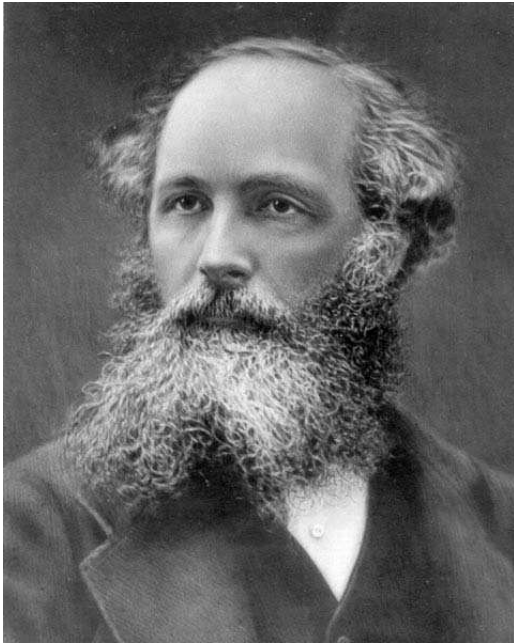


3. Particle-like properties of E&M radiation

3.1. Maxwell's equations...



Maxwell (1831–1879) studied the following equations^a:

Gauss's Law of Electricity: $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

Gauss's Law of Magnetism: $\vec{\nabla} \cdot \vec{B} = 0$

Faraday's Law: $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Ampère-Maxwell Law: $\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$

^aIn Chapter 1 these equations were given in cgs units. Here, the more conventional SI units are used.

Gauss's Law of Electricity describes how charges have associated with them, a divergent electric field $\vec{E}$.

Gauss's Law of Magnetism describes the divergence of magnetic fields, $\vec{B}$, and state that there is no magnetic "charge". All magnetic charges are made up of magnetic dipoles

Faraday's Law shows how a time varying magnetic field can induce an electric field.

The Ampère-Maxwell Law shows how a time varying electric field, as well as a current, $\vec{J}$, can induce a magnetic field.

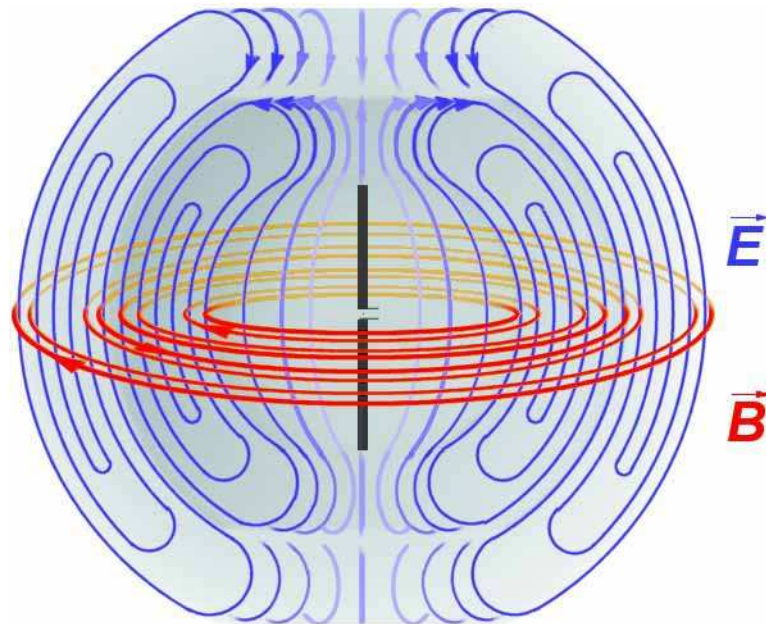
The classical wave equation ...

One of Maxwell's greatest discoveries was that the simplest form (without time varying currents) of the last two equations, namely:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad ; \quad \vec{\nabla} \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} , \quad (3.1)$$

where $c = 1/\sqrt{\mu_0 \epsilon_0}$.

(3.1) describes how an EM wave propagates. Time-varying electric fields induce time-varying magnetic fields that induce time-varying electric fields. and so on. The initiator can be a vertical antenna with vertical and descending swarms of electrons that give rise to axial magnetic fields that induce radial electric fields.



