

2. The Theory of Special Relativity

2.5. Relativistic Kinematics

In this section we concern ourselves, primarily, with two-body scattering of relativistic particles, including photons. We start with some review of kinematic variables.

Symbol	= Expression	Interpretation
v		speed of a particle with mass
c		speed of light, speed of massless particles
β , or β_v	v/c	speed (in units of c) of a particle with mass $0 \leq \beta < 1$
γ or γ_v	$(1 - \beta^2)^{-1/2}$	“energy factor”, “dilation factor”, “contraction factor” γ is often used as a symbol to represent a photon
	mc^2	rest mass energy of a particle with mass
E	$mc^2\gamma$	total energy of a particle with mass (rest + kinetic energy)
E_γ		total (or kinetic) energy of a photon (or massless particle)
K	$mc^2(\gamma - 1)$	kinetic energy of a particle with mass
$\vec{p}$	$mc\vec{\beta}\gamma, m\vec{v}\gamma$	momentum of a particle with mass
$\vec{p}_\gamma$	$\hat{n}E_\gamma/c$	magnitude of momentum of a photon, or particle without mass $\hat{n}$ is a unit vector in the direction of motion
$(mc^2)^2$	$E^2 - (pc)^2$	fundamental relation linking m , E , and $ \vec{p} ^2$
1	$\gamma^2(1 - \beta^2)$	useful property of γ and β (really worth memorizing!)

Non-relativistic limit ...

Relativistic calculations are a little more complicated than classical ones.

You can verify your expressions by taking the the non-relativistic limit.

This is done by making a Taylor expansion (See Chapter 18.) in β , and keep the leading order expressions that express the non-relativistic limit.

The next-order term is the relativistic correction, which gives you an estimate of the error you would make if you used the classical limit only.

Factors of β that remain should be replaced by cv , and the final result should resemble:

$$\lim_{\beta \rightarrow 0} (\text{Relativistic expression}) = (\text{Non-relativistic expression}) + \mathcal{O}(1/c^n) , \quad (2.1)$$

where $n \geq 1$. Finally the non-relativistic limit is obtained by setting the $O(1/c^n)$ expressions to zero. Note that some expressions are intrinsically relativistic and not reducible to non-relativistic limits. For example, rest mass energy, and photon kinematic variables are concepts that you may have encountered, but they have no “classical” meaning.

Non-relativistic limit ...

For example, it would not make sense to ask, "What is the classical limit of the rest mass energy?". There was no answer in classical physics. However, classical and relativistic makes use of "kinetic energy" which is the energy associated with the motion of an object. If you toss a log into a pond, you can be sure that the log will eventually come to rest, and its kinetic energy was converted into heat! Another example ... the burning of meteors in the atmosphere!

Let's make the kinaction (pun intended!) between the relativistic and classical forms. In relativistic physics, the kinetic energy of a particle of mass m , is:

$$K = mc^2(\gamma - 1) .$$

To obtain the classical limit, we take the limit $\beta = |\vec{v}|/c$ in a Taylor expansion (this is a good time to review that topic):

In the limit that $\beta \rightarrow 0$ K , you can show:

$$\lim_{\beta \rightarrow 0} K = \lim_{\beta \rightarrow 0} mc^2(\gamma - 1) = \frac{1}{2}mv^2 \left(1 + \frac{3}{4}\beta^2 + \mathcal{O}(\beta^4) \right) , \quad (2.2)$$

... Non-relativistic limit

Imagine an object going at speed $\beta = 0.1$, about 30,000 km/s¹. The “relativistic fraction” only amounts to about 3/4%.

The relativistic correction for momentum can be found similarly to be:

$$\lim_{\beta \rightarrow 0} m\vec{v}\gamma = m\vec{v} \left(1 + \frac{1}{2}\beta^2 + \mathcal{O}(\beta^4) \right) . \quad (2.3)$$

¹The fastest speed measured for a meteor that entered the earth's atmosphere, was going 28.6 km/s (Sutter's Hill meteor, California), that's about $\beta = 0.0001$ only! The Chelyabinsk meteor that exploded over Russia (it was about the size of a school bus and weighed about 10 tons) on February 15, 2013, was only (!) going 19.16 ± 0.15 km/s. It exploded 12–15 miles in altitude, with an explosive force of about 500 kilotons of TNT.

Why are Taylor series important?

Let's work out an example:

Q: If an object has total energy $E/mc^2 = \gamma = 1.00 \times 10^{20}$, how close is its speed to c ?
i.e. Evaluate $(c - v)/c = 1 - \beta$.

A: You can show that $1 - \beta = 1 - \sqrt{1 - \gamma^{-2}}$. A calculator will give you a 0 (!!!!)
With a Taylor expansion, you can show that $1 - \beta = 5.00 \times 10^{-41} + \mathcal{O}(10^{-80})$

Relativistic Collision Kinematics

We now repeat the discussion of the previous Chapter but include the effect of relativistic speeds.

Consider the collision of two moving particles with masses m_1 and m_2 , producing particles m_a and m_b following the collision. We conserve *total* energy and momentum, to obtain the following equations:

CoE $\Rightarrow$

$$m_1 c^2 \gamma_1 + m_2 c^2 \gamma_2 = m_a c^2 \gamma_a + m_b c^2 \gamma_b , \quad (2.4)$$

We note that Q is the zero-speed limit of (2.4) and is included automatically in the subsequent analysis.

Co $\vec{M}$ $\Rightarrow$

$$m_1 c \vec{\beta}_1 \gamma_1 + m_2 c \vec{\beta}_2 \gamma_2 = m_a c \vec{\beta}_a \gamma_a + m_b c \vec{\beta}_b \gamma_b . \quad (2.5)$$

Solution Strategies ...

How we manipulate (2.4) and (2.5) depends on what information we know, and what information we wish to extract. We shall discuss the most common situation now, and leave some of the special cases to the examples and problems.

