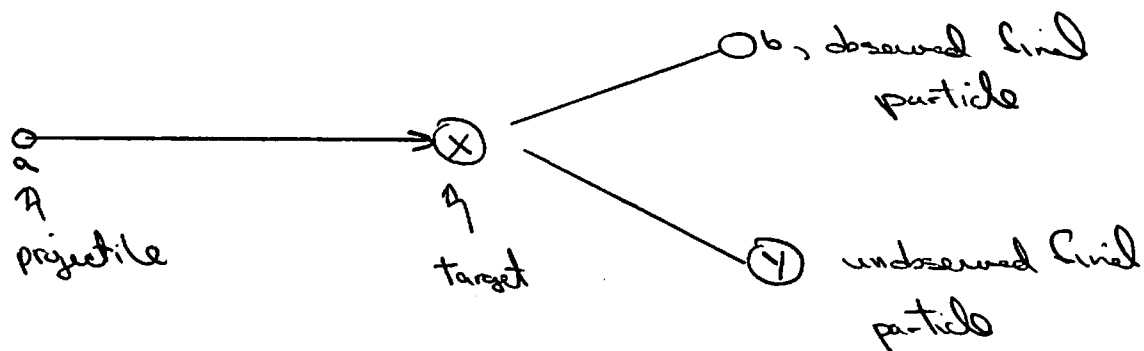


11.1 Types of reactions and Conservation laws

Typical nuclear reaction

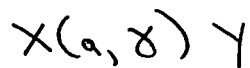
We restrict ourselves to light projectiles ($A \leq 4$) and heavier targets ($A \geq 4$), projectile energies: $E_a \leq 10 \text{ MeV}$, 2 particles in the final state.



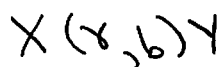
i.e. $a + X \rightarrow b + Y$

or $\underbrace{X}_{\text{particle in}}(a, \underbrace{b}_{\text{particle out}})Y$ shorthand for above

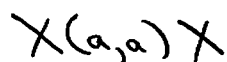
Some nomenclature



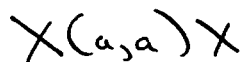
"radiative capture" of a by nucleus X , producing Y & a γ
 e.g. (n, γ) reactions



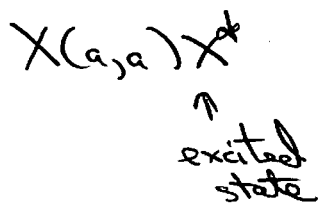
"nuclear photoeffect"
 e.g. (γ, p) reaction



nuclear scattering
 e.g. Rutherford scattering

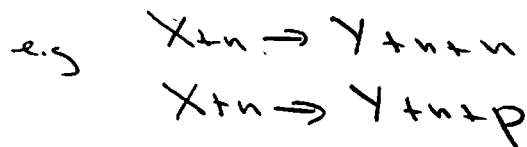


elastic scattering if
 X & a remain in their
 ground states
 e.g. Rutherford scattering

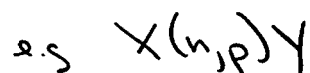


inelastic scattering

knock out reaction



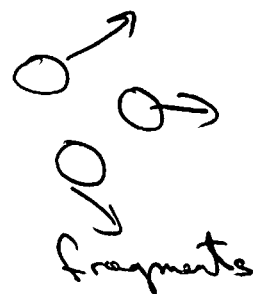
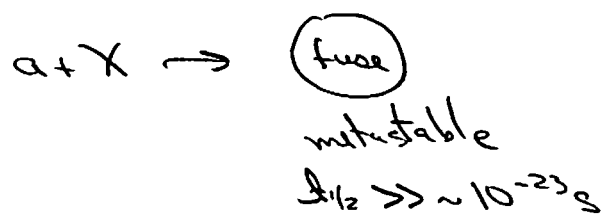
transfer reaction



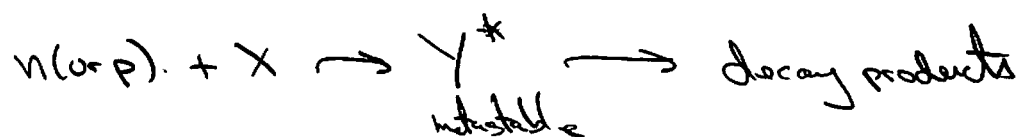
direct reactions

few nucleus participate,
 most nucleus are passive
 bystanders
 (shell model exploration)

compound reaction



resonance reaction

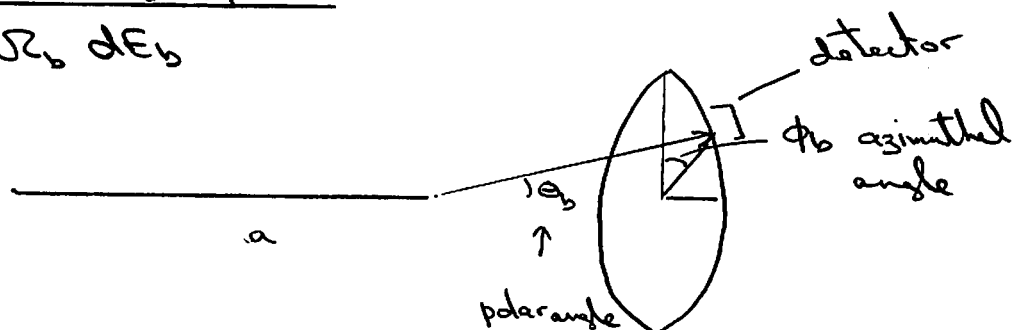


Observables

$$X(a, b)Y$$

generally, measure scattering angle and energy of b . The differential cross section is measured.

$$\textcircled{1} \quad \frac{d\sigma(\Theta_b, \phi_b, E_b)}{d\Omega_b dE_b}$$



Otherwise, integrate over all energies, E_b

$$(2) \quad \frac{d\sigma(\theta_b, \phi_b)}{d\Omega_b} = \int dE_b \frac{d\sigma(\theta_b, \phi_b, E_b)}{d\Omega_b dE_b}$$

Integrate over all angles

$$(3) \quad \frac{d\sigma(E_b)}{dE_b} = \int_{4\pi} d\Omega_b \frac{d\sigma(\theta_b, \phi_b, E_b)}{d\Omega_b}$$

(4) Integrate over all angles and energies to get the total cross section

$$\sigma = \int dE_b \int_{4\pi} d\Omega_b \frac{d\sigma(\theta_b, \phi_b, E_b)}{d\Omega_b dE_b}$$

Aside

$$\frac{1}{\sigma} \frac{d\sigma(\theta_b, \phi_b, E_b)}{d\Omega_b dE_b} \quad (5)$$

can be regarded as a probability distribution, joint in Ω_b, E_b from which average values can be determined

$\overline{E_b}$ average energy

$\overline{\cos\theta_b}$ average scattering cosine

Conservation laws

Conservation of Energy & Linear Momentum

- using this deduce properties of $X \rightarrow Y$

Conservation of neutron & proton number

(low energies, below meson creation thresholds)

direct nuclear reactions take $t \sim 10^{-23}$ s,

β decay is much slower

Conservation of angular momentum and parity

- allows us to deduce spins

$(-1)^l$ rule

11.2 Energetics of nuclear reactions

General Considerations, relativistic formalism

For the $X(a,b)Y$ reaction

$$\text{Conservation of Energy} \Rightarrow m_X c^2 + m_a c^2 + T_a + T_x = m_Y c^2 + m_b c^2 + T_b + T_Y$$

$$\text{or } \underbrace{(m_X + m_a - [m_Y + m_b]) c^2}_{\equiv Q} + T_x + T_a = T_Y + T_b \quad (6)$$

$$Q + T_x + T_a = T_Y + T_b \quad \text{reaction } Q \text{ value} \quad (7)$$

interpretation: The kinetic energies of the products is the sum of Q and the kinetic energies of the initial reactants.

