

Classical Rutherford Scattering Cross Section

We have, from page 172 (Chapter 6) of the online notes, a relationship between b , the impact parameter, and the scattering angle, θ .

From eqn 10,

$$b = \left(\frac{zZ}{2K_0} \right) \left[\frac{e^2}{4\pi\epsilon_0} \right] \cot \frac{\theta}{2} \quad (1)$$

short hand () []

electrostatics term

z = Atomic # of projectile
 Z " " " target
 K_0 KE of projectile

assumptions

z, Z are point particles

Z is "bare" (no atomic e^- 's)

Z does not recoil i.e. $m_Z \gg m_z$

non-relativistic

no wave mechanics!

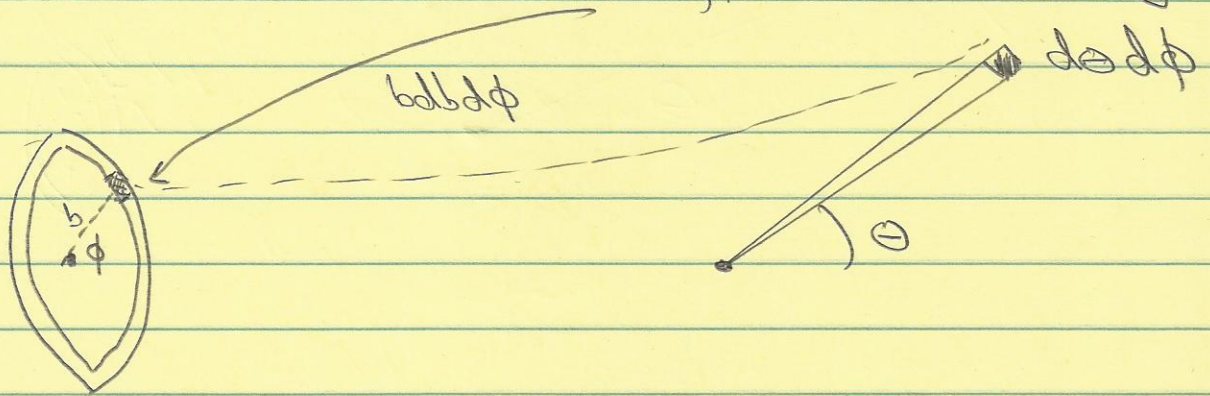
Now consider

$$|b/db| d\phi = \left(\left(\frac{zZ}{2K_0} \right) \left[\frac{e^2}{4\pi\epsilon_0} \right] \right)^2 \left(-\frac{1}{2} \frac{1}{\sin^2 \frac{\theta}{2}} \right) \cot \frac{\theta}{2} |d\theta d\phi|$$

$$|b/db| d\phi = \frac{1}{2} \left(\frac{zZ}{K_0} \right)^2 \left[\frac{e^2}{4\pi\epsilon_0} \right]^2 \frac{\cos \frac{\theta}{2}}{\sin^3 \frac{\theta}{2}} \quad (2)$$

What is $|b(db)d\phi|$?

It is the elemental area, that scatters into $d\phi$



In more popular vernacular, it is the effective "size" of the scatterer, to produce a deflection into $d\phi$

Using the concept of solid angle, $\times \frac{1}{r^2} \div \text{by } \sin\theta$ gives

$$|b(db)d\phi| = \frac{1}{4} \left(\frac{r}{\sin^2 \theta} \right)^2 dR$$

This is called $d\sigma$, the effective size of the scatterer producing a deflection to dR

Here dR just stands for two things,
 $\sin\theta d\theta \div d\phi$

Finally

$$\frac{d\sigma}{d\Omega} = \left(\frac{zZ}{4K_0} \right)^2 \left(\frac{e^2}{4\pi\epsilon_0} \right)^2 \left(\frac{1}{\sin^4 \frac{\theta}{2}} \right)$$

