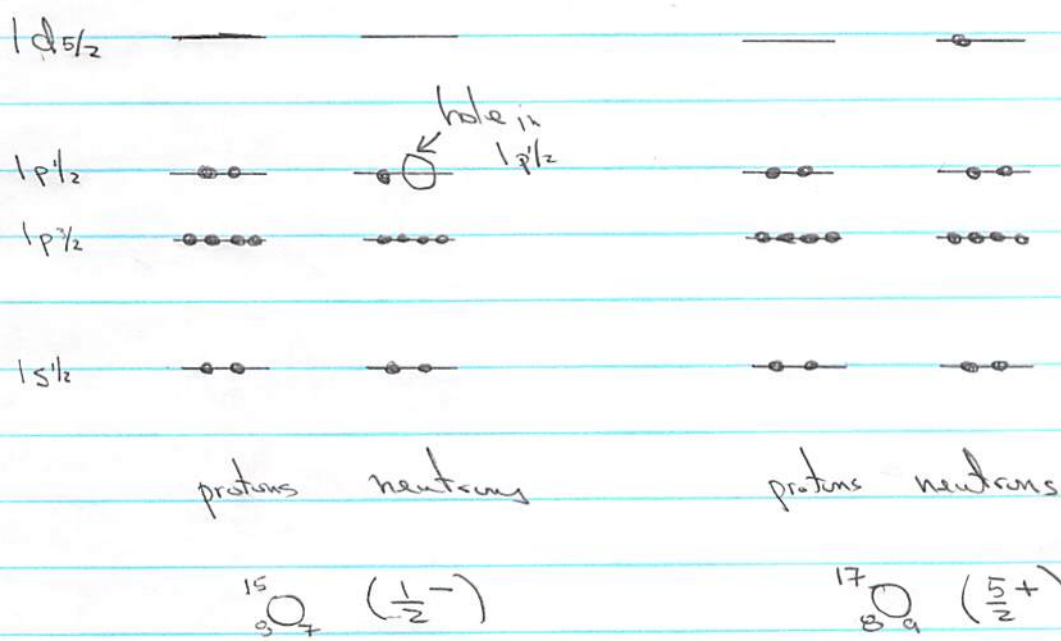
$$j^2 = l^2 + 2\vec{l} \cdot \vec{s} + s^2$$

$$\langle \vec{l} \cdot \vec{s} \rangle = \frac{1}{2} \langle j^2 - l^2 - s^2 \rangle$$

$$j = l \pm \frac{1}{2}$$



# Recall from last time



This way of assigning  $I^\pi$  is based on the Extreme Independent Particle Model (EIPM)

"Characteristics of the nucleus is determined by the last particle in (valence neutron)"  
 (inner nucleons pair up & cancel)

|                       |                 |                      |        |
|-----------------------|-----------------|----------------------|--------|
| $^{14}\text{O}_6$     | $0^+$           | } $\epsilon$ capture |        |
| $^{15}\text{O}_7$     | $\frac{1}{2}^-$ |                      |        |
| $^{16}\text{O}_8$     | $0^+$           |                      | 97.76% |
| $^{17}\text{O}_9$     | $\frac{5}{2}^+$ | } $\beta^-$ decay    | 0.033% |
| $^{18}\text{O}_{10}$  | $0^+$           |                      | 0.204% |
| $^{19}\text{O}_{11}$  | $\frac{5}{2}^+$ |                      |        |
| $^{20}\text{O}_{12}$  | $0^+$           |                      |        |
| all predicted by EIPM |                 |                      |        |

|                                             |                 |                          |
|---------------------------------------------|-----------------|--------------------------|
| <sup>13</sup> <sub>5</sub> B <sub>8</sub>   | $\frac{3}{2}^-$ | all predicted<br>by EIPM |
| <sup>14</sup> <sub>6</sub> C <sub>8</sub>   | 0 <sup>+</sup>  |                          |
| <sup>15</sup> <sub>7</sub> N <sub>8</sub>   | $\frac{1}{2}^-$ |                          |
| <sup>16</sup> <sub>8</sub> O <sub>8</sub>   | 0 <sup>+</sup>  |                          |
| <sup>17</sup> <sub>9</sub> F <sub>8</sub>   | $\frac{5}{2}^+$ |                          |
| <sup>18</sup> <sub>10</sub> Ne <sub>8</sub> | 0 <sup>+</sup>  |                          |

### EIPM predictions of magnetic moments of the nucleus

Recall  $\mu = \mu_N (g_L l_z + g_S s_z)$

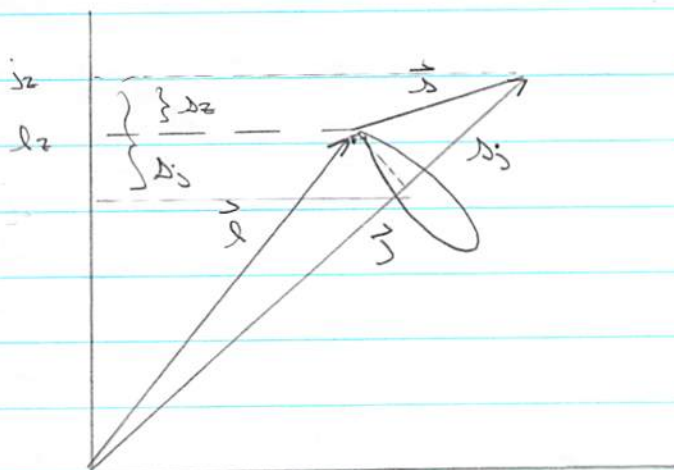
$l_z, s_z$  have  
max +ve  
values

Let  $\vec{j} = \vec{l} + \vec{s}$

$l_z = j_z - s_z$

①  $\mu = \mu_N (g_L (j_z - \langle s_z \rangle) + g_S \langle s_z \rangle)$

$\vec{s}$  'precesses' about the  $\vec{j}$  axis



(72)

$\vec{S}$  precesses around the  $\vec{J}$  axis, hence only  $S_J$  is known

$$\langle S_z \rangle = \langle S_J \rangle = \frac{\vec{S} \cdot \vec{J}}{|\vec{J}|}$$

$$\vec{S} = \frac{\vec{J}(\vec{S} \cdot \vec{J})}{|\vec{J}|^2}$$

$$S_J = \frac{j}{j(j+1)} \frac{1}{2} \left( j(j+1) - l(l+1) - s(s+1) \right)$$

exactly as in  
spin orbit  
splitting

For  $j = l + \frac{1}{2}$

$$S_J = \frac{1}{l + \frac{3}{2}} \cdot \frac{1}{2} \left[ \underbrace{(l + \frac{1}{2})(l + \frac{3}{2}) - l(l+1) + \frac{1}{2}(\frac{3}{2})}_{l + \frac{3}{2}} \right]$$

$$S_{l+1/2} = \frac{1}{2}$$

$$S_{l-1/2} = \frac{1}{l + \frac{1}{2}} \cdot \frac{1}{2} \left[ \underbrace{(l - \frac{1}{2})(l + \frac{1}{2}) - l(l+1) + \frac{1}{2}(\frac{3}{2})}_{-l + \frac{1}{2}} \right]$$

$$= -\frac{1}{2} \frac{l - \frac{1}{2}}{l + \frac{1}{2}} = -\frac{1}{2} \frac{j}{j+1}$$