$Q > 0$ exothermic (exoergic)
energy is released by the transformation

$Q < 0$ endothermic (endoergic)
energy is required to make the reaction go. (Supplied by KE of a $\frac{1}{2} X$)

Recall that this ⁽⁷⁾ is a relativistic expression if we use the relativistic version of T

$$T = mc^2(\gamma - 1) \quad (8)$$

m_e, m_p, m_n, m_γ

non-relativistic limit

$$\gamma = [1 - \beta^2]^{-1/2} \rightarrow 1 - \frac{1}{2}(-\beta^2) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-\beta^2)^2 + O(\beta^6)$$

$$\gamma \rightarrow 1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + O(\beta^6) \quad (9)$$

(9) $\rightarrow$ (8) $\Rightarrow$

$$\begin{aligned} T &= mc^2 \left[1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + O(\beta^6) - 1 \right] \\ &= \frac{1}{2} m (pc)^2 \left[1 + \frac{3}{4}\beta^2 + O(\beta^4) \right] \\ &= \frac{1}{2} m v^2 \left[1 + \frac{3}{4}\beta^2 \right] \quad (10) \end{aligned}$$

Can we ignore this?

worst possible case

$$T = 10 \text{ MeV}$$

$m =$ proton or neutron mass

$$\beta^2 \approx \frac{2T}{mc^2} \approx 0.02$$

$$\frac{3}{4}\beta^2 \approx 0.015$$

1.5% correction
in the worst case

From now on we will ignore this.

But, keep in mind that large energies and small masses could benefit from this, when high accuracy is desired.

If photons are involved, we must use

T_X for energy and $\frac{T_X}{c}$ for X momentum

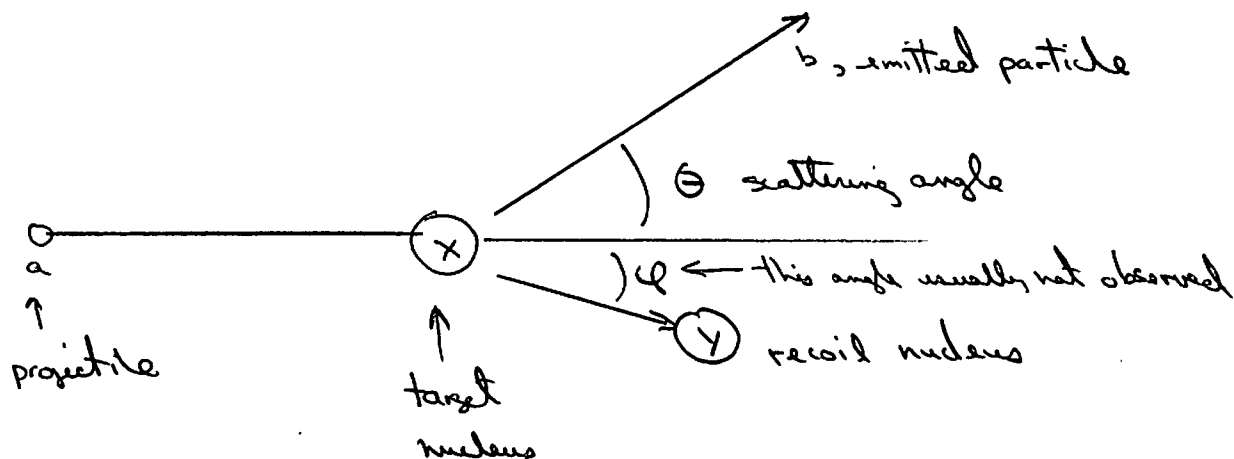
Laboratory frame, non-relativistic analysis

$X(a,b)Y$, X is considered to be at rest

Any thermal motion of X can be ignored.

Nuclear Reactions (Ch. 11)

11.2 Energetics of Nuclear Reactions



Approximations

$$T_a \lesssim 10 \text{ MeV}$$

a's kinetic energy

$$T_x = 0$$

X is at rest (thermal motion can be ignored)

Y is unobserved

(we eliminate its dynamics from resultant equations)

$$1 \leq m_a, m_b \leq 4 (u)$$

$$m_x, m_y \geq 4 (u)$$

generally, equations more general
generally "

relativistic corrections are ignored

(only a 1.5% correction for 10 MeV p or n)

With these approximations

$$\text{CoE: } Q + T_a = T_Y + T_b \quad (11)$$

$$\text{CoM } \vec{p}_a = \vec{p}_b + \vec{p}_Y \quad (12)$$

Eliminate T_b and $\vec{p}_Y$ using $\vec{p}_Y^2 = 2m_Y T_Y$
So,

$$2m_Y * (11) \Rightarrow$$

$$2m_Y(Q + T_a - T_b) = p_Y^2 \quad (13)$$

$$(12)^2 \Rightarrow$$

$$p_a^2 - 2p_a p_b \cos \Theta + p_b^2 = p_Y^2 \quad (14)$$

$$2m_Y(Q + T_a - T_b) - [2m_a T_a - 2\sqrt{4m_a m_b} \sqrt{T_a T_b} \cos \Theta + 2m_b T_b] = 0$$

reorganizing

$$(15) \quad T_b (m_Y + m_b) - 2\sqrt{m_a m_b} T_a \cos \Theta \sqrt{T_b} - [m_Y(Q + T_a) - m_a T_a] = 0$$

This is a quadratic equation in $T_b^{1/2}$ in terms of T_a, Q , and the masses.

Solving the quadratic equation gives

(16)

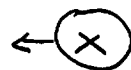
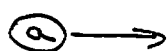
$$T_b^{1/2} = \frac{(m_a m_b T_a)^{1/2} \cos \theta \pm \sqrt{m_a m_b T_a \cos^2 \theta + (m_y + m_b) [m_y Q + (m_y - m_a) T_a]}}{(m_y + m_b)}$$

This, (16), is a complicated expression, but very rich in content.