The most common situation involves the scattering of a known projectile from a known target, where initial masses and velocities are known, to a set of final particles whose masses are known, but only the lighter product particle leaves the collision area. (For example, a proton scattering from a stationary nucleus, with a transformed nucleus and a neutron in the final state.) Since the heavier product particle stays in the collision area, it is unobserved, hence its velocity is not measurable, and we strive to eliminate it. We proceed as follows.

Reorganize (2.4) and (2.5) as follows, to put the kinematics of the “b” particle on the right hand side (RHS) of the equations:

From the CoE equation:

$$m_1 c^2 \gamma_1 + m_2 c^2 \gamma_2 - m_a c^2 \gamma_a = m_b c^2 \gamma_b , \quad (2.6)$$

and the Co $\vec{M}$ equation:

$$m_1 c \vec{\beta}_1 \gamma_1 + m_2 c \vec{\beta}_2 \gamma_2 - m_a c \vec{\beta}_a \gamma_a = m_b c \vec{\beta}_b \gamma_b . \quad (2.7)$$

... Solution Strategies ...

Dividing the square of (2.6) by c^4 and subtracting the square of (2.7) divided by c^2 gives:
 $(2.6)^2/c^4 - (2.7)^2/c^2 \Rightarrow$

$$(m_1\gamma_1 + m_2\gamma_2 - m_a\gamma_a)^2 - (m_1\vec{\beta}_1\gamma_1 + m_2\vec{\beta}_2\gamma_2 - m_a\vec{\beta}_a\gamma_a)^2 = m_b^2\gamma_b^2(1 - \beta_b^2) . \quad (2.8)$$

The motivation for this arithmetical manipulation is now evident: no factors of c appear, and most importantly, we may exploit the $\beta\gamma$ relation, $\gamma^2(1 - \beta^2)$ to great effect. Doing so results in:

$$m_1^2 + m_2^2 - m_a^2 - m_b^2 + 2m_1m_2\gamma_1\gamma_2(1 - \vec{\beta}_1 \cdot \vec{\beta}_2) = 2m_1m_a\gamma_1\gamma_a(1 - \vec{\beta}_1 \cdot \vec{\beta}_a) + 2m_2m_a\gamma_2\gamma_a(1 - \vec{\beta}_2 \cdot \vec{\beta}_a) . \quad (2.9)$$

We see that we have isolated the only unknown quantity, $\vec{\beta}_a$, and by inference γ_a on the RHS of (2.9). We may further reduce this equation by noting that the mass term on the LHS may be rewritten as follows:

$$m_1^2 + m_2^2 - m_a^2 - m_b^2 = (m_1 + m_2 - m_a - m_b)(m_1 + m_2 + m_a + m_b) = (\Delta M)M , \quad (2.10)$$

where $M = M_i + M_f = m_1 + m_2 + m_a + m_b$ is the sum of the masses of the initial and final particles, while $\Delta M = M_i - M_f = m_1 + m_2 - m_a - m_b$ is the difference of the sum of the initial masses and the sum of the final masses of the particles participating in the reaction.

... Solution Strategies ...

We also note that ΔMc^2 is the reaction Q -value discussed previously. Note how it appears naturally in the analysis, while it has to be “tacked on” in an *ad hoc* fashion in the non-relativistic analysis.

So, finally we write:

$$(\Delta M)M + 2m_1m_2\gamma_1\gamma_2(1 - \vec{\beta}_1 \cdot \vec{\beta}_2) = 2m_1m_a\gamma_1\gamma_a(1 - \vec{\beta}_1 \cdot \vec{\beta}_a) + 2m_2m_a\gamma_2\gamma_a(1 - \vec{\beta}_2 \cdot \vec{\beta}_a). \quad (2.11)$$

Having derived a relativistic result, we should check that it gives the correct non-relativistic limit. To do this, we note that we can rewrite

(2.8) as:

$$\vec{P}^2 - 2m_b(K + Q) = (Q + K)^2/c^2, \quad (2.12)$$

where

$$\vec{P} \equiv m_1c\vec{\beta}_1\gamma_1 + m_2c\vec{\beta}_2\gamma_2 - m_ac\vec{\beta}_a\gamma_a,$$

and

$$K \equiv m_1c^2(\gamma_1 - 1) + m_2c^2(\gamma_2 - 1) - m_ac^2(\gamma_a - 1).$$

(2.12) is fully relativistic.

... Solution Strategies

Obtaining the non-relativistic form is tantamount to replacing $\vec{P}$ and K with their non-relativistic counterparts (given in Chapter 1) and setting the $1/c^2$ on the RHS of (2.12) to zero.

This agrees with the non-relativistic form given in Chapter 1, and we have verified the non-relativistic limit of our relativistic expression. It is not absolutely foolproof, however, verifying non-relativistic limits is a very important verification tool.

Zero-Momentum Frame ...

We can also perform any calculation in the zero-momentum frame.

In these set of notes, we don't exploit the zero-momentum frame extensively, since the laboratory frame makes more sense for nuclear engineering and radiological applications (fixed targets, β - and γ -decay).

However, high-energy physics exploit the zero-momentum frame extensively, since particle-antiparticle colliders operate in the zero-momentum frame.