So,

$$\mu_{j=1+1/2} = \mu_N \left[ g_p \left( j - \frac{1}{2} \right) + \frac{g_s}{2} \right]$$

$$\textcircled{2} \quad \mu_{j=1+1/2} = \frac{\mu_N}{2} \left[ g_p (2j-1) + g_s \right]$$

$$\mu_{j=1-1/2} = \mu_N \left[ g_p \left[ j + \frac{1}{2} \frac{j}{j+1} \right] - \frac{g_s}{2} \frac{j}{j+1} \right]$$

$$\textcircled{3} \quad \mu_{j=1-1/2} = \frac{\mu_N j}{2(j+1)} \left[ g_p (2j+3) - g_s \right]$$

Note a) K 5.9

$$\mu_{j=1-1/2} = \mu_N \left[ \frac{g_p j(j+\frac{3}{2})}{(j+1)} - \frac{g_s}{2(j+1)} \right]$$

add "j"  
↓

b) agrees with wiki!

Plotting  $\mu$  vs  $j$  Schmidt lines

Show graph 5.9 on p.127 &amp; 128

Experimental values are generally lower for the  $g_s$  part

Reason: Hypothesis (polarization of the nuclear matter in the nucleus)

$$g_s (\text{in nucleus}) \approx 0.6 g_s (\text{free})$$

EIPM predictions of the quadrupole moment

Recall  $Q_2 = \int d\vec{r} \psi^*(\vec{r}) (3z^2 - r^2) \psi(\vec{r})$

↑  
single nucleon wave fn

Quantum Mechanical calculation

$$\langle Q_{sp} \rangle = - \frac{2j-1}{2(j+1)} \langle r^2 \rangle$$

↑  
single  
proton

$$\langle Q_n \rangle = \langle Q_{sp} \rangle \left[ 1 - \frac{2(n-1)}{2j-1} \right]$$

one particle  
 $1 \leq n \leq 2j$  full shell less one particle

$$= \langle Q_{sp} \rangle \left[ \frac{2(j-n)+1}{2j-1} \right]$$

note:

$$\langle Q_{1/2} \rangle = \langle Q_{sp} \rangle$$

single particle

$$\langle Q_{2j} \rangle = -\langle Q_{sp} \rangle$$

single hole

↑  
 $2j+1$  particles  
 to fill a shell

$$\langle Q_{sp} \rangle = 0 \quad \text{for neutrons}$$

Show data in Fig. 5.10 p 130

Generally EIPM prediction is  $\sim 2-3$   
times too small.

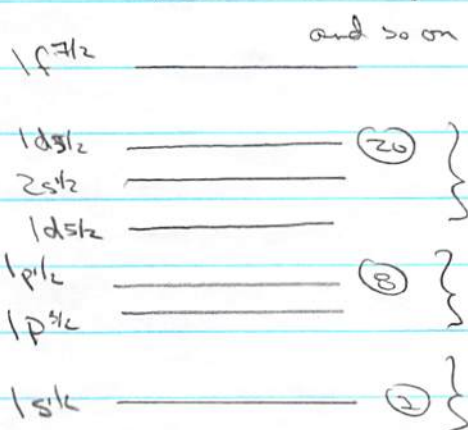
In certain ranges  $150 < A < 190$   $A > 220$   
a factor of 10 too small.

Evidence for collective effects.

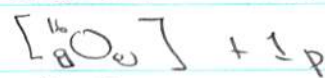
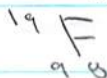
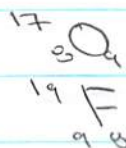
## Shell model predictions of excitation levels

If the EIPM model were true, we could measure the shell model ground state levels, by looking at excited states, just as we do in atomic structure.

### Recall shell model

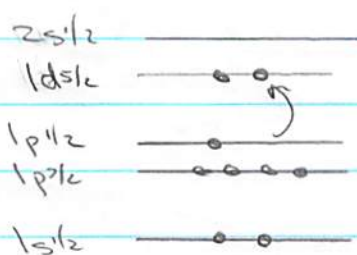
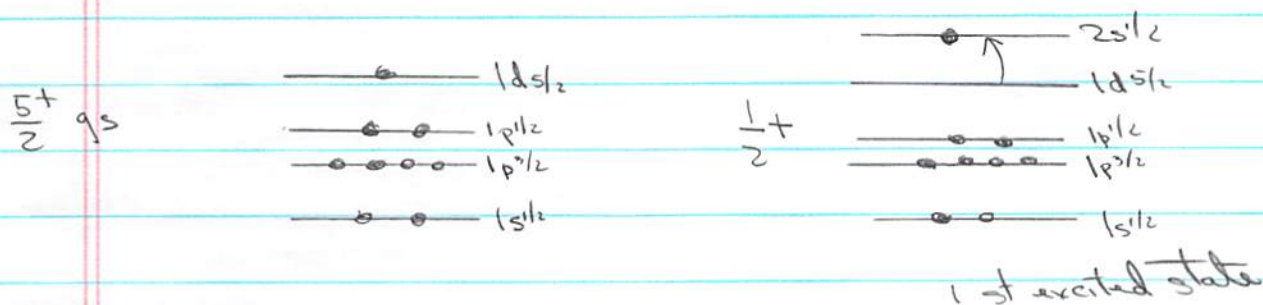
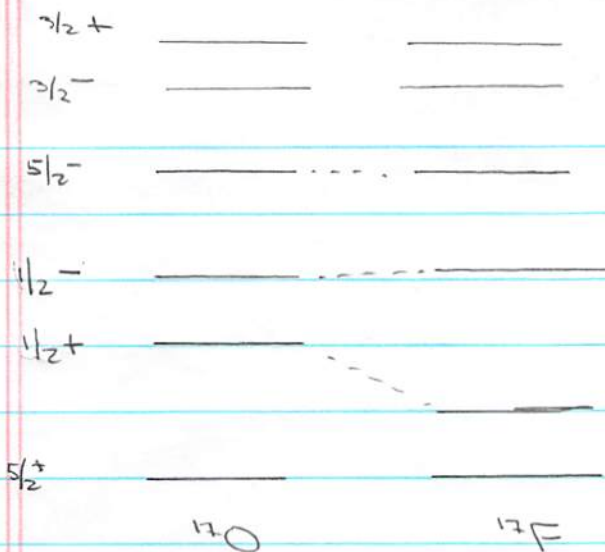