1) If $Q > 0$, we must always choose the + sign in (16), else $T_b^{1/2}$ goes negative

2) If $Q < 0$, there exists the possibility that, for given T_a , T_b can have two possible values. There is a threshold energy for this to occur. At this threshold $\cos \theta = 1$ ($\theta = 0$). This is most easily seen by considering the collision in the Zero Momentum frame (aka Center of Mass (COM) frame)

COM frame



① } at rest in COM frame
② }

in Lab frame

① → } same velocity, forward
② → } directed

Looking at (16), at the threshold $\sqrt{} = 0$
 which implies

$$0 = m_a m_b T_{th} + (m_y + m_b) m_y Q + (m_y + m_b)(m_y - m_a) T_{th}$$

$$T_{th} = \frac{(-Q) m_y (m_y + m_b)}{(m_y + m_b)(m_y - m_a) + m_a m_b}$$

$$= \frac{(-Q) m_y (m_y + m_b)}{m_y^2 + m_y m_b - m_y m_a}$$

$$\boxed{T_{th} = \frac{(-Q)(m_y + m_b)}{m_y + m_b - m_a}}$$

(17)

The upper limit for this double-valued behavior occurs when the - sign solution in (16) falls to zero. This energy is called T_a' . By inspection of (16)

$$\boxed{T_a' = (-Q) \frac{m_y}{m_y - m_a}}$$

(18)

Range of double valued behavior

$$\begin{aligned}
 T_{a'} - T_m &= (-Q) \left\{ \frac{m_y}{m_y - m_a} - \frac{m_y + m_b}{m_y + m_b - m_a} \right\} \\
 &= -Q \left\{ \frac{m_y(m_y + m_b - m_a) - (m_y + m_b)(m_y - m_a)}{(m_y - m_a)(m_y + m_b - m_a)} \right\} \\
 &= -Q \frac{m_a m_b}{(m_y - m_a)(m_y + m_b - m_a)} \\
 &\approx -Q \frac{m_a m_b}{(m_y - m_a)(m_x)} \quad m_x = m_y + m_b - m_a
 \end{aligned}$$

as $m_x, m_y \gg m_a, m_b$, effect becomes less prominent.

Maximum angle for double valued behavior

From (16)

$$(19) \quad \cos^2 \Theta_m = \frac{(m_y + m_b)}{m_a m_b T_a} \left\{ m_y(-Q) - (m_y - m_a) T_a \right\}$$

$\Rightarrow$ for $T > T_{a'}$, single valued behavior

Equation (15) can be rewritten as a solution for Q , in terms of T_a, T_b

$$(20) \quad Q = T_b \left(1 + \frac{m_b}{m_y} \right) - T_a \left(1 - \frac{m_a}{m_y} \right) - 2 \left(\frac{m_a m_b}{m_y^2} T_a T_b \right)^{1/2}$$

We can use the measurement of T_b at some θ to determine Q , and thereby determine m_y , given m_a, m_b, m_x

Note, however that m_y has a weak dependence on Q , since

$$(21) \quad Q = (m_x + m_a - [m_y + m_b])c^2$$

To account for this rigorously, put (21) $\Rightarrow$ (20) at a special measurement angle of $\theta = \frac{\pi}{2}$

So at 90°

$$(22) \quad Q = (T_b - T_a) + \frac{m_b c^2 T_b + m_a c^2 T_a}{[m_x + m_a - m_b]c^2 - Q}$$

or

$$(23) \quad [(m_x + m_a - m_b)c^2 - Q]Q = ([m_x + m_a - m_b]c^2 - Q)(T_b - T_a) + m_b c^2 T_b + m_a c^2 T_a$$

results in a quadratic equation for Q

To save writing, write (23) as

$$(24) \quad \{(\quad) - Q\}Q = \{(\quad) - Q\}\Delta T + [\]$$

$$\text{where } (\quad) \equiv (m_x + m_a - m_b)c^2$$

$$[\] \equiv m_b c^2 T_b + m_a c^2 T_a$$

$$\Delta T \equiv T_b - T_a$$

Rewrite (24) as

$$0 = Q^2 - Q\{(\quad) + \Delta T\} + \{(\quad)\Delta T + [\]\}$$

Solving for Q

$$Q = \frac{1}{2} \left\{ (\quad) + \Delta T \pm \sqrt{\{(\quad) + \Delta T\}^2 - 4\{(\quad)\Delta T + [\]\}} \right\}$$

The - sign applies in this case

$$Q = \frac{1}{2} \left\{ (\quad) + \Delta T \right\} \left\{ 1 - \sqrt{1 - \frac{4\{(\quad)\Delta T + [\]\}}{\{(\quad) + \Delta T\}^2}} \right\}$$

This quantity is small.

$$\text{Using } (1-\epsilon)^{1/2} = 1 + \frac{1}{2}(-\epsilon) + \frac{1}{2}\left(-\frac{1}{2}\right)\epsilon^2$$

$$= 1 - \frac{\epsilon}{2} - \frac{\epsilon^2}{8}$$

$$Q = \frac{1}{2} \left\{ () + \Delta T \right\} \left\{ \cancel{1} - \cancel{[]} - \frac{2 \{ () \Delta T + [] \}}{\{ () + \Delta T \}^2} - \frac{2 \{ () \Delta T + [] \}^2}{\{ () + \Delta T \}^4} \right\}$$

$$Q = \frac{() \Delta T + []}{() + \Delta T} \left\{ 1 + \frac{() \Delta T + []}{\{ () + \Delta T \}^2} + \dots \right\}$$

Now, use $() \gg \Delta T$ to get the lowest order correction

$$Q = \{ \Delta T + [] / () \} \left\{ 1 - \frac{\Delta T}{()} \right\} \left\{ 1 + \frac{\Delta T}{()} + \frac{[]}{()^2} \right\}$$

$$Q = \underbrace{\{ \Delta T + [] / () \}}_{Q_0} \left\{ 1 + \underbrace{\frac{[]}{()^2}}_{\frac{\partial Q}{\partial Q_0}} \right\}$$

$Q_0 = (22)$ setting $Q=0$ on RHS

$$\frac{\Delta Q}{Q_0} = \frac{m_b x^2 T_b + m_a c^2 T_a}{[(m_x + m_a - m_b x^2)^2]} \quad (25)$$

example

$$m_a = m_b \equiv m \quad m_x = M$$

$$\frac{\Delta Q}{Q_0} = \left(\frac{m}{M} \right) \left(\frac{T_a + T_b}{Mc^2} \right)$$

worst possible case $m = M = 4A$
 $T_a \approx T_b = 10 \text{ MeV}$

$$\frac{\Delta Q}{Q_0} \approx 5 \times 10^{-3}$$

Generally, accurately measured Q s are of the order of $\frac{\Delta Q}{Q} \approx 1 \times 10^{-3}$

So, this correction is only required for light nuclei.

Effect of ^{atomic} binding energy?

ignoring the above higher order correction

$$Q = (T_b - T_a) + \frac{m_b c^2 T_b + m_a c^2 T_a}{(m_x + m_a - m_b) c^2}$$

The masses are nuclear masses.

Recall the definition of Q

$$Q = (m_a + m_x - [m_b + m_y])c^2$$

Substituting for atomic masses

$$Q = [m(a) + m(A_x) - m(b) - m(A'_y)]c^2$$

$$- \left\{ \underbrace{\sum_{n=1}^{Z_a} B_n^{(a)} - \sum_{n=1}^{Z_b} B_n^{(b)}}_{\text{if the projectiles are small } A,} + \underbrace{\sum_{n=1}^{Z_x} B_n^{(x)} - \sum_{n=1}^{Z_y} B_n^{(y)}}_{\text{if the nuclei are heavy, we can not ignore this, especially when } Z_x \neq Z_y \text{ are quite different}} \right\}$$

if the projectiles
are small A ,
we can
ignore
this

if the nuclei are
heavy, we
can not ignore
this, especially
when
 $Z_x \neq Z_y$ are
quite different

From last time, we can determine Q in a $X(a,b)Y$ reaction from

$$Q = \underbrace{T_b - T_a}_{\text{this is what we would have if } X, Y \text{ were only heavy}} + \underbrace{\frac{m_a T_a + m_b T_b}{m_x + m_a - m_b}}_{\text{mass correction}} \quad (1)$$

(1) is an approximation that should be corrected for if X is "light". See previous notes

m_x, m_a, m_b are nuclear masses. What approximation are we making by using atomic masses?