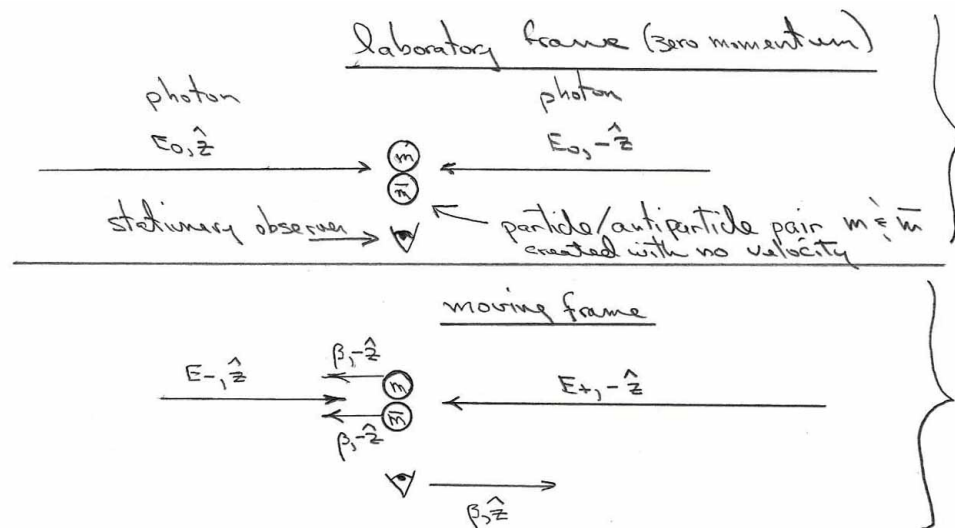
We shall exploit it, however, for two important illustrations:

... Zero-Momentum Frame ...

Example: Particle/antiparticle creation with mass

Consider a collision of two photons, going in exact opposite directions, $\pm\hat{z}$, each with energy E_0 . E_0 is arranged so that after the collision, a particle and antiparticle, each with mass m , is at rest. Thus, by CoE, $E_0 = mc^2$.

Now consider that a different observer, moving along the direction of one of the photons, with velocity $+\beta\hat{z}$, observes the event. In that frame of motion, the particle/antiparticle pair is moving in the direction opposite to him.



... Zero-Momentum Frame ...

In the observer's frame of motion, the expressions for CoE and $\text{Co}\vec{M}$ are:

$$2mc^2\gamma = E_+ + E_- \quad (2.13)$$

$$-2mc\beta\gamma = (-E_+ + E_-)/c, \quad (2.14)$$

where β is the observer's velocity with respect to the zero-momentum frame, E_+ is the higher energy photon he observes, with E_- is the lesser energy photon in his frame. By manipulating the equations in the now familiar way, we may relate the energy of the photon in the moving frame, relative to the rest frame.

The result is:

$$E_{\pm} = E_0 \sqrt{\frac{1 \pm \beta}{1 \mp \beta}}, \quad (2.15)$$

where we consistently use only the upper or lower signs in the expressions involving $\pm$ or $\mp$.

Students who have read the optional section on special relativity, will recognize this as the “Doppler effect”, that characterizes the change in wavelength of photons because of the motion of the photon source, or the observer, or both!

The Doppler effect has enjoyed an important history in astrophysics, and a checkered one in traffic enforcement (radar).

Other relativistic kinematic examples ...

Example: Particle/antiparticle decay into photons

Here we consider a particle with mass m , β_0 , γ_0 on a collision course with its antiparticle, moving in the exact opposite direction. They annihilate, producing two photons, each with energy E_0 , moving in exact opposite directions, along the original direction of motion.

An observer, moving with parameters β and γ , along the original direction of motion, observes the same annihilation, and his CoE and Co $\vec{M}$ equations take the forms:

$$mc^2\gamma_+ + mc^2\gamma_- = E_+ + E_- \quad (2.16)$$

$$-mc\beta_+\gamma_+ + mc\beta_-\gamma_- = (-E_+ + E_-)/c . \quad (2.17)$$

Here, the “+” refers to the more energetic particle and photon in the frame of the observer. The arithmetic is a little more involved than in the previous example, but, after some work, we can conclude that:

$$\gamma_{\pm} = \gamma_0\gamma(1 \pm \beta\beta_0) , \quad (2.18)$$

which expresses a “Doppler shift”, but for particles with mass.

... Zero-momentum frame ...

Applying (2.18), imagine that the observer is traveling at exactly β_0 , putting one of the charged particles in the rest frame.

The higher energy electron will have a “ γ -shift” of approximately $2\gamma_0^2$.

For example, the Stanford Linear Accelerator produces electrons and positrons with energies of about 50 GeV, a γ -shift of about 10^5 . The collision of these particles in the zero-momentum frame, is equivalent to a fixed target γ -shift of 2×10^5 . It is no wonder that particle-antiparticle colliders are such an important research tool.

Inelastic collisions ...

Sticky collisions/exploding masses

Finally, we end this section with a discussion on inelastic collisions.

In the last chapter, we inferred the Q -value of a sticky collision. Let's reformulate the problem in a relativistic framework.

Imagine that a particle of mass m_0 , with speed v_0 , strikes an identical particle at rest, and they fuse.

You can not balance the CoE and $\text{Co}\vec{M}$ equations if the masses are allowed to remain unchanged.

One finds, in this case that the fused mass is

$$m = 2m_0 \sqrt{\frac{1 + \gamma_0}{2}}.$$

The increase in mass is due to the increase in internal energy of the mass m .

Inelastic collisions ...

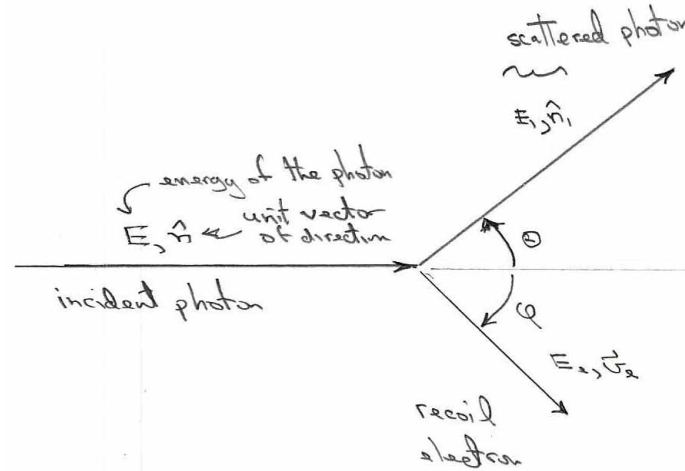
Similarly, if a mass m explodes into two equal masses, m_0 , you may show that

$$m = 2m_0\gamma_0 .$$

In other words, internal energy is converted into kinetic energy of the resultant particles.