Let's see how this works for the most suitable candidates

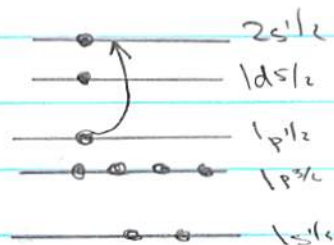


doubly  
magic

one valence  
nucleon

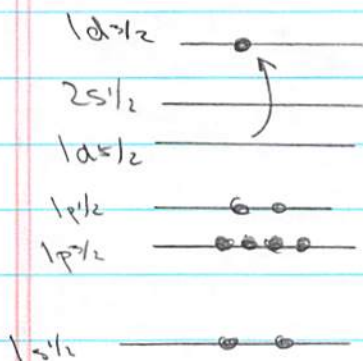


$\frac{1}{2}^-$  2nd excited state



$\frac{5}{2}^-$  3rd excited state

and  $\frac{3}{2}^-$  4th excited state



$\frac{3}{2}^+$

$$\frac{3}{2} < \left( \frac{1}{2} + \frac{1}{2} + \frac{5}{2} \right) < \frac{7}{2}$$

EIPD works quite well





**Figure 5.12** Energy levels of nuclei with odd particles in the  $1f_{7/2}$  shell.

Show graph on p 132

ESPM works quite well 1 away from closed shells.

More valence nucleons ... not so much

Q. How reliable is a single particle wavefunction when a nucleon is in close proximity to the core?

A. When  $S \neq 0$ , we expect ... probably ok.

For an S state consider

$$\langle \rho_p \rangle \quad \text{proton density} \quad - \quad \langle \rho_p \rangle$$

$\begin{matrix} 205 \\ 81 \text{ Tl} \end{matrix}_{124}$ 
 $\begin{matrix} 209 \\ 82 \text{ Pb} \end{matrix}$

$\uparrow$   
 $3s^{1/2}$  shell  
 only has one  
 proton

$\uparrow$   
 closed magic shell

that extra S shell  
 proton goes right  
 through the  
 nucleus in  
 a classical  
 sense

Show fig 5.13 p 133

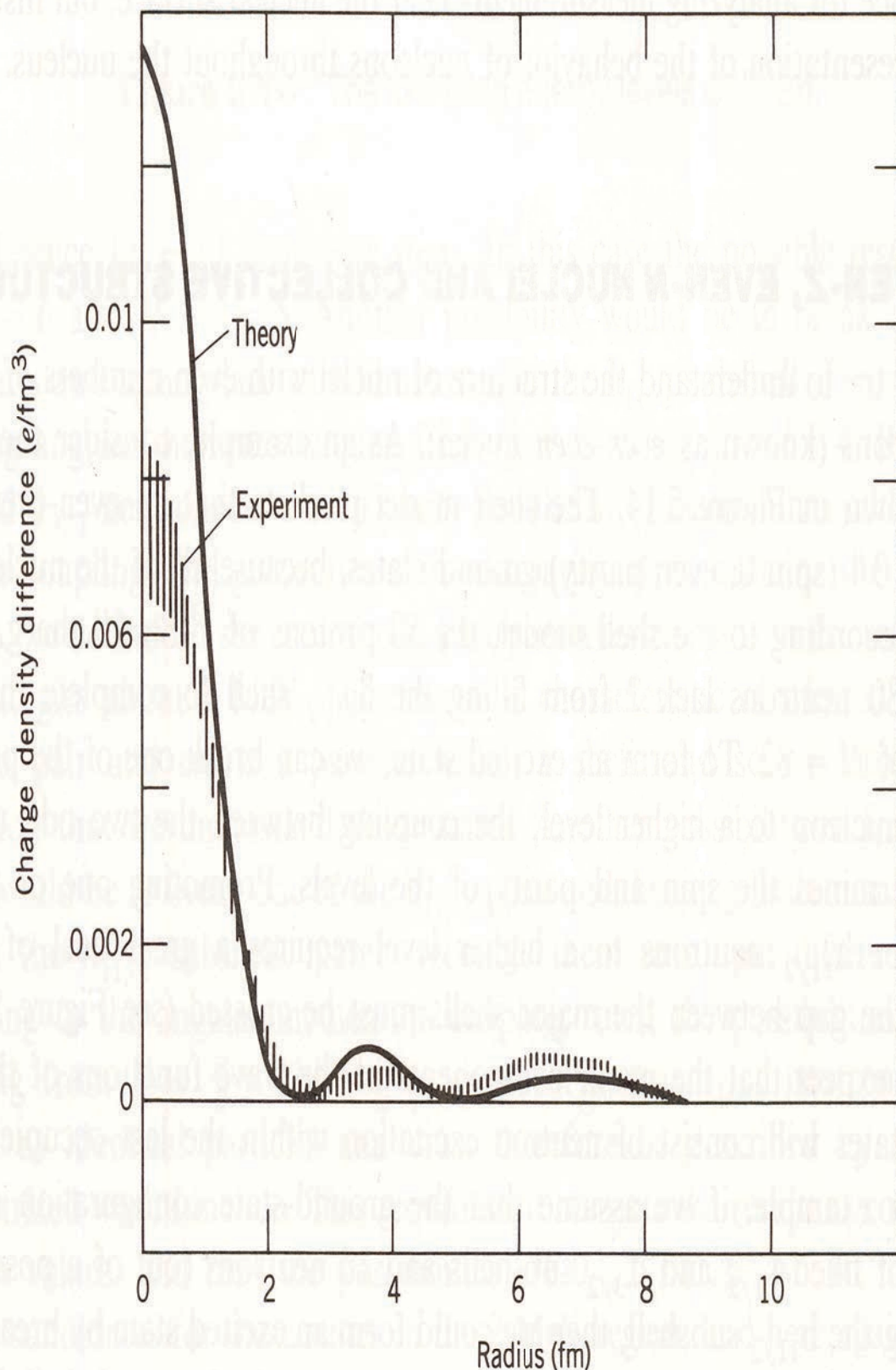


**Figure 5.13** The difference in charge density between  $^{205}\text{Tl}$  and  $^{206}\text{Pb}$ , as determined by electron scattering. The curve marked “theory” is just the square of a harmonic oscillator 3s wave function. The theory reproduces the variations in the charge density extremely well. Experimental data are from J. M. Cavedon et al., *Phys. Rev. Lett.* **49**, 978 (1982).