Using atomic masses and the relationship wrong sign

$m_e \equiv$ electron mass

$B_i^x \equiv$ binding energy of i th electron in X 's atomic shell

$$M_x = m(X) + \sum_{i=1}^Z m_e - \frac{1}{c^2} \sum_{i=1}^Z B_i^x \equiv B_{Zx}^x$$

$$Q = T_b - T_a + \frac{m_a c^2 T_a + m_b c^2 T_b}{[m(X) + (Z + Z_a - Z_b)m_e]c^2 - [B_{Zx}^x + B_{Z_a}^a - B_{Z_b}^b]} \quad \text{can ignore this}$$

$$= \underbrace{Q_a}_{\substack{\uparrow \\ \text{using atomic mass}}} + \underbrace{\frac{m_a c^2 T_a + m_b c^2 T_b}{[m(X) + m(a) - m(b)]c^2} * \frac{(Z_x + Z_a - Z_b)m_e}{m(X) + m(a) - m(b)}}_{\Delta Q} \quad (2)$$

worst possible case $X = \alpha, a = \alpha, b = d$

for $T_a = T_b = 10 \text{ MeV}$

$$\Delta Q \approx 2(T_a + T_b) * 5 \times 10^{-4}$$

$$= 0.02 \text{ MeV} \quad 20 \text{ keV}$$

Major Conclusion

Thus, this formula for Q must be used with caution for light nuclei nuclear reactions

usually γ is left as γ^* and decays long after b has left

Q_0 is the ground state Q

E_{ex} is the excitation energy

Accounting for the nuclear reaction leaving γ in an excited state?

$$Q = (m_x + m_a - m_{\gamma^*}^* - m_b)c^2$$

$$= \underbrace{(m_x + m_a - m_{\gamma} - m_b)c^2}_{\equiv Q_0} - \underbrace{(m_{\gamma^*}^* - m_{\gamma})c^2}_{E_{ex}}$$

$$Q = Q_0 - E_{ex} \quad (3)$$

Must account for this

11.3 Isospin

The strong force does not distinguish between protons and neutrons (mostly)

Therefore, consider there to be a more general particle, called a nucleon with two possible spin states

$T_3 \Rightarrow$ 3rd component, like S_z

$$T_3 \text{ for a proton} = +\frac{1}{2}$$

$$T_3 \text{ " " neutron} = -\frac{1}{2}$$

T is called the "isospin"

historically it has been called isotopic spin (T_3 value $\Rightarrow$ isotope)

or isobaric spin (range of T_3 describes an isobaric multiplet)

"isospin" is now the preferred name.

See evidence for isobaric energy levels ~

Fig. 11.5

e.g. the dinucleon

same arithmetic as
normal spin

$T = 1$ state $T_3 = -1, 0, 1$

$T = 0$ state $T_3 = \text{singlet}$

The triplet

these states
are not seen
in nature

$$\psi = \frac{1}{\sqrt{2}} \left\{ \psi_1(\vec{r}_1) \psi_2(\vec{r}_2) - \psi_1(\vec{r}_2) \psi_2(\vec{r}_1) \right\}$$

must be antisymmetric to
account for Pauli's Exclusion principle.

$$1, 2 = n \quad ; \quad 1, 2 = p \quad ; \quad 1 = p, 2 = n \text{ or } 1 = n, 2 = p$$

this is observed
in nature as
the deuteron

The dinucleon singlet

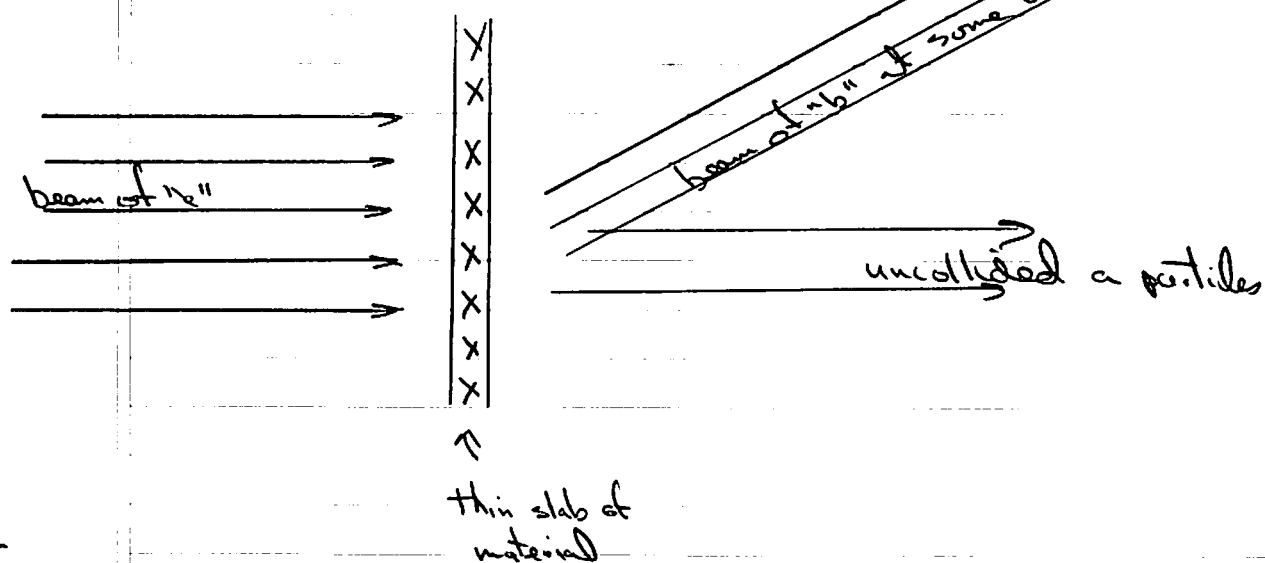
$$\psi = \frac{1}{\sqrt{2}} \left\{ \psi_n(\vec{r}_1) \psi_p(\vec{r}_2) + \psi_p(\vec{r}_1) \psi_n(\vec{r}_2) \right\}$$

(read rest of 11.3 on your own)

11.4 Reaction Cross sections

Much of this is 311 Review

α scattering



the slab is thin
so that α escapes
without scattering
or being absorbed

Intensity of α is I_α (#/area/time)

The cross section is defined as

(4)

$$\sigma = \frac{R_b}{I_\alpha N}$$

R_b rate of b particles
produced (over 4π)

of target nuclei
irradiated by the
 α -beam

Differential cross section

$$\frac{d\sigma}{d\Omega_b} = \frac{r_b(\theta, \phi)}{4\pi I_a N}$$

r_b is rate of b particles received at Ω_b

r_b is normalized so that

$$\int_{4\pi} d\Omega r_b(\theta, \phi) = 4\pi R_b$$

(5)

If $r_b = 1 \Rightarrow$ isotropic scattering

Differential forms

σ total reaction cross section measures all b 's

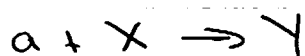
$\frac{d\sigma}{d\Omega_b}$ measures only those b 's at a given angle