Compton scattering ...

Compton scattering is the name given to the deflection of a photon caused by a “free” (i.e. at rest, unbound) electron, producing a lower-energy scattered photon, and a recoil electron:



CoE and Co $\vec{M} \Rightarrow$:

$$E + mc^2 = E_1 + mc^2\gamma \quad (2.19)$$

$$\frac{E}{c}\hat{n} = \frac{E_1}{c}\hat{n}_1 + m\vec{v}_e\gamma, \quad (2.20)$$

... Compton scattering ...

$(2.19)/mc^2$, $(2.20)/mc$, and some reorganization results in:

$$\alpha + 1 - \alpha_1 = \gamma \quad (2.21)$$

$$\alpha \hat{n} - \alpha_1 \hat{n}_1 = \vec{\beta} \gamma, \quad (2.22)$$

where the α 's are photon energies divided by the electron rest mass energy. Squaring both equations and subtracting them eliminated the electron from the equations, by exploiting the relationship $\gamma^2(1 - \beta^2) = 1$. With a little arithmetic, we can derive one of the most famous equations in all of physics:

$$\frac{1}{\alpha_1} - \frac{1}{\alpha} = 1 - \cos \theta \quad ; \quad \alpha_1 = \frac{\alpha}{1 + \alpha(1 - \cos \theta)} \quad (2.23)$$

that relates the scattered photon energy to its scattering angle.

... Compton scattering

Some interesting consequences:

1. Assuming E and $\hat{n}$ are known, we have a relationship that expresses E in terms of its scattering angle.
2. Indeed, any two unknowns can be expressed in terms of each other.
3. If $\cos(\theta) = 1$, *i.e.* $\theta = 0$, $E_1 = E$. This can be interpreted as a “miss”, or the limiting case as $\theta \rightarrow 0$.
4. If $\cos(\theta) = -1$, *i.e.* $\theta = \pi, 180^\circ$, the photon backscatters. In this case:

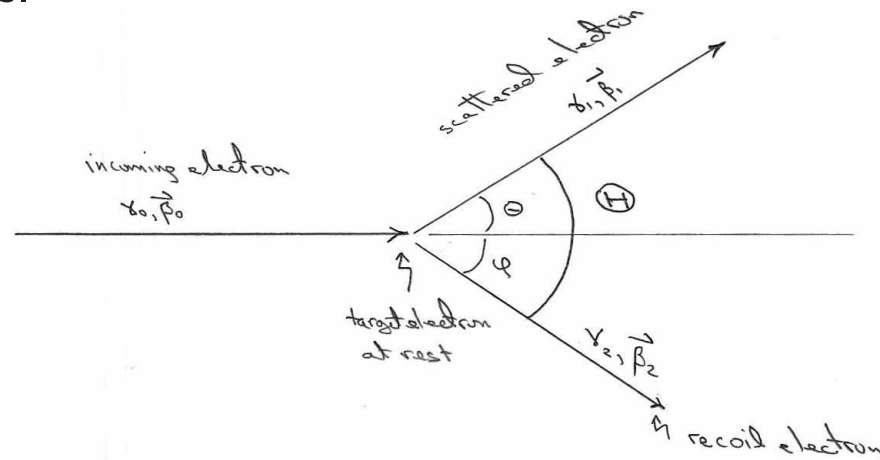
$$\frac{1}{\alpha_1} = 2 + \frac{1}{\alpha} \quad ; \quad \alpha_1 = \frac{\alpha}{1 + 2\alpha}$$

The implications of this are interesting! If $\alpha \rightarrow 0$, then $\alpha_1 \rightarrow 0$.

If $\alpha \rightarrow \infty$, then $\alpha_1 \rightarrow 1/2$.

Møller scattering ...

Møller scattering is the scattering of a projectile electron from another electron that is at rest. The applications of this are in nuclear reactors (much of the damage in fuel bundles is due to recoil electrons, as well as radiological sciences, since electrons cause ionization and radiological changes.



After the collision the higher energy is called the “scattered” electron, and the lower-energy electron is the “recoil” electron.

In order to draw a connection to our non-relativistic calculation, we will find the opening angle of the resultant particles, when a relativistic particle of mass m , collides with an equal mass, at rest, and show explicitly the transition to the well-known non-relativistic limit.

... Møller scattering ...

- Set up the CoE and Co $\vec{M}$ equations assuming “2” is at rest:

$$mc^2\gamma_0 + mc^2 = mc^2\gamma_1 + mc^2\gamma_2 \quad (2.24)$$

$$mc\vec{\beta}_0\gamma_0 = mc\vec{\beta}_1\gamma_1 + mc\vec{\beta}_2\gamma_2 \quad (2.25)$$

- We require the angle between the resultant trajectories. The cosine of this angle is obtained by $\vec{\beta}_1 \cdot \vec{\beta}_2$. To isolate this: $(2.24)^2/(mc^2)^2 - (2.25)^2/(mc)^2 \Rightarrow$

$$\gamma_1\gamma_2\vec{\beta}_1 \cdot \vec{\beta}_2 = \gamma_1\gamma_2 - \gamma_0 .$$

If we let Θ represent the opening angle of the outgoing particles, we may manipulate the above (Show this!) equation to be:

$$\cos \Theta = \sqrt{\frac{K_1 K_2}{(K_1 + 2mc^2)(K_2 + 2mc^2)}} , \quad (2.26)$$

explicitly showing the dependence on the outgoing kinetic energies.