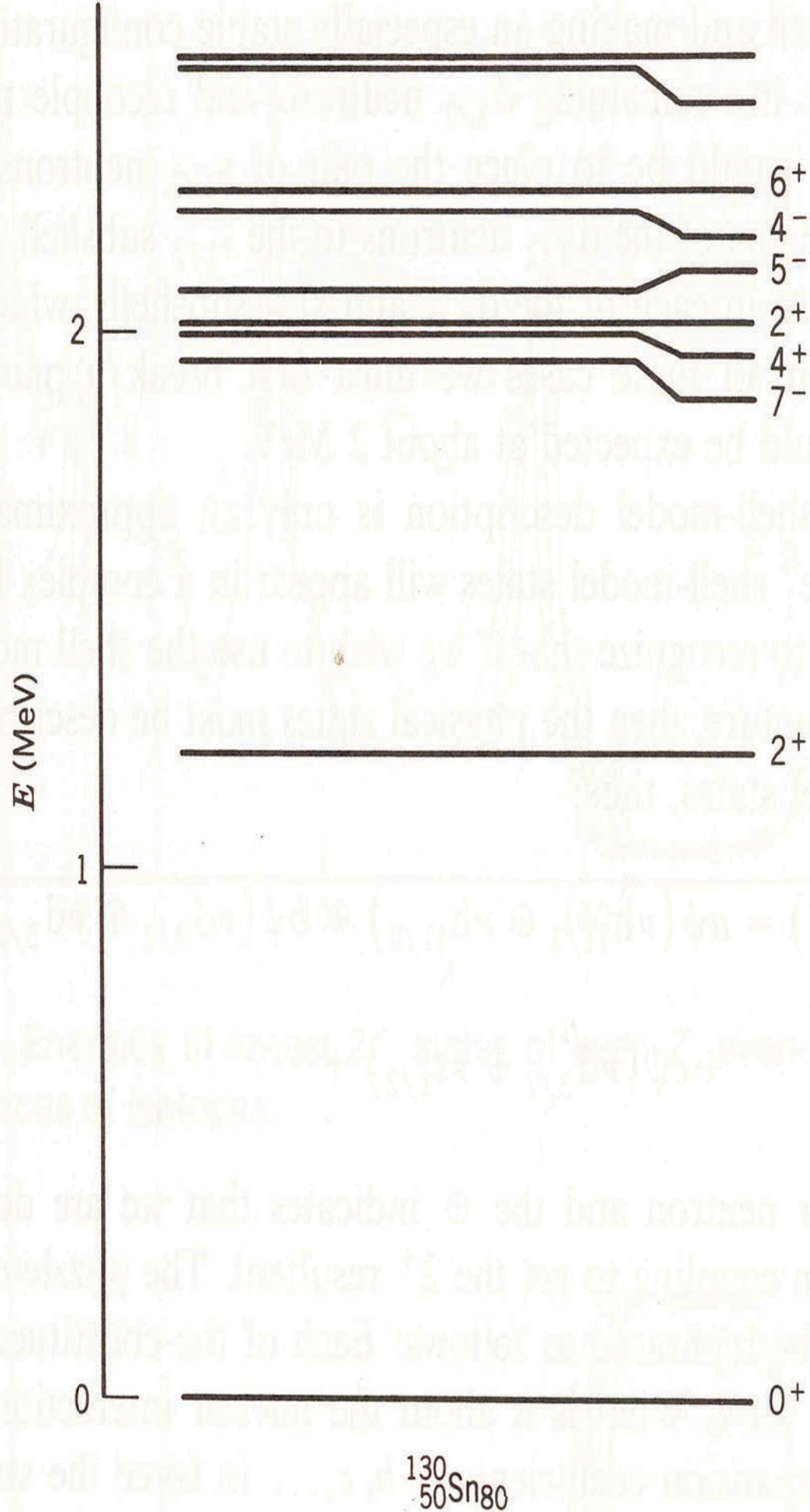


**Figure 5.14** The low-lying energy levels of  $^{130}\text{Sn}$ .

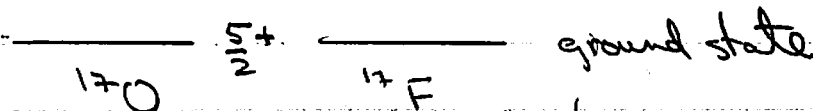
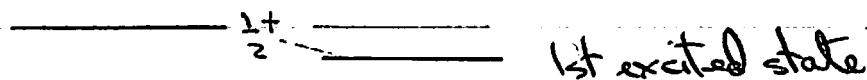
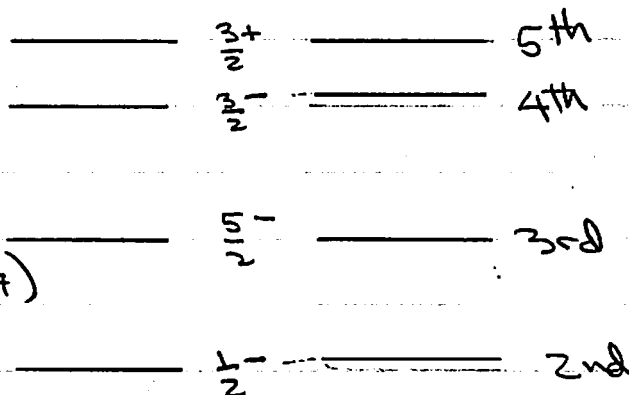
Excited states of  $^{17}\text{O}_9$ ,  $^{17}\text{F}_8$  explained  
by the EIPM & the Shell model.  
"Valence nucleons" Kpp 131-134