$\frac{d\sigma}{dE_b}$ measures all b 's with a given energy

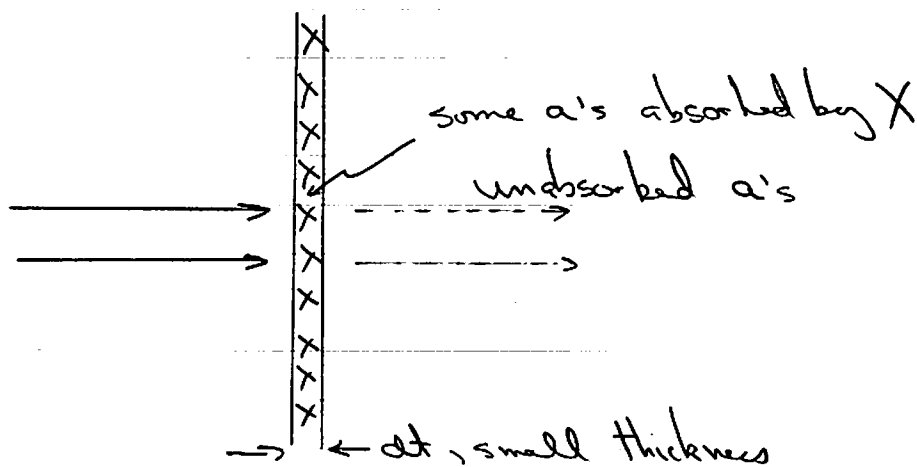
$\frac{d\sigma}{dE_b d\Omega_b}$ b 's have given energy and momentum

$\frac{d\sigma}{dE_a dE_b d\Omega_b}$ b 's set in motion by a with given energy, at angle Ω_b and E_b

Absorption cross sections (attenuation)



target is thin so that only a small fraction of a 's are absorbed



$$dI_a = -I_a n \sigma_a dt$$

$n = \text{density of } X\text{'s}$

$$\sigma_a = \frac{|dI_a|}{I_a dt n}$$

This cross section can also be differential

Read the rest of 11.4

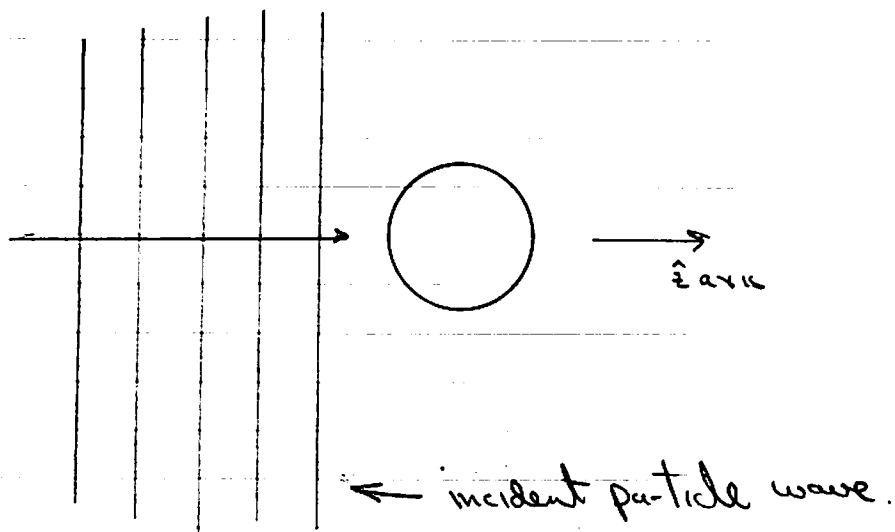
$$\frac{d\sigma_a}{dE_a}$$

11.5 (not covered)

11.6 (done in 3.11)

11.7 these ideas were covered in Chapter 3

11.8 Scattering and Reaction Cross sections



$$\psi_{\text{inc}} = A e^{ikz} \quad (6)$$

We know (6) is a solution to the Schrodinger equation. (SE)

If it is a sol'n to the SE, we should be able to write it in spherical polar coordinate, and identify the components of angular momentum.

If we did this, we could show

$$\psi_{\text{inc}} = A e^{ikz} = A \sum_{l=0}^{\infty} i^l (2l+1) j_l(kr) P_l(\cos\theta)$$

$l\hbar = p, \text{ momentum}$

← regular Legendre polynomials

Thus, the plane wave is resolved into separate components of l , each with angular momentum $l\hbar$

(7)

j_l 's are the spherical Bessel functions encountered where we solved the spherical well potential

The magnetic quantum number, m , does not appear because the incoming wave is azimuthally symmetric. It is easy to do this, but not really necessary for this discussion.

(7) is called the "partial wave expansion" and manipulations of it is called "partial wave analysis".

Partial wave analysis

An aside

Semi-classical estimate of cross section

an "impact parameter" can be defined as

$$L\hbar = pb$$

$$\text{or } b = \frac{L\hbar}{p} = \frac{L\lambda}{2\pi} = L\lambda$$

$$L = 0 \rightarrow 1 \text{ covers an area } \pi\lambda^2$$

$$L = 1 \rightarrow 2 \quad " \quad " \quad (4-1)\pi\lambda^2$$

$$L \rightarrow L+1 \quad " \quad " \quad [(L+1)^2 - L^2]\pi\lambda^2$$

$$= (2L+1)\pi\lambda^2$$

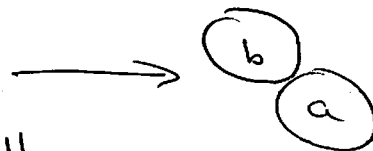
$$\frac{R}{\lambda} \equiv \text{maximum } L$$

$$\sigma = \sum_{L=0}^{R/\lambda} (2L+1)\pi\lambda^2 \quad (9)$$

gives a sense of both target and projectile size.

Remember when we calculated the classical cross section for two spheres of unequal size?

$$\sigma = \pi(a+b)^2$$



it's the same thing

Continuing with partial wave analysis

In scattering, we are always measuring the waves very far from the source of scattering.

Thus $kr \gg 1$

In this regime

$$j_l(kr) \rightarrow \frac{\sin(kr - \frac{1}{2}l\pi)}{kr} \quad \text{for } kr \gg 1$$

$kr \gg 1$
is a stronger
constraint than

$kr \gg 1$, but,
we expect only
a few l 's to
really matter

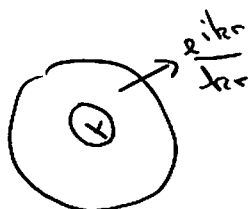
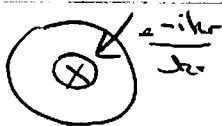
Using this limit

$$\frac{j_l(kr)}{kr} = \frac{i \left(e^{-i(kr - l\pi/2)} - e^{i(kr - l\pi/2)} \right)}{2kr}$$

and $(*) \Rightarrow$

$$\psi_{\text{inc}} = \frac{A}{2kr} \sum_{l=0}^{\infty} i^{l+1} (2l+1) \left[e^{-i(kr - l\pi/2)} - e^{i(kr - l\pi/2)} \right] \neq P_2(\cos\theta)$$

(9)



One should note here the similarity in language with the 3D scattering we studied so carefully in 311

The $\frac{e^{-ikr}}{kr}$ part represents a spherical wave converging on the nucleus