... Møller scattering ...

Taking $c \rightarrow \infty$ yields the expected result, that the opening angle is $\pi/2$, in a non-relativistic analysis. This is tantamount to saying that $K_1 \ll mc^2$ and $K_2 \ll mc^2$.

However, (2.26) contains even more information.

If either outgoing particle is non-relativistic, that is, $K_1 \ll mc^2$ or $K_2 \ll mc^2$, the opening angle tends to $\pi/2$.

If either outgoing particle is at rest, the opening angle is $\pi/2$, exactly as in the non-relativistic case, and also true for the relativistic case.

These results arise from the conservation of energy and momentum in both non-relativistic and relativistic formalisms.

In the case that both outgoing particles are relativistic, (2.26) demonstrates that the opening angle is less than $\pi/2$.

... Møller scattering ...

Since $K_1 = K_0 - K_2$, it also follows that there must be a particular sharing of the initial kinetic energy, K_0 , with that of the outgoing particles, that minimizes the opening angle. With $K_1 = K_0/2 + \Delta$ & $K_2 = K_0/2 - \Delta$ we may write (2.26) as:

$$\cos \Theta = \sqrt{\frac{(K_0 + 2\Delta)(K_0 - 2\Delta)}{(K_0 + 2\Delta + 4mc^2)(K_0 - 2\Delta + 4mc^2)}} . \quad (2.27)$$

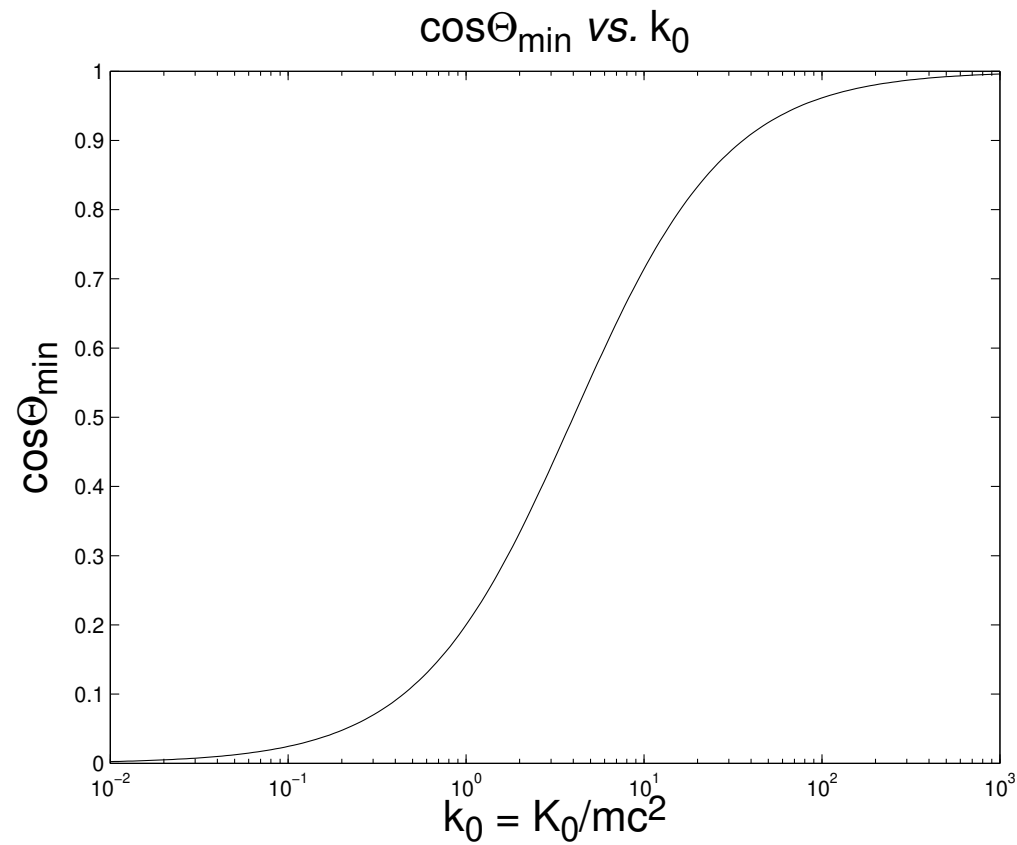
From this (2.26) symmetry between K_1 & K_2 under the interchange $1 \leftrightarrow 2$, it follows that the point $\Delta = 0$ is an extremum (it turns out to be a maximum), and the $\cos \theta$ is symmetric about that point.

You can prove this mathematically².

See the plot on the next page, for a logarithmic spacing of K_1/K_0 between 10^{-2} and 10^3 .

²Here's the proof: Consider a function, $f(x)$, that is symmetric about the origin. (Any different point of symmetry may be used, but it can be translated back to the origin.) It follows that the function's first derivative is antisymmetric, and must pass through the origin. Hence, the origin is the location of an extremum of $f(x)$. Q.E.D.

... Møller scattering ...



... Møller scattering ...