Observed structure

The small  
differences  
relate to

$$V_c(9,12) - V_c(8,17)$$

$$V_p(8,16) - V_n(8,16)$$



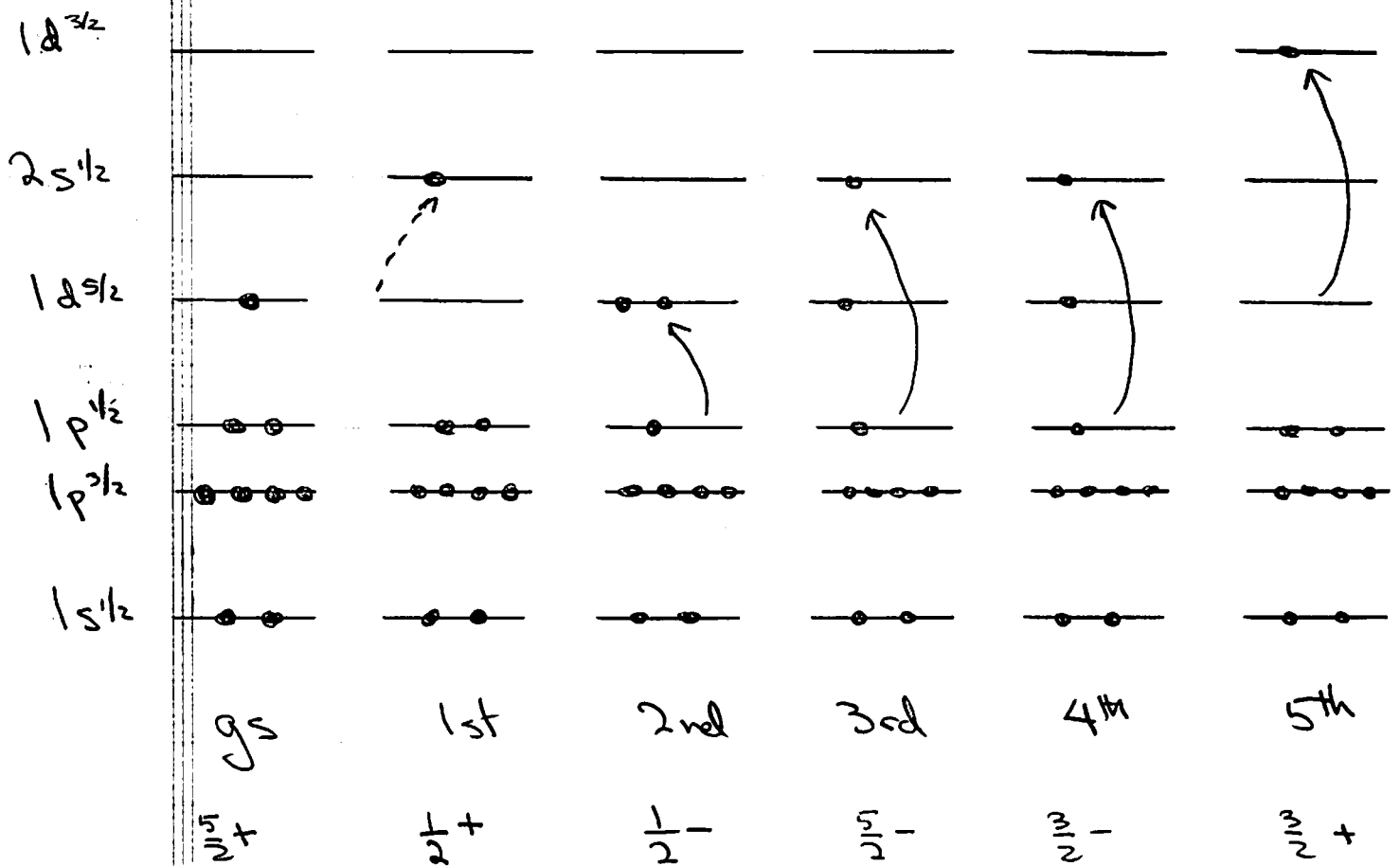
1st excited state

ground state

( see implied  
level occupancies  
on next page

● represents n in the case of  $^{17}\text{O}$  (8)  
 ● " p " " " " "  $^{17}\text{F}$

from the shell model



gs: Discussed before

1st:  $1d^{5/2} \rightarrow 2s^{1/2}$   $\frac{1}{2}^+$  ✓

2nd:  $1p^{1/2} \rightarrow 1d^{5/2}$   $\frac{1}{2}^-$  (due to <sup>single</sup> vacancy in  $1p^{1/2}$ )

3rd:  $1p^{1/2} \rightarrow 2s^{1/2}$   $\frac{5}{2}^-$   $\frac{3}{2} \leq I \leq \frac{7}{2}$ ;  $\frac{5}{2}$  most favorable  
 $\pi = (-1)^1 * (-1)^2 * (-1)^0 = -$

4th:  $1p^{1/2} \rightarrow 2s^{1/2}$   $\frac{3}{2}^-$   $\frac{3}{2} \leq j \leq \frac{7}{2}$ ;  $\frac{3}{2}$  preferred over  $\frac{7}{2}, \frac{5}{2}$  taken  
 $\pi = \text{same as 3rd}$

$1d^{5/2} \rightarrow 1d^{3/2}$   $\frac{3}{2}^+$  ✓

5th

## Even-even nuclei and Collective Structure

All even/even nuclei are  $0^+$  (pairing force)

All even/even nuclei have an anomalously small 1st excited state at  $2^+$  that can not be explained by the shell model (EIPM or not). Read K pp 134-139

Show Fig. 5.15 a

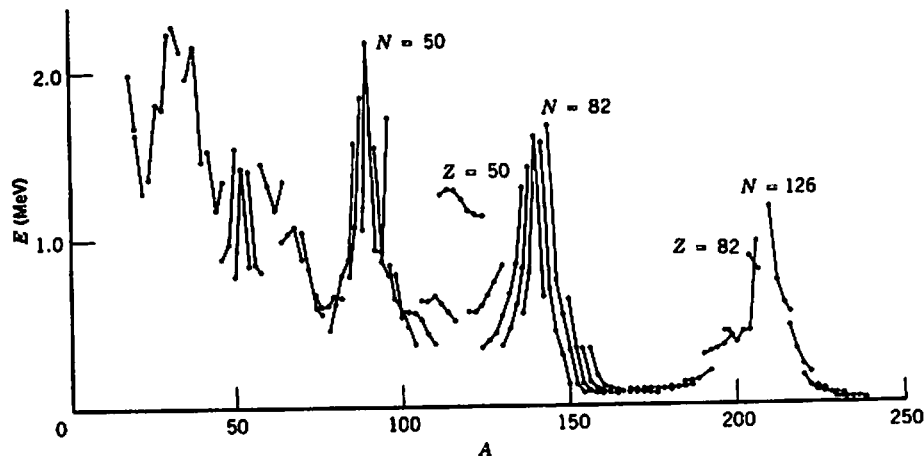


**Figure 5.15a** Energies of lowest  $2^+$  states of even-Z, even-N nuclei. The lines connect sequences of isotopes.

Except near closed shells,  
smooth downward trend in  $E(2^+)$ .  
Regions  $150 < A < 190$   $A > 220$  seem  
very small & consistent