The $\frac{e^{ikr}}{kr}$ part represents a spherical wave moving away from the nucleus

The scattering center can only modify the outgoing wave. Hence, we can write the solution in terms of a phase shift of the outgoing part. Thus the solution must be of the form

$$\psi = \frac{A}{2kr} \sum_{l=0}^{\infty} i^{l+1} (2l+1) \left[e^{i(kr - l\pi/2)} - \eta_l e^{i(kr - l\pi/2)} \right] P_l(\cos\theta)$$

↑
phase shift

This is a superposition of the scattered wave and incident wave

$$\psi = \psi_{inc} + \psi_{sc}$$

$$\psi_{sc} = \psi - \psi_{inc} = \frac{A}{2kr} \sum_{l=0}^{\infty} i^{l+1} (2l+1) (1 - \eta_l) e^{i(kr - l\pi/2)} P_l(\cos\theta)$$

note that $e^{-il\pi/2} = (-i)^l$

$$\text{So } \psi_{sc} = \frac{A i e^{ikr}}{2kr} \sum_{l=0}^{\infty} (2l+1) (1 - \eta_l) P_l(\cos\theta)$$

As in 3D, we examine the probability current density

scattered current density

$$(13) \quad j_x = \frac{\hbar}{2mi} \left(\psi_x^* \frac{\partial \psi_x}{\partial r} - \frac{\partial \psi_x^*}{\partial r} \psi_x \right)$$

you can show

$$(14) \quad j_x = |A|^2 \frac{\hbar}{4\pi m k^2} \left| \sum_{l=0}^{\infty} (2l+1)(1-\eta_l) P_l(\cos\theta) \right|^2$$

and

$$(15) \quad j_{inc} = |A|^2 \frac{\hbar k}{m}$$

$$(16) \quad \boxed{\frac{d\sigma}{d\Omega} = \left(\frac{j_x}{j_{inc}} \right)^2 r^2 = \frac{1}{4k^2} \left| \sum_{l=0}^{\infty} (2l+1)(1-\eta_l) P_l(\cos\theta) \right|^2}$$

Total cross section

$$\begin{aligned} \sigma &= \int_{4\pi} d\Omega \frac{d\sigma}{d\Omega} = \frac{1}{4k^2} \sum_{l=0}^{\infty} \sum_{l'=0}^{\infty} (2l+1)(2l'+1) (1-\eta_l^*)(1-\eta_{l'}) \\ &\quad \underbrace{\int d\Omega P_l(\cos\theta) P_{l'}(\cos\theta)}_{= \frac{4\pi}{2l+1} \delta_{ll'}} \\ &= \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) |1-\eta_l|^2 \end{aligned}$$

Note similarity with (8)

(17)

$$\boxed{\sigma = \pi \lambda^2 \sum_{l=0}^{\infty} (2l+1) |1-\eta_l|^2}$$

$$\sigma_{classical} = \pi \lambda^2 \sum_{l=0}^{\infty} (2l+1)$$

elastic scattering

In elastic scattering $|M_\ell| = 1$, no loss of probability
the amplitude of each partial wave is unchanged

Convention is to write M_ℓ in this case as $e^{2i\delta_\ell}$

$$M_\ell = e^{2i\delta_\ell} \quad |M_\ell| = 1$$

$$|1 - M_\ell|^2 = |1 - e^{2i\delta_\ell}|^2$$

$$= \left| \frac{e^{i\delta_\ell} - e^{-i\delta_\ell}}{-e^{-i\delta_\ell}} \right|^2$$

$$= 4 \sin^2 \delta_\ell$$

(18)

$$\text{So } \boxed{\sigma_{sc} = 4\pi k^2 \sum_{\ell=0}^{\infty} (2\ell+1) \sin^2 \delta_\ell}$$

elastic cross section

reaction cross sections

In general $|M_\ell| \leq 1$, there can be absorption.

For elastic scattering $|R_{in}| = |R_{out}|$

$\therefore$ Use the deficit to defining the reaction (anything not elastic) $j_{in} = j_{out}$

$$\sigma_r = \left(\frac{|j_{in}| - |j_{out}|}{|j_{in}|} \right) r^2$$

~

(19)

$$\sigma_r = \pi \lambda^2 \sum_{l=0}^{\infty} (2l+1) (1 - |n_l|^2)$$

total cross section

$$\sigma_t = \sigma_a + \sigma_r = \pi \lambda^2 \sum_{l=0}^{\infty} (2l+1) \left\{ 1 - |n_l|^2 + (1 - |n_l|^2) \right\}$$

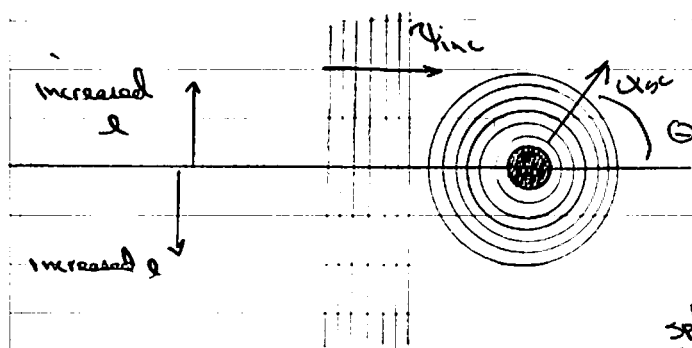
(20)

$$\sigma_t = 2\pi \lambda^2 \sum_{l=0}^{\infty} (2l+1) (1 - \text{Re } n_l)$$

Partial wave analysis (cont)

Summarizing what we did last time

→ detector



(7)

↑
numbering
from
last time

$$\psi_{inc} = A e^{ikz} = A \sum_{l=0}^{\infty} i^l (2l+1) j_l(kr) P_l(\cos\theta)$$

↑
rectilinear
coord system

spherical
Bessel

Legendre
polynomials

↑
spherical polar coord system,
(next rewrite)

Since the detector is very far from the source of scattering, use

$$j_l(kr) \xrightarrow{kr \gg l} \frac{\sin(kr - \frac{1}{2}l\pi)}{kr}$$

$$\psi_{inc} = \frac{A}{2kr} \sum_{l=0}^{\infty} (i)^{l+1} (2l+1) \left[e^{-i(kr - \frac{1}{2}l\pi)} - e^{-i(kr - \frac{1}{2}l\pi - \pi)} \right] P_l(\cos\theta)$$

incoming
spherical
wave

outgoing
spherical
wave

↑
only this
can be changed
by the scatterer

If we allow the outgoing part to be modified by a factor η_l , we can immediately discover the form of the scattered wave

$$(12) \quad \psi_{sc} = \frac{A e^{ikr}}{2kr} \sum_{l=0}^{\infty} (2l+1)(1-\eta_l) P_l(\cos\theta)$$