Therefore at the midpoint, $K_1 = K_2 = K_0/2$, and the minimum opening angle can be shown to be given by:

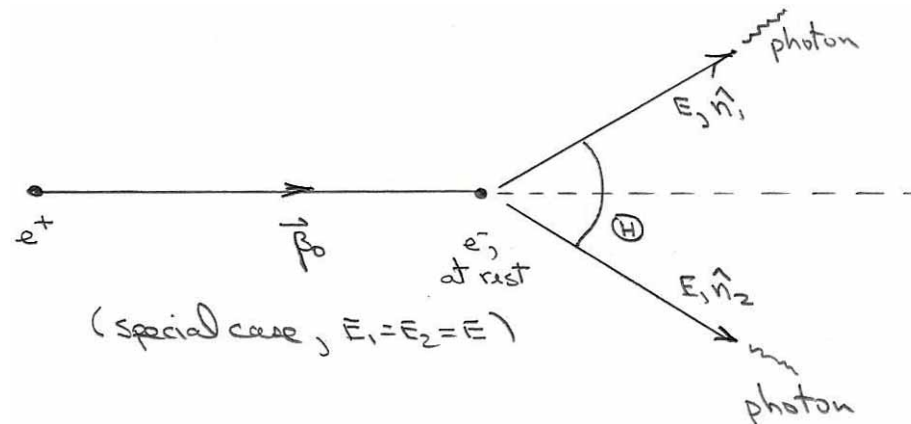
$$\cos \alpha_{\min} = \frac{K_0}{K_0 + 4mc^2} . \quad (2.28)$$

We note from (2.28) that the expected non-relativistic limit is obtained again. However, as the incoming kinetic energy is extended upwards into the relativistic range, the energy is increasingly carried into the forward direction. This is the principle upon which particle ray-guns operate. It is also responsible for spectacular accidents when charged high energy particle beams are mistakenly steered into beam pipes and magnets.

Positron annihilation ...

The positron is the antiparticle of the electron. If they encounter each other, they annihilate, usually into photons, converting all of their energy, (total and kinetic) into the energy of the photons. What do the conservation laws tell us about this case?

Let's study the specific case where the photons share the energy equally, but are going in different directions. Let see what happens to the angle between the directions of the photons.



... Positron annihilation ...

CoE and Co $\vec{M} \Rightarrow$:

$$mc^2\gamma_0 + mc^2 = 2E \quad (2.29)$$

$$mc\vec{\beta}_0\gamma_0 = (E/c)(\hat{n}_1 + \hat{n}_2) , \quad (2.30)$$

or, in our unitless notation

$$\gamma_0 + 1 = 2\alpha \quad (2.31)$$

$$\vec{\beta}_0\gamma_0 = \alpha(\hat{n}_1 + \hat{n}_2) . \quad (2.32)$$

Squaring and subtracting gives us (fill in the blanks):

$$\cos \Theta = \frac{\gamma_0 - 3}{\gamma_0 + 1} \quad (2.33)$$

... Positron annihilation ...

Does it make sense?

When γ_0 , the positron is at rest, along with the electron. In this case, the photons are colinear, with an opening angle of π , headed in opposite directions, each with energy equal to the rest mass energy of the electron, about 0.511 MeV.

Yes! It does make sense.

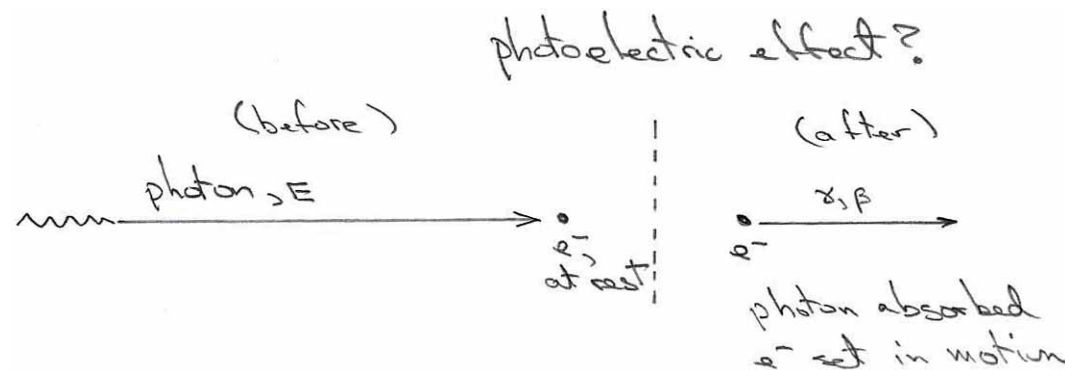
When $\gamma_0 \rightarrow \infty$, the opening angle gets smaller and smaller. This is a general feature of relativistic particles, and we have seen this before in connection with relativistic billiard balls.

The power of conservation laws ...

As we have seen, the conservation laws are important tools to understanding what is happening in nature.

They have another important property, they absolutely rule out what does not happen! To illustrate this, let's dream up of few interactions, and see if they are permitted.

Can the following happen?



... The power of conservation laws ...

Let's see! CoE and Co $\vec{M} \Rightarrow$:

$$E + mc^2 = mc^2\gamma \quad (2.34)$$

$$(E/c) = mc\beta\gamma, \quad (2.35)$$

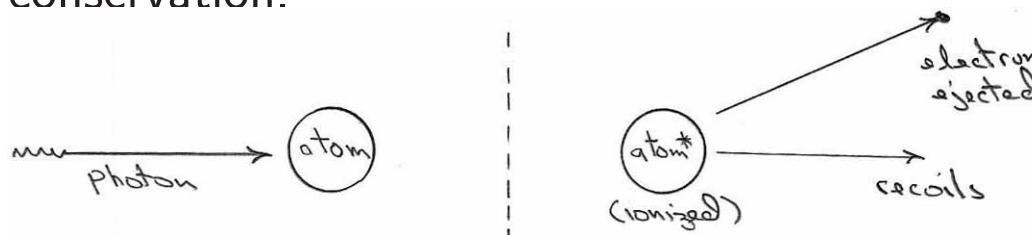
or, in our unitless notation

$$\alpha + 1 = \gamma \quad (2.36)$$

$$\alpha = \beta\gamma_0. \quad (2.37)$$

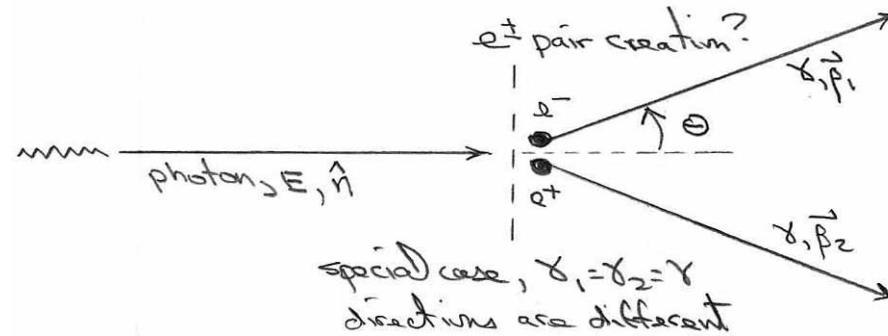
Solving this for α yields the solution $\alpha = 0$. No photon energy! This interaction can not happen!