## Quadrupole moments

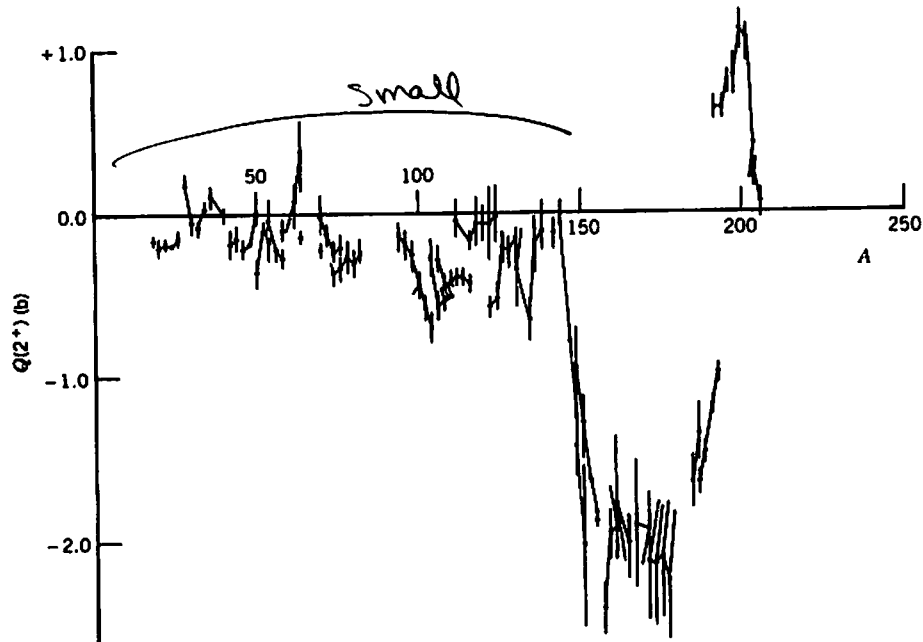


**Figure 5.16b** Electric quadrupole moments of lowest  $2^+$  states of even-Z, even-N nuclei. The lines connect sequences of isotopes.

$Q_2$  is small for  $A < 150$

$Q_2$  large and negative for  
 $150 < A < 200$  suggesting "oblate"  
 deformation

$$E(4^+)/E(2^+)$$

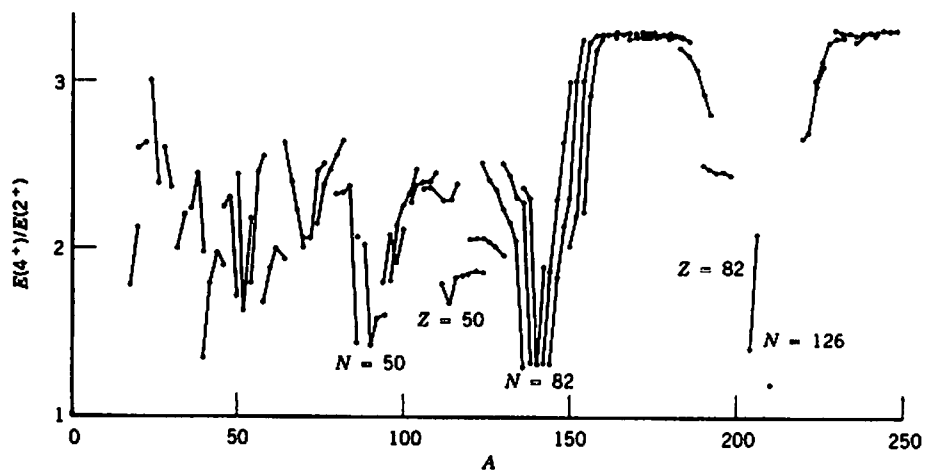


**Figure 5.15b** The ratio  $E(4^+)/E(2^+)$  for the lowest  $2^+$  and  $4^+$  states of even- $Z$ , even- $N$  nuclei. The lines connect sequences of isotopes.

Region

between  $150 < A < 190$

$A > 220$

markedly different

## Types of collective behavior

"liquid drop model"

Distortions can be

(phonons)

i) vibrational (about a spherical or ellipsoidal shape) (all A)

(rotons)

ii) rotational, associated with rotations of the entire nucleus

## Vibrations about a spherical shape

phonons (as the wave)

radius

$$R(\theta, \phi, t) = \underset{\substack{\uparrow \\ \text{average}}}{R_{av}} + \sum_{\lambda \geq 1} \sum_{\mu = -\lambda}^{\lambda} \alpha_{\lambda}(t) \underset{\substack{\uparrow \\ \text{units of} \\ \text{angular} \\ \text{momentum}}}{Y_{\lambda, \mu}}(\theta, \phi) \underset{\substack{\uparrow \\ \text{spherical} \\ \text{harmonic}}}{Y_{\lambda, \mu}(\theta, \phi)}$$

$\lambda \hbar$  is the angular momentum

$$| \lambda |^2 = \lambda(\lambda+1) \hbar^2 \quad \text{same quantization rule}$$

$$-\lambda \leq \mu \leq \lambda \quad \quad \quad " \quad " \quad "$$

Reflection symmetry  $\Rightarrow \alpha_{\lambda\mu} = \alpha_{\lambda-\mu}$

$\lambda = 1$ ; "Dipole" vibration

A dipole vibration is a vibration of the center of mass.

Not allowed, no external forces  
(= noise wave)

$\pi = +1$      $\lambda = 2$ ; "Quadrupole" vibration

double wave, diametrically opposite

$\pi = -1$      $\lambda = 3$ ; "Octopole" vibration

Coupling 2 quadrupole phonons

$$\vec{\Lambda} = \vec{2} + \vec{2} \Rightarrow$$

|               | $M_n$                           | $d$ |
|---------------|---------------------------------|-----|
| $\Lambda = 0$ | 0                               | 1   |
| $\Lambda = 1$ | 0, $\pm 1$                      | 3   |
| $\Lambda = 2$ | 0, $\pm 1, \pm 2$               | 5   |
| $\Lambda = 3$ | 0, $\pm 1, \pm 2, \pm 3$        | 7   |
| $\Lambda = 4$ | 0, $\pm 1, \pm 2, \pm 3, \pm 4$ | 9   |

So, in principle 5 different ways.  
But  $\Lambda = 1, 3$  have odd parity and not allowed for a 2-boson wf. Hence.

$$\Lambda = 0^+, 2^+, 4^+$$

So 2 quadrupole phonons, give a triplet  $0^+, 2^+, 4^+$  at  $2 \times$  the energy of the single  $2^+$  excitation.