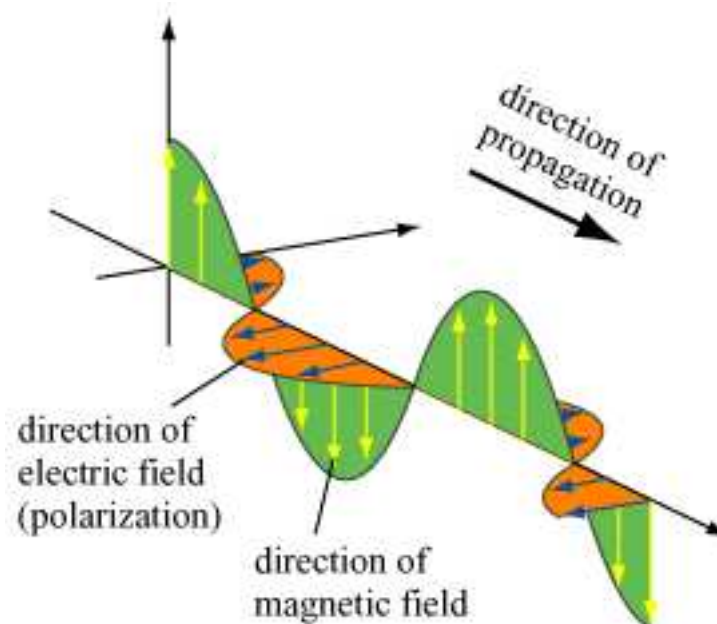
... The classical wave equation ...

The simplest solution for this is in terms of a “plane wave”, a single-frequency wave going down the $\hat{z}$ -axis, having infinite lateral extent:

$$\vec{E} = E_0 \hat{x} \sin(kz - \omega t) , \quad (3.2)$$

$$\vec{B} = B_0 \hat{y} \sin(kz - \omega t) , \quad (3.3)$$

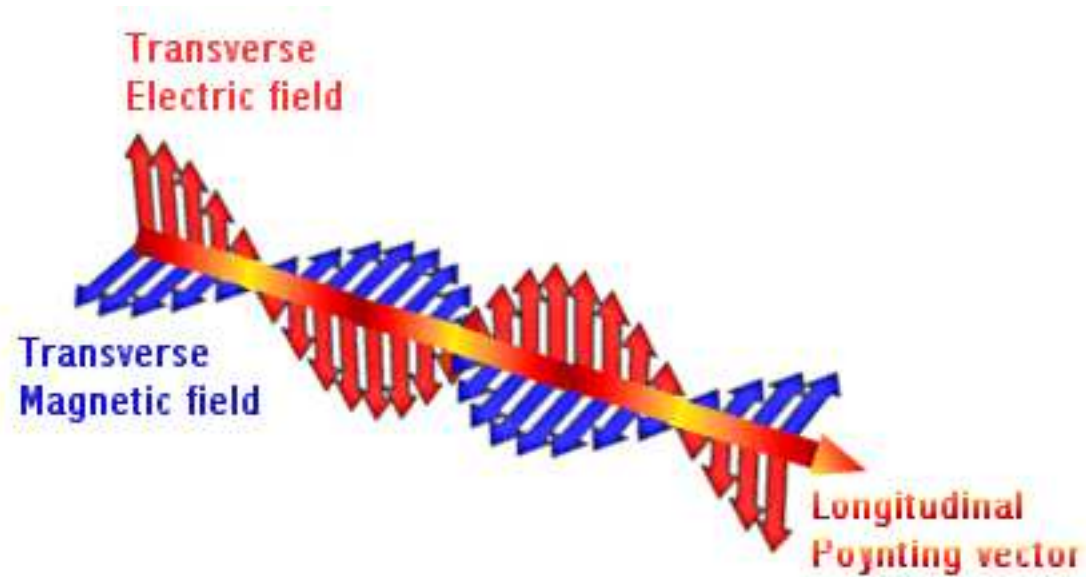
where $w = ck$, $E_0 = B_0 c$. $k = 2\pi/\lambda$ is called the “wave number” and λ is the “wavelength”. ω is the frequency.



Energy intensity...