Yet, we do know of the photoelectric effect, whereby a photon liberates an electron from an atomic orbital. The extra object in the vicinity, the rest of the atom, must be there to allow for CoE/Co $\vec{M}$ conservation.



... The power of conservation laws ...

Let's try one more example. Can this happen? It resembles, somewhat, $e^\pm$ pair creation.



CoE and Co $\vec{M} \Rightarrow$ (for equal $e^\pm$ energies):

$$E = 2mc^2\gamma \quad (2.38)$$

$$(E/c)\hat{n} = mc(\vec{\beta}_1 + \vec{\beta}_2)\gamma, \quad (2.39)$$

or, in our unitless notation

$$\alpha - \gamma = \gamma \quad (2.40)$$

$$\alpha\hat{n} - \vec{\beta}_1\gamma = \vec{\beta}_2\gamma. \quad (2.41)$$

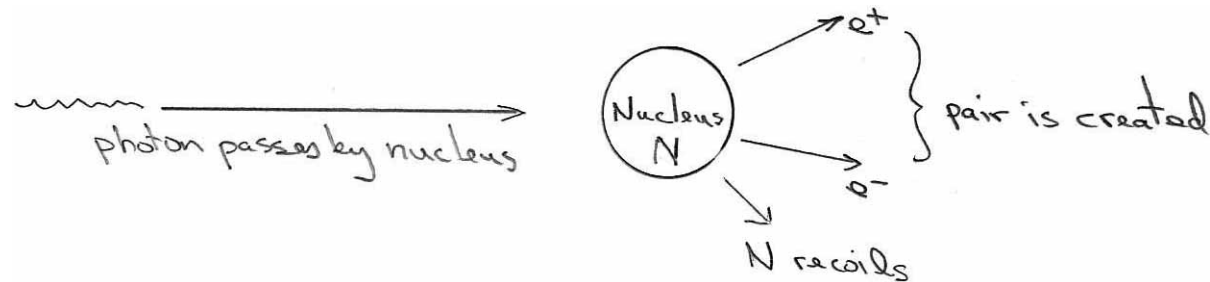
Solving this gives:

$$\cos\theta = \frac{1}{\beta_1}, \quad (2.42)$$

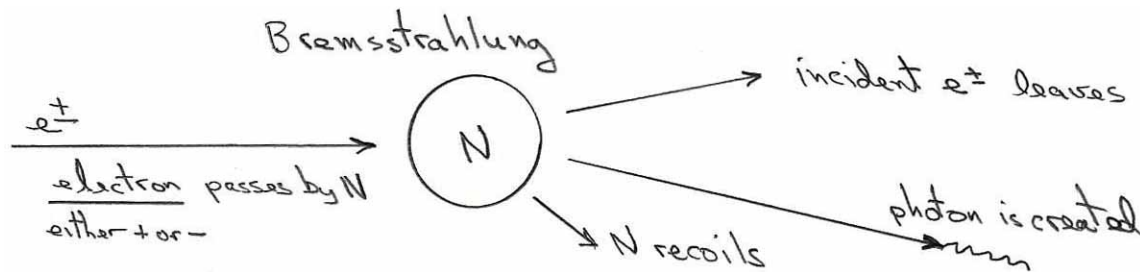
which does NOT make sense.

... The power of conservation laws ...

Yet, pair creation does take place!



Pair creation has an analog ... bremsstrahlung photon creation ...



In both cases there are two electrons, a photon and a nucleus in the interaction. A nucleus is needed to soak up the momentum.

There is a common theme here. If a conservation law rules out a process, it can not happen.

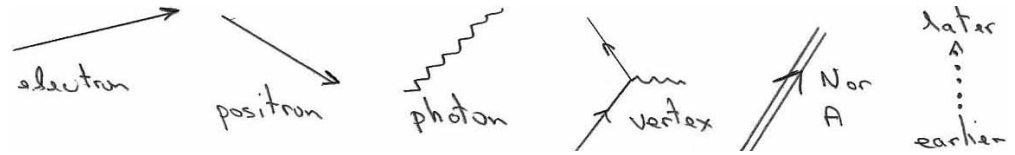
On the other hand, if a conservation law does not rule it out, then it does happen!

Maybe not in abundance, but it can happen.

*Feynman diagrams, Quantum Electrodynamics

And now, for a cultural aside³!