In a similar way

$$\vec{\Lambda} = \vec{2} + \vec{2} + \vec{2} \Rightarrow \underbrace{0^+, 2^+, 3^+, 4^+, 6^+}_{\substack{\text{5 fold degeneracy} \\ \text{at } \approx 3 \times \\ \text{the single } 2^+ \\ \text{energy.}}}$$

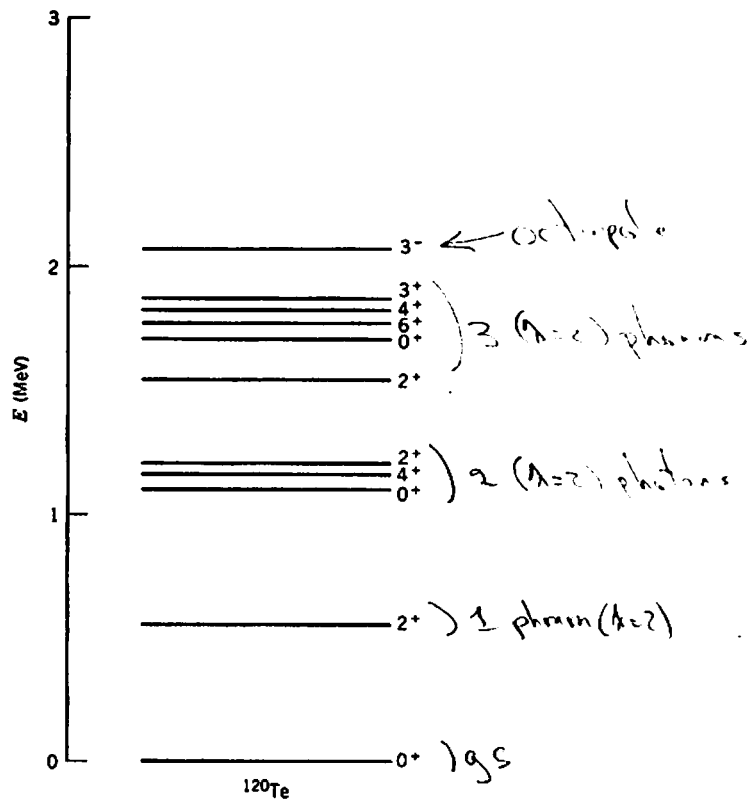


**Figure 5.19** The low-lying levels of  $^{120}\text{Te}$ . The single quadrupole phonon state (first  $2^+$ ), the two-phonon triplet, and the three-phonon quintuplet are obviously seen. The  $3^-$  state presumably is due to the octupole vibration. Above 2 MeV the structure becomes quite complicated, and no vibrational patterns can be seen.

## Nuclear Rotations (Rotons)

$$R(\theta, \phi) = R_{\text{av}} [1 + \beta Y_{20}(\theta, \phi)]$$

↑  
fixed,  
not a  
function of time

$$\beta = \frac{4}{3} \sqrt{\frac{11}{5}} \frac{\Delta R}{R_{\text{av}}}$$

$\Delta R =$  difference  
between  
semi major  
and semi minor  
axes

Recall,

$$\text{(oblate)} \quad \frac{x^2 + y^2}{R_{\text{max}}^2} + \frac{z^2}{R_{\text{min}}^2} = 1$$

$$\text{(prolate)} \quad \frac{x^2 + y^2}{R_{\text{min}}^2} + \frac{z^2}{R_{\text{max}}^2} = 1$$

} eqns of  
an ellipsoid

$$\Delta R = R_{\text{max}} - R_{\text{min}}$$

↑ semi-major      ↑ semi-minor

Recall, the quadrupole moment

$$Q_2 = \frac{3}{\sqrt{5}} R_{\text{av}}^2 (1 + 0.36\beta)$$

$\sim \frac{2}{7} \left(\frac{5}{11}\right)^{1/2}$

### Energy of rotation

Classically  $E_{\text{rot}} = \frac{1}{2} I \omega^2$

← moment of inertia

↑ script I

$I\omega = \text{angular momentum}$

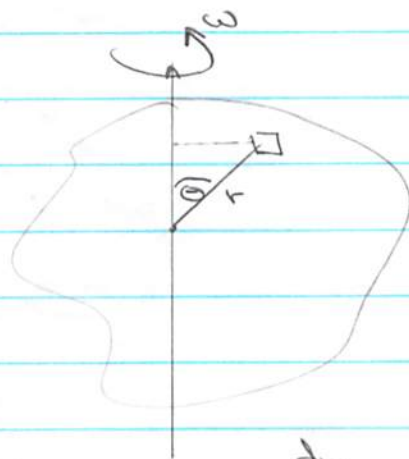
### Quantum transition

$$E_{\text{rot}} = \frac{1}{2} \frac{1}{I} \underbrace{(I\omega)^2}_{\substack{\uparrow \\ (\text{angular} \\ \text{momentum})^2}}$$

$$= \frac{1}{2} \frac{1}{I} I(I+1)$$

## Moment of inertia?

Imagine that an object is spinning around the z-axis, which cuts through its center of mass



classical calculation

$$dE_{\text{rot}} = \frac{1}{2} \underbrace{\rho(\vec{x}) d\vec{x}}_{\text{mass}} |\underbrace{\vec{v}(\vec{x})}_{\vec{\omega} r \sin\theta}|^2$$

$$\vec{v}(\vec{x}) = \vec{\omega} r \sin\theta$$

$$dE_{\text{rot}} = \frac{1}{2} \rho(\vec{x}) d\vec{x} \omega^2 r^2 \sin^2\theta$$

$$E_{\text{rot}} = \frac{1}{2} \underbrace{\int_{\text{Volume}} d\vec{x} \rho(\vec{x}) r^2 \sin^2\theta}_{\text{Volume}} \omega^2$$

For a uniform sphere  $\rho(\vec{x}) = \frac{M}{\frac{4}{3}\pi R^3}$

$$I = \frac{M}{\frac{4}{3}\pi R^3} \int_0^{2\pi} d\phi \int_0^\pi \sin\phi d\phi \sin^2\theta \int_0^R r^2 dr r^2$$

$$= \frac{3}{2} \frac{M}{R^3} \underbrace{2(1-\frac{1}{3})}_{\frac{4}{3}} \frac{R^5}{5}$$

$$= \frac{2}{5} MR^2$$

For the ellipsoidal nucleus

$$I = \int d\vec{x} \rho(\vec{x}) r^2 \sin^2 \theta$$

$$= \frac{M}{\frac{4}{3}\pi R_{av}^3} \int_0^{2\pi} d\phi \int_0^\pi \sin\theta d\theta \sin^2\theta \int_0^{R_{av}(1+\beta Y_{20})} r^2 dr r^2$$

$$= \frac{3}{2} \frac{M}{R_{av}^3} \int_{-1}^1 d\mu (1-\mu^2) \int_0^{R_{av}(1+\beta Y_{20})} r^4 dr$$