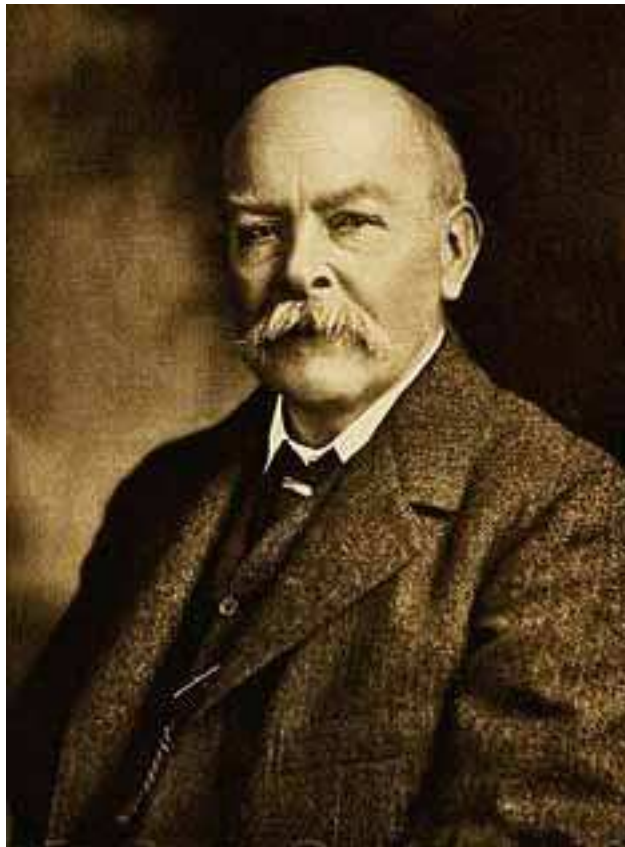
The energy intensity in W/m^2 is given by:

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \quad . \quad (3.4)$$



...Energy flow

The energy intensity vector is also known as the Poynting vector, named after John Henry Poynting, 1852–1914



Successes and Failures

The spectacular successes of Classical E&M theory

- Electricity and Magnetism were found to be part of the same theory
- Many behaviors of light were explained
- Circuits, electronics, heat and light generators
- Dielectrics, polarization, reflection, absorption
- Electromagnets and motors ...

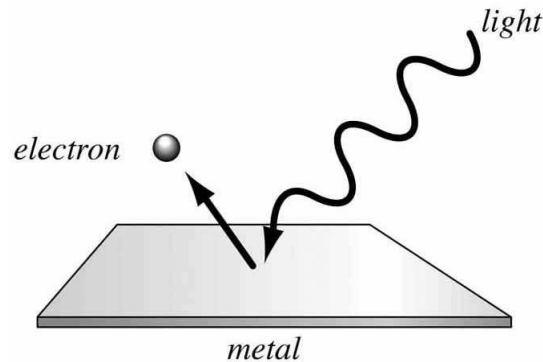
The spectacular failures of Classical E&M theory

- The Photoelectric effect
- The Compton effect
- Blackbody radiation and the ultraviolet catastrophe

Resolving these ushered in the era of Modern Physics.

Photoelectric effect ...

Consider the following experiment:



The post-Maxwell theory, pre-Modern Physics physicists theorized that:

1. Any wavelength of light would liberate electrons from a metal plate, given enough time for the electrons to absorb the energy.
2. If the energy were increased, the K_{e^-} will increase as well.

After all, this is what Maxwell's equations predict!

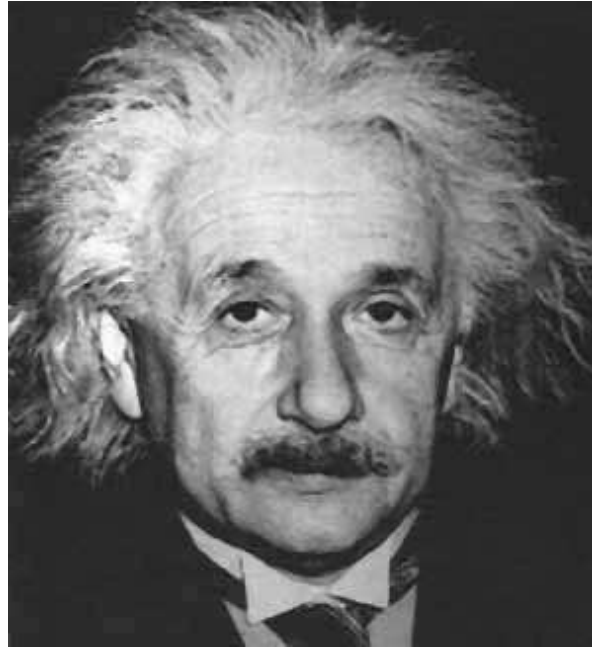
... Photoelectric effect ...