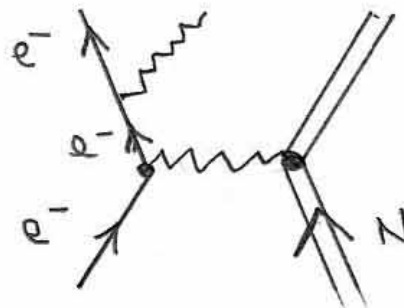
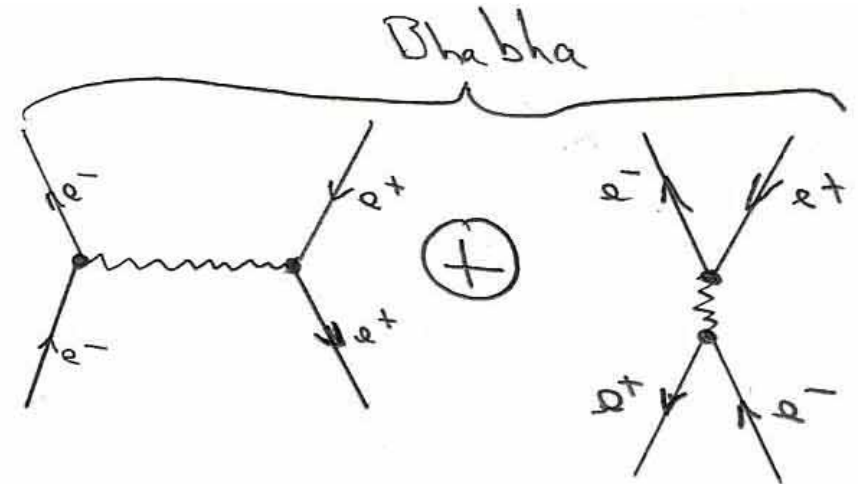
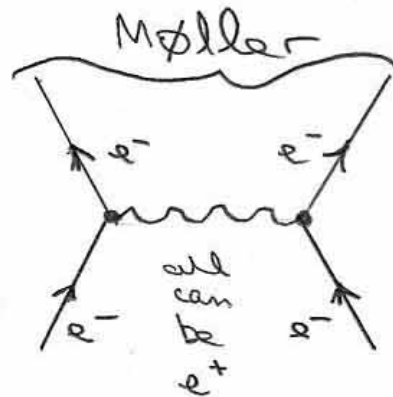
There are several features to remember:



1. The progress of time goes from the bottom of the diagram to the top.
2. A thin solid line represents an electron, if it is going forward in time, or a positron, if it is going backward in time.
3. A thick solid line represents a nucleus, N , or atom, A , that may be neutral, in a state of excitation but still neutral, e.g. N^* , or ionized, e.g. A^+ .
4. A squiggly line represents a photon. Photons have no direction because they are going at the speed of light. Or, you also say that it is going forward AND backwards in time.
5. A vertex describes the interaction between a photon and an electron or positron. All electromagnetic forces require a vertex.
6. Any line that is connected to two vertices violates conservation laws, but are allowed due to the Heisenberg Uncertainty relations.

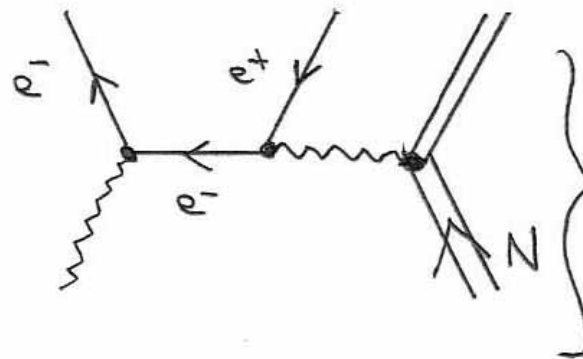
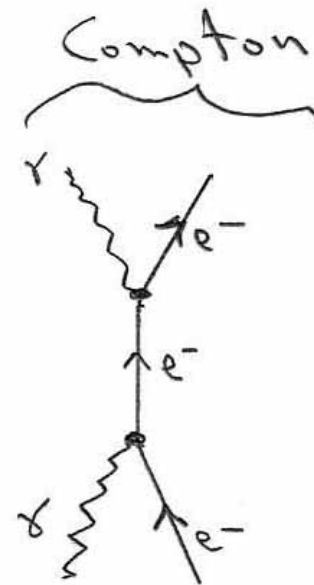
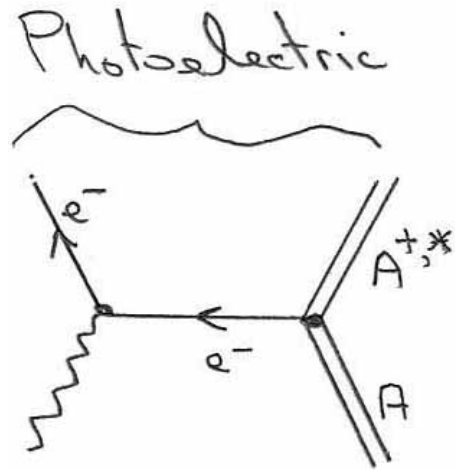
³Henceforth, all slides with a leading asterisk (*) in the title line, is scientific “culture” — information that you won’t be graded on, but knowledge that I feel compelled to tell you about.

A catalog of $e^\pm$ processes



Bremsstrahlung

A catalog of γ processes



Pair Creation

What is a photon?

A photon is a curious object.

- In the frame of reference of the photon, the size of the universe is zero.
- In the lab frame, if the photon had a clock, it would appear to the observer that the time never changed.
- It behaves both as a wave, and a particle.
- It feels the pull of gravity, despite having no mass.
- No matter how fast the observer is moving, its speed is always c .
- Yet, using classical and quantum electrodynamics, we can describe how it interacts with anything.

Besides being able to calculate its effect on other things, we really don't understand it.

What is a particle?

A particle is a curious object as well is a curious object.

- A particle is both a “particle” and a wave.
- It can go through two places simultaneously.

No one really understands exactly what a fundamental particle is.

*Final words on “Quantum Mechanics”

- Those who are not shocked when they first come across it cannot possibly have understood it⁴
- If you are not completely confused by it, you do not understand it⁵
- It is safe to say that nobody understands it⁶
- If it is correct, it signifies the end of physics as a science⁷
- I do not like it, and I am sorry I ever had anything to do with it⁸

In other words, if you think you understand Quantum mechanics, you have another think coming!

⁴Niels Bohr

⁵John Wheeler

⁶Richard Feynman

⁷Albert Einstein

⁸Erwin Schrödinger