Notes:

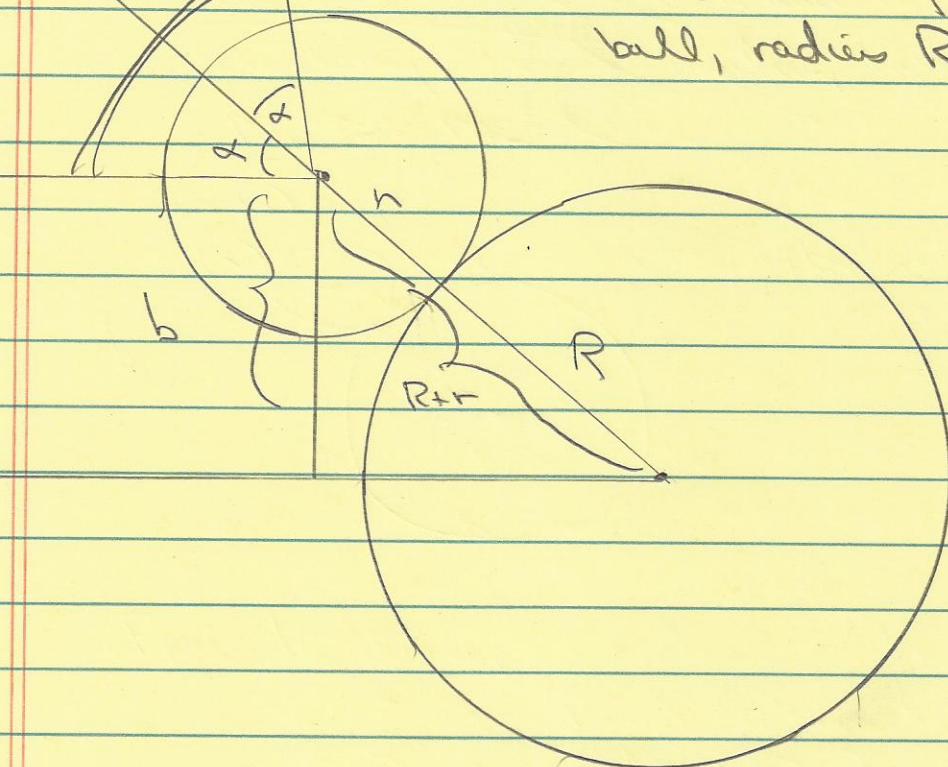
$\left(\frac{d\sigma}{d\Omega} \right) \uparrow$	as $z \uparrow$	✓	} all verified by experiment
$\left(\frac{d\sigma}{d\Omega} \right) \uparrow$	as $Z \uparrow$	✓	
$\left(\frac{d\sigma}{d\Omega} \right) \downarrow$	as $K_0 \uparrow$	✓	

$$\int d\Omega \frac{d\sigma}{d\Omega} \Rightarrow \infty !$$

No surprise! A bare nucleus will produce a scattering angle for any b .

(4)

Cross section for a tennis ball,
radius r , scattering from a bowling
ball, radius R



From the drawing $\Theta = 2\alpha$ $b = (R+r)\sin\alpha$

$$b = (R+r)\sin\frac{\Theta}{2}$$

← the whole challenge
is finding Θ , given b

$$b db d\phi = (R+r)^2 \sin\frac{\Theta}{2} \left(\frac{1}{2}\right) \cos\frac{\Theta}{2} d\Theta d\phi$$

$$= \frac{1}{4} (R+r)^2 \sin\Theta d\Theta d\phi$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{4} (R+r)^2$$

← no angular
dependence $\Rightarrow$
isotropic

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \pi(R+r)^2$$

← scattering

as expected

σ is the effective size of the projectile - scatterer duo.

(5)

Problem: Calculate the $\bar{I}_2$ for an ellipse

with 3 unequal axes, in any

orientation. Extra credit!