In 1887, Heinrich Rudolf Hertz (1857–1894) actually did the experiment finding:

1. If $\lambda > \lambda_0$ (λ is the wavelength of the light), no e^- 's are emitted, no matter how intense the light is.
 2. If $\lambda < \lambda_0$, e^- 's are emitted, instantly, no matter how weak the source is.
 3. If the intensity of the light is increased, the number of e^- 's increases, not their energy.
- These findings were completely contradictory to the accepted theory.

... Photoelectric effect ...



Albert Einstein (1879–1955) provided the answer (paraphrase): *EM waves are really made up of single particles called photons.*

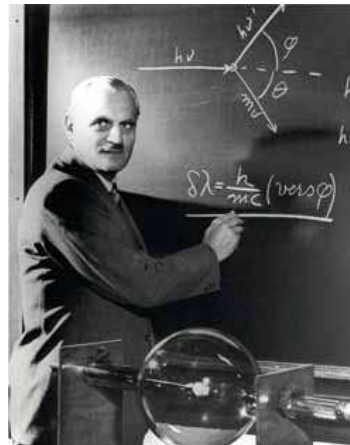
He received the Nobel Prize in 1921 for his work on the photoelectric effect.

We know now that the metals hold electrons in a potential called the “work function” that is from 2-6 eV, bridging the visible light region (2–3 eV) to the ultraviolet (3 eV – 1 keV), explaining why the photons had to cross a wavelength threshold, to liberate the electrons.

Thomson/Compton scattering ...



Sir Joseph John Thomson (1856–1940) *ca.* 1897, described the scattering of light from electrons according to Maxwell's equations.



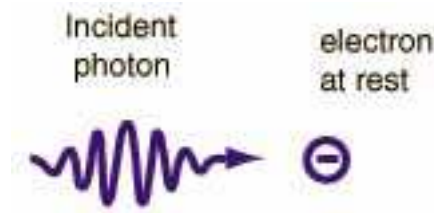
Arthur Holly Compton (1892–1962) *ca.* 1923 found the relativistic formula¹.

¹This is a great picture of Compton, with his famous equation. The function $\text{vers}()$ is an outdated shorthand for $1 - \cos()$.

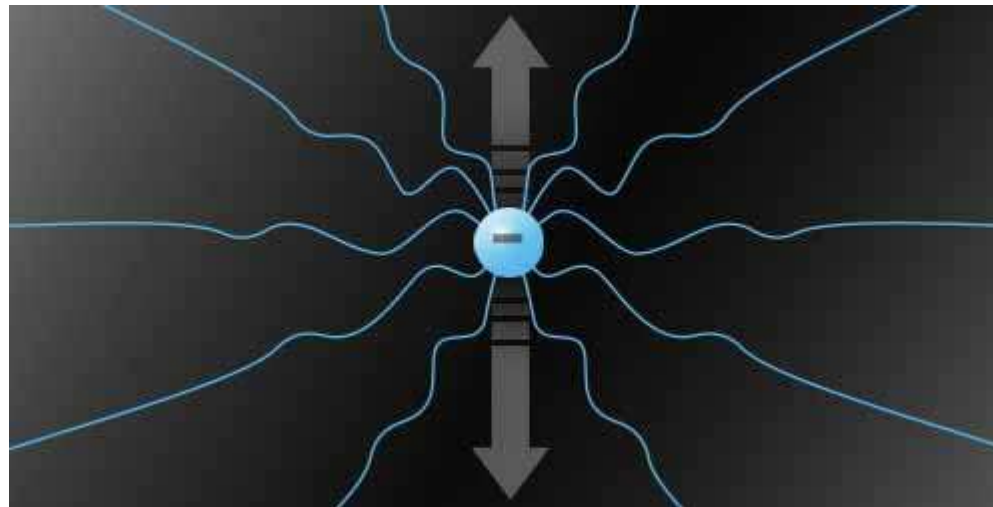
... Thomson/Compton scattering ...