$$= \frac{3}{2} \frac{M}{R_{av}^3} \int_{-1}^1 d\mu (1-\mu^2) \frac{1}{5} R_{av}^5 (1+\beta Y_{20})^5$$

$$= \frac{3}{10} MR_{av}^2 \int_{-1}^1 d\mu (1-\mu^2) \left(1 + 5\beta \left(\frac{5}{16\pi}\right)^{1/2} (3\mu^2 - 1)\right)$$

$$= \frac{3}{10} MR_{av}^2 \left\{ \frac{4}{3} + 5\beta \left(\frac{5}{16\pi}\right)^{1/2} 2 \int_0^1 d\mu (1-\mu^2) (3\mu^2 - 1) \right\}$$

$$= \frac{2}{5} MR_{av}^2 \left\{ 1 + \frac{15}{2} \left(\frac{5}{16\pi}\right)^{1/2} \underbrace{\int_0^1 d\mu [4\mu^2 - 1 - 3\mu^4]}_{\frac{4}{3} - 1 - \frac{3}{5} = -\frac{4}{15}} \right\}$$

$$= \frac{2}{5} MR_{\text{core}}^2 \left\{ 1 - \left( \frac{5}{4\pi} \right)^{1/2} \beta \right\}$$

$$J_N = \frac{2}{5} MR_{\text{core}}^2 \{ 1 - 0.63\beta \}$$

with  $\beta = 0.29$  about a 20% factor

Rotational bands

$$E = \frac{1}{2I} I(I+1)$$

$$E(0^+) = 0 \quad \text{ground state}$$

$$E(2^+) = 3\hbar^2/I$$

$$E(4^+) = 10\hbar^2/I$$

$$E(6^+) = 42\hbar^2/I$$

$$E(8^+) = 72\hbar^2/I$$

Using  $I_{\text{rigid}}$  spacing is off by about 2-3

$$\text{Using } I_{\text{fluid}} \left( \frac{9}{8\pi} MR_{\text{core}}^2 \beta \right),$$

too high by about 2-3

$$J_{\text{fluid}} < J_N < J_{\text{rigid}}$$

(23)

# Moment of inertia of the ellipsoidal nucleus

moment of inertia  $I = \int d\vec{r} \rho(\vec{r}) r^2 \sin^2 \theta$

$$\rho(\vec{r}) = \text{constant} = M/V ; 0 \leq R \leq R_{\text{av}} (1 + \beta \gamma_{20})$$

$$V = \int d\vec{r} = \int_0^{2\pi} d\phi \int_0^{\pi} \sin \theta d\theta \int_0^{R_{\text{av}}(1+\beta\gamma_{20})} r^2 dr$$

$$= 2\pi \int_0^{\pi} \sin \theta d\theta \frac{R_{\text{av}}^3}{3} (1 + \beta \gamma_{20})^3$$

$$= \frac{2\pi}{3} R_{\text{av}}^3 \int_0^{\pi} \sin \theta d\theta \left( 1 + \beta \left( \frac{5}{16\pi} \right)^{1/2} (3\cos^2 \theta - 1) \right)^3$$

$$= \frac{2\pi}{3} R_{\text{av}}^3 \int_{-1}^1 d\mu \left( 1 + \beta \left( \frac{5}{16\pi} \right)^{1/2} (3\mu^2 - 1) \right)^3$$

$$= \frac{4\pi}{3} R_{\text{av}}^3 \int_0^1 d\mu \left\{ 1 + \beta \left( \frac{5}{16\pi} \right)^{1/2} (3\mu^2 - 1) \right\}^3$$

$$= \left( 1 + \frac{3}{4\pi} \beta^2 + \frac{\sqrt{5}}{20\pi^{3/2}} \beta^3 \right) \frac{4\pi}{3} R_{\text{av}}^3$$

$$J = \int d\vec{r} g(\vec{r}) r^2 \sin^2 \theta$$

$$= \frac{M}{V} \int_0^{2\pi} d\phi \int_0^\pi \sin \theta d\theta \sin^2 \theta \int_0^{R_{av}(1+\beta Y_{20}(\theta))} r^2 dr r^2$$

$$= \frac{M}{V} 2\pi \int_{-1}^1 d\mu (1-\mu^2) \frac{R_{av}^5}{5} (1+\beta Y_{20})^5$$

$$= \frac{M}{V} \frac{2\pi}{5} R_{av}^5 \int_{-1}^1 d\mu (1-\mu^2) \left(1 + \left(\frac{5}{16\pi}\right)^{1/2} \beta (3\mu^2 - 1)\right)^5$$

$$= \frac{M}{V} \frac{4\pi}{5} R_{av}^5 \int_0^1 d\mu (1-\mu^2) \left(1 + \left(\frac{5}{16\pi}\right)^{1/2} \beta (3\mu^2 - 1)\right)^5$$

$$J = \frac{2}{5} M R_{av}^2 \left\{ 1 - \frac{1}{2} \sqrt{\frac{5}{11}} \beta + \frac{71}{288\pi} \beta^2 + O(\beta^3) \dots \right\}$$

$$= \frac{2}{5} M R_{av}^2 \left\{ 1 - 0.63\beta + 0.91\beta^2 \dots \right\}$$

< 1%

$$= \frac{2}{5} M R_{av}^2 \left\{ 1 - 0.18 + 0.07 \dots < 1\% \right\}$$

$$J = \frac{M 4\pi R_{av}^5}{5} \int_0^1 d\mu (1-\mu^2) \left(1 + \left(\frac{5}{16\pi}\right)^{1/2} \beta (3\mu^2 - 1)\right)^5$$


---


$$\frac{4\pi R_{av}^3}{3} \int_0^1 d\mu (1 + \left(\frac{5}{16\pi}\right)^{1/2} \beta (3\mu^2 - 1))^3$$

## Rotational Bands

$$E = \frac{1}{2I} I(I+1) \hbar^2$$

$$E(0^+) = 0 \quad \text{ground state}$$

$$E(2^+) = 3\hbar^2/I$$

$$E(4^+) = 10\hbar^2/I$$

$$E(6^+) = 42\hbar^2/I$$

$$E(8^+) = 72\hbar^2/I$$

Using above  $I$ , relative spacing is fine  
but absolute is too low by  $2-3 \times$   
 $\Rightarrow I$  is too high

Using a fluid model

$$I_{\text{fluid}} = \frac{2}{5} M R^2 \beta$$

} no idea how  
this calculation  
is done

$\Delta E$  too high by about  $2-3$

(shell of deformation circling)  
a static core

$$\text{So } I_{\text{fluid}} < I_N < I_{\text{rigid}}$$

Can model this as a surface that drags  
part of the inner core with it