Using the formula for current density (radial portion)

$$(13) \quad j_r = \frac{\hbar}{2mi} \left(\psi_{sc}^* \left(\frac{\partial}{\partial r} \psi_{sc} \right) - \left(\frac{\partial \psi_{sc}^*}{\partial r} \right) \psi_{sc} \right)$$

$$(14) \quad j_r = \frac{|A|^2 \hbar}{4\pi k r^2} \left| \sum_{l=0}^{\infty} (2l+1)(1-\eta_l) P_l(\cos\theta) \right|^2$$

$$(15) \quad \text{With } j_{inc} = |A|^2 \frac{\hbar k}{m}$$

$$(16) \quad \frac{d\sigma}{d\Omega} = \left(\frac{j_r}{j_{inc}} \right) r^2 = \frac{1}{4k^2} \left| \sum_{l=0}^{\infty} (2l+1)(1-\eta_l) P_l(\cos\theta) \right|^2$$

$$(17') \quad \sigma = \int_{4\pi} d\Omega \frac{d\sigma}{d\Omega} = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) |1-\eta_l|^2$$

$$\text{or } \frac{1}{k^2} = \frac{\lambda^2}{4\pi} = \lambda \leftarrow \text{reduced wavelength}$$

(17)

$$\sigma = \pi \lambda^2 \sum_{l=0}^{\infty} (2l+1) |1-\eta_l|^2$$

elastic scattering

In this case $|H_l| = 1 \Rightarrow$ no loss of outgoing probability

Hence we can write $H_l = e^{2i\delta_l}$ (just a phase shift)
and show

(19)

$$\sigma_{sc} = 4\pi\lambda^2 \sum_{l=0}^{\infty} (2l+1) \sin^2 \delta_l$$



we reserve the name σ_{sc} to elastic scattering

reaction cross sections

In general $|H_l| < 1$, as the incoming beam can be absorbed

We'll call $\left(\frac{|j_{in}| - |j_{out}|}{j_{in}} \right) r^2$

the reaction cross section, σ_r

(19)

$$\sigma_r = \pi\lambda^2 (1 - |H_l|^2)$$

Total cross section, σ_t

(20)

$$\sigma_t = 2\pi\lambda^2 \sum_{l=0}^{\infty} (2l+1) (1 - \text{Re} H_l)$$

Interpretation

Recall

$$(17) \quad \sigma_{sc} = \pi \lambda^2 \sum_{l=0}^{\infty} (2l+1) |1 - \eta_l|^2$$

$$(19) \quad \sigma_r = \pi \lambda^2 \sum_{l=0}^{\infty} (2l+1) (1 - |\eta_l|^2)$$

If totally elastic scattering $|\eta_l| = 1 \quad \sigma_r = 0$

However, there is no way to make $\sigma_{sc} = 0$ and keep σ_r non-zero.

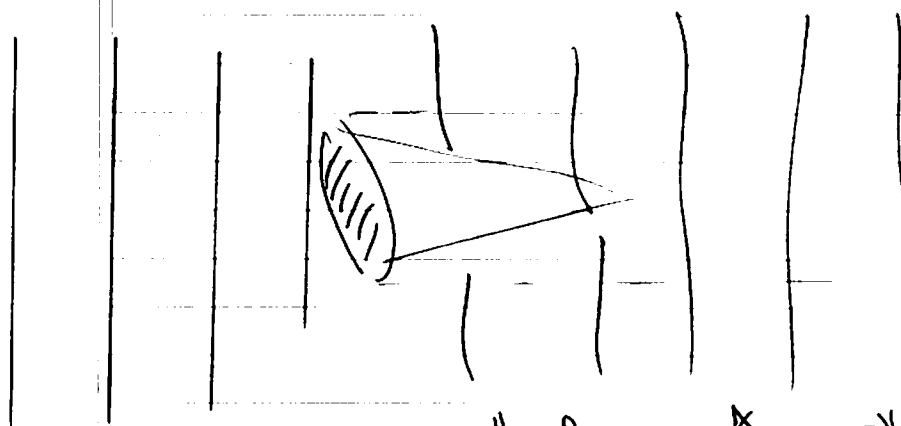
e.g. $\left\{ \begin{array}{l} \eta_l = 0 \text{ for } l = 1, 3, \dots, L \\ \eta_l = 1 \text{ for } l > L \end{array} \right.$
 absorbing disk

$$\sigma_{sc} = \pi \lambda^2 \sum_{l=0}^L (2l+1)$$

$$\sigma_r = \pi \lambda^2 \sum_{l=0}^L (2l+1)$$

} same!

explanation



wave will eventually fill in

the larger λ is, the faster the shadow gets filled in

Using $\sum_{l=0}^L l = \frac{L(L+1)}{2}$

$$\sum_{l=0}^L 1 = L+1$$

$$\sum_{l=0}^L (2l+1) = L(L+1) + (L+1) = (L+1)^2$$

Semi-classical approximation

$$\begin{aligned}\sigma_{sc} = \sigma_r &= \pi \lambda^2 \sum_{l=0}^L (2l+1) \\ &= \pi \lambda^2 (L+1)^2 \\ &= \pi ((\lambda L)^2 + \lambda^2)\end{aligned}$$

$$L \equiv \frac{R}{\lambda} \leftarrow \text{radius of the scatterer}$$

(21)

$$\text{Thus } \sigma_{sc} = \sigma_r = \pi (R^2 + \lambda^2)$$

Recall in 311, we worked out the cross section



$$\sigma = \pi (b^2 + a^2) \quad ?$$

Here we see that λ plays the role of the effective size of the projectile.

Aside How are the η_e 's calculated?

Generally, this is done numerically by integrating the radial portion of the SE wave equation, guarantee value and slope continuity everywhere, and determine the η_e 's from the asymptotic form of the solution, given by $\psi_{inc} + \psi_{sc}$.

11.9 The Optical Model

$$U(r) = V(r) + iW(r) \quad (1)$$

$\nearrow$ the potential $\nearrow$ real part (elastic scattering) $\nearrow$ imaginary part (absorption)

We studied this potential at some length in 311. At least,

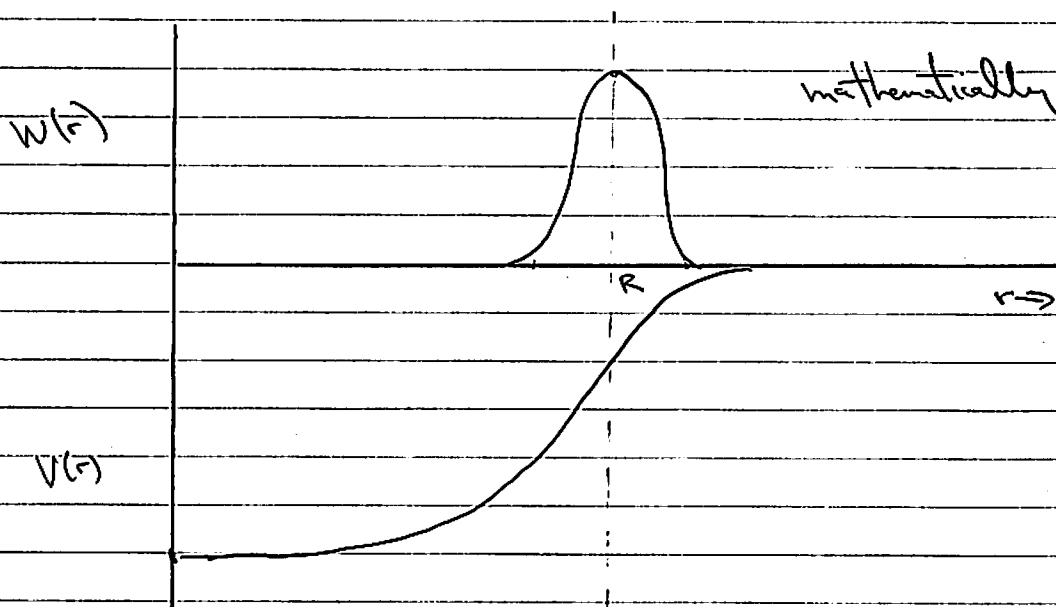
$$V_0 = V_0 + iW_0, \text{ constant forms}$$