The classical interpretation of photon-electron scattering (Thomson scattering) is:

1. A wave impinges upon a stationary electron,



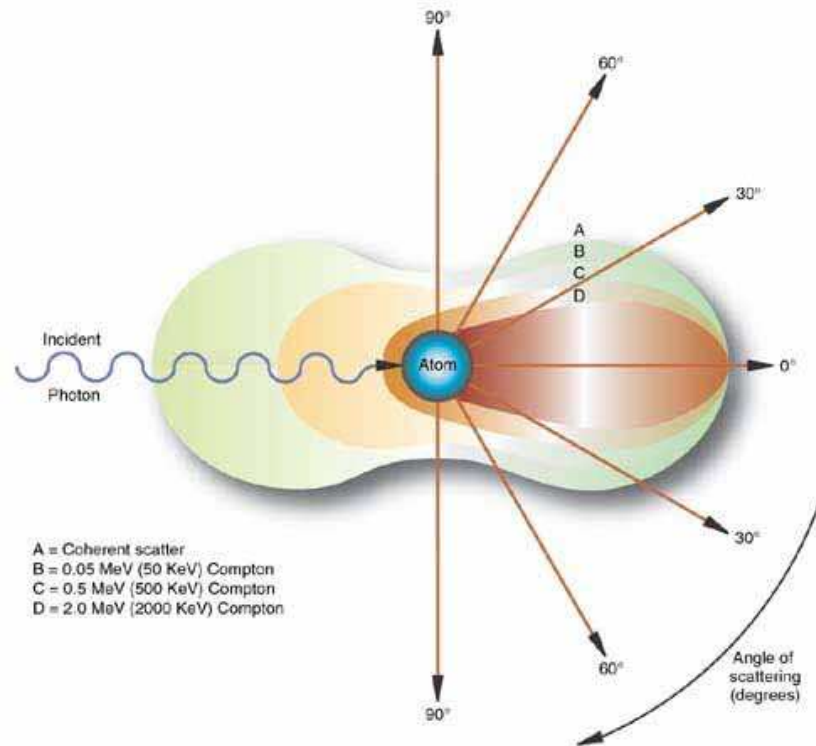
2. causing it to vibrate up and down (following the oscillating electric field lines),



3. radiating a symmetric pattern of radiation from the accelerated charge.
(The classical form of bremsstrahlung radiation.)

... Thomson/Compton scattering

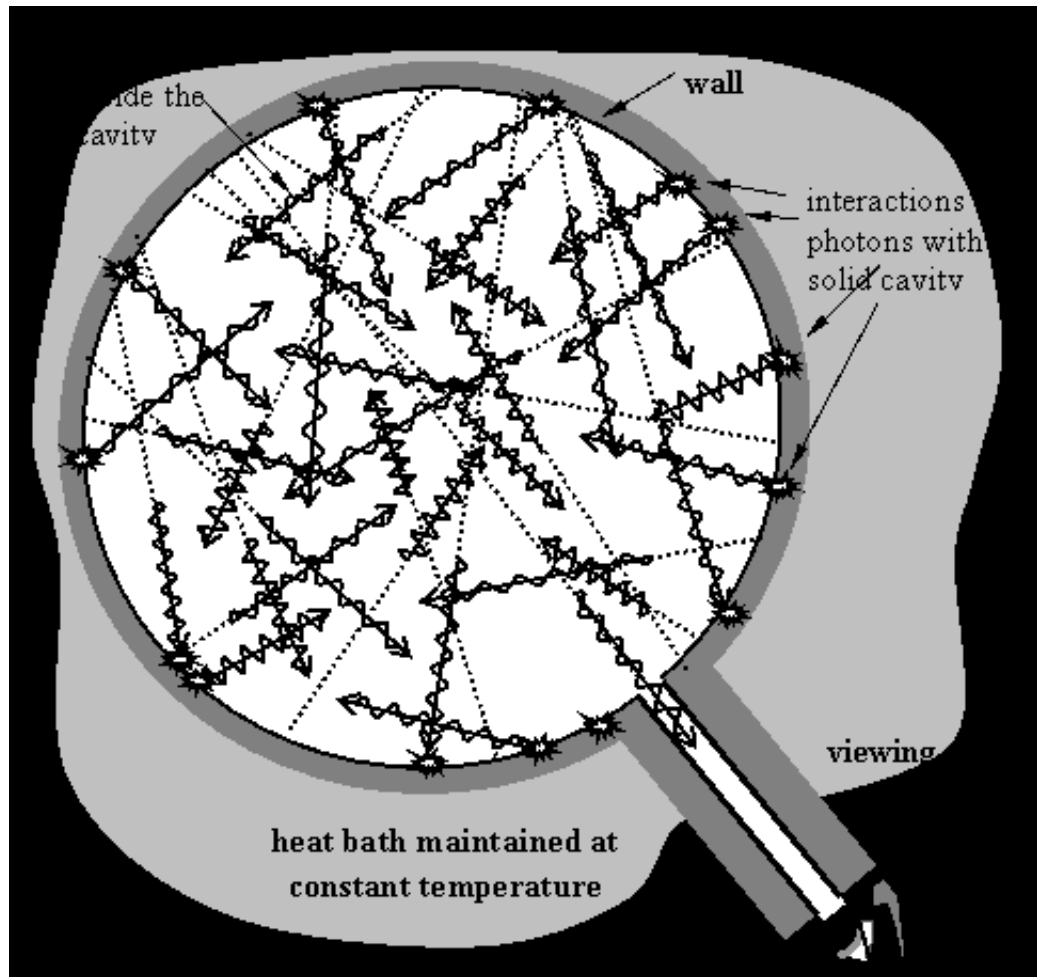
However, we know from the previous chapter that photons carry momentum, the so-called Compton effect, or Compton scattering. The figure below shows this.



The classical limit is called "Coherent scatter" in the figure, because no energy is lost to electron recoil in the classical limit.

The Compton angular distribution is easily measured, and the relativistic form has been verified.

Blackbody radiation and the ultraviolet catastrophe ...



If one heats up an object, as depicted above, it emits “blackbody radiation”, with a characteristic spectrum that shifts to lower wavelengths (higher photon energy) with higher temperature.

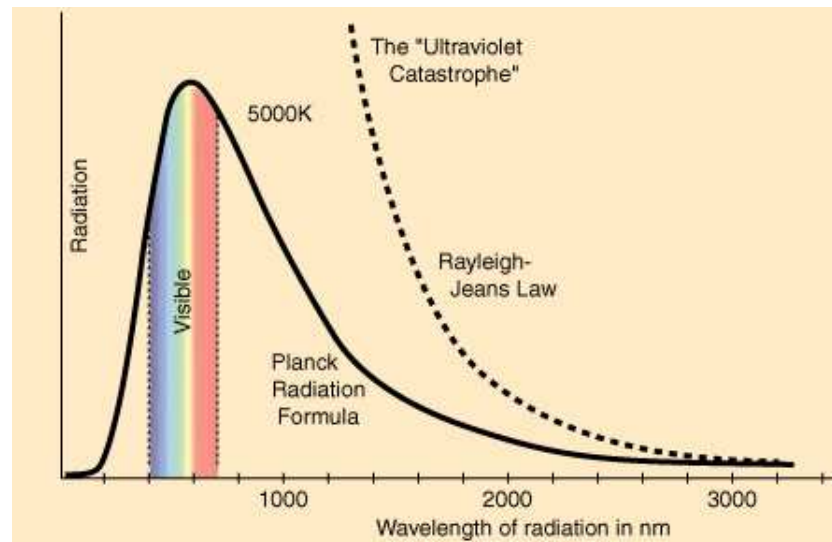
... Blackbody radiation and the ultraviolet catastrophe ...

According to Maxwell's equations, the “radiancy” (energy density that passes through a unit volume per unit time) is given by:

$$R(\lambda) = \frac{2\pi ck_B T}{\lambda^4}, \quad (3.5)$$

where k_B is Boltzmann's constant ($8.6173324(78) \times 10^{-5} \text{ eV T}^{-1}$), and T is the temperature in $^\circ\text{K}^2$.

Unfortunately, if you integrate (3.5) over the volume of your oven, and over all wavelengths that are produced, to find out how much energy is there, you would get ∞ ! We know this can not be true. It was termed the “ultraviolet catastrophe”, shown below.



²This is called Rayleigh-Jeans theory. I suppose, for consistency, I should have photos of Rayleigh, Jeans and Boltzmann!. Perhaps I will do that at a later time.

... Blackbody radiation and the ultraviolet catastrophe ...



The work of Max Karl Ernst Ludwig Planck (1858–1947), *ca.* 1900 resolved this difficulty, and the Quantum Age truly began. Planck received the Nobel Prize for this in 1918.

Planck's result revised the radiancy formula:

$$R(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right) - 1}, \quad (3.6)$$

where $h = 4.135667516(91) \times 10^{-15}$ eV s is Planck's constant, a fundamentally new quantity that describes how everything is quantized.

... Blackbody radiation and the ultraviolet catastrophe ...

Implications

- (3.6) shows that at $\lambda \rightarrow 0$, $R(\lambda) \rightarrow 0$. The ultraviolet catastrophe is averted!
- The classical limit is obtained by taking the limit $h \rightarrow 0$ in (3.6).