more realistic forms

$$V(r) = -V_0 \frac{1 + e^{-(r-R)/a}}$$

Vo depth
R radius
a skin

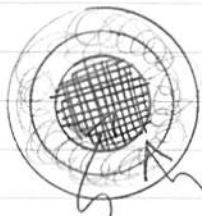
$$W(r) = \frac{dV}{dr} \quad \text{why?}$$



physically

particle's eye view, cross section of the nucleus

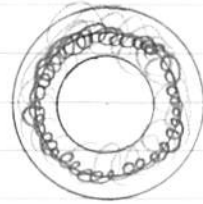
$V(r)$



dense
core

soft
surface

$W(r)$



$\Rightarrow$ absorption
only occurs in
the soft surface

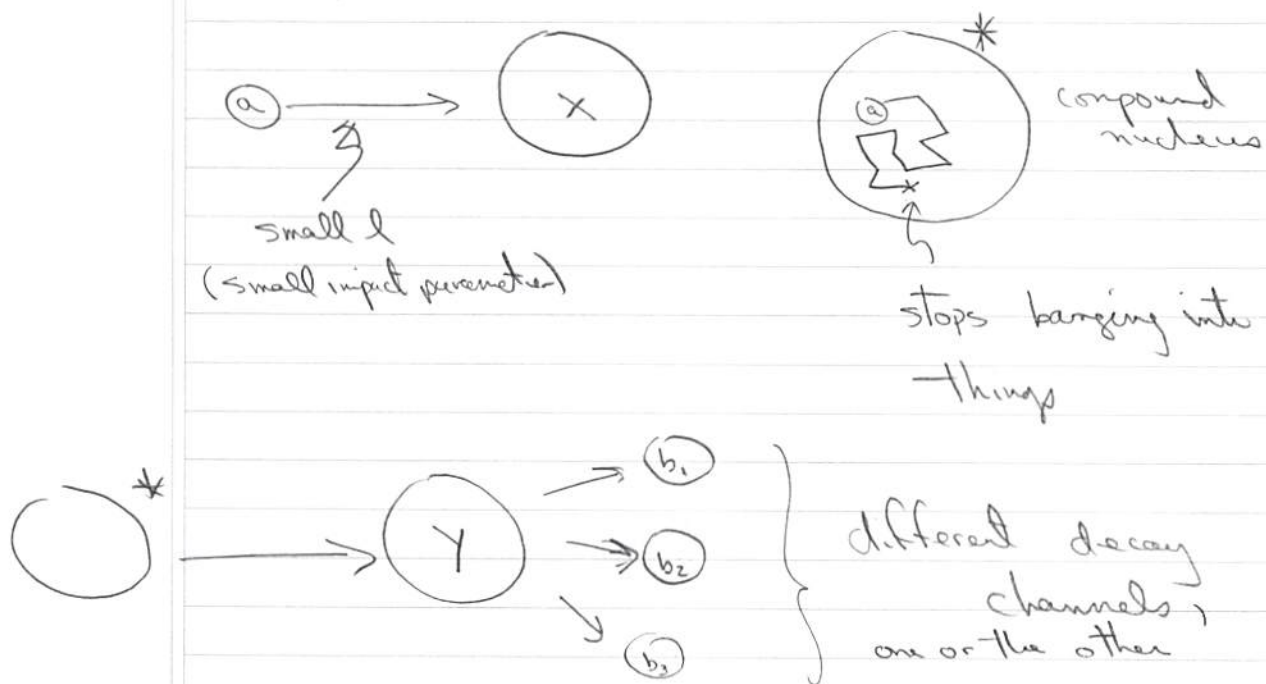
In the core, all the states are occupied.
In the surface, valence nucleons want to
close off shells.

More sophisticated models introduce
spin-orbit and Coulomb terms.

See Fig 11.17 for simple sphere
models

Fig 11.19 for sophisticated optical-model
comparisons

11.10 Compound Nucleus Reactions



Note how a enters the nuclear core and knocks other nuclei around, putting them into higher energy levels. This intermediate state is called a compound nucleus.

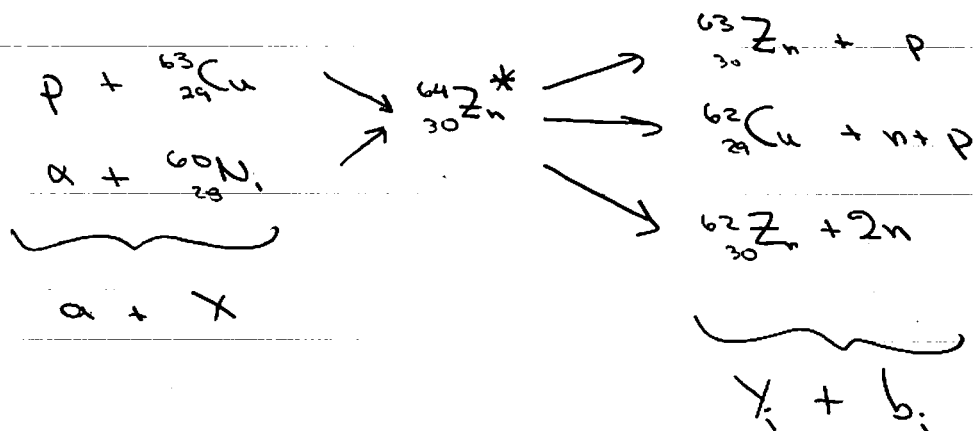
Symbolically the reaction is



in this model the reaction products lose track of the nature of how C^* was created.

Consequence: b_i is isotropic
different a, X to create C^*
decay the same

Specific example



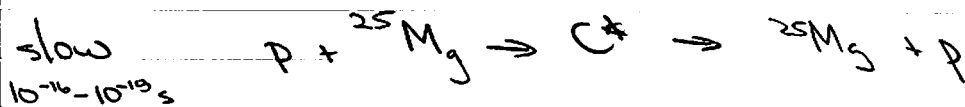
Show Fig. 11.19
p. 417

Fig. 11.19 shows how different ways of creating ${}^{64}\text{Zn}^*$ leads to (mostly) the same cross sections for the 3 decay processes.

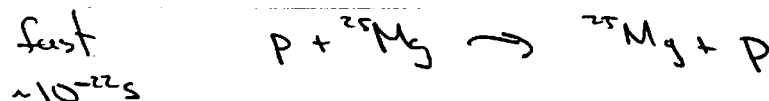
Show Fig. 11.20

Fig. 11.20 shows the isotropic nature of the process ${}^{25}\text{Mg}(p,p){}^{25}\text{Mg}$ at 6 MeV that has both a compound-nucleus channel, and a direct reaction channel (knockout reaction).

Compound reaction



Direct reaction



Energy range of importance

The compound reaction works best in the regime 10-20 MeV where the internal collisions stay within the nuclear core

(268)

← effective radius of the projectile

$$\lambda = \frac{1}{k} = \frac{\hbar}{p} = \frac{\hbar c}{\sqrt{2mc^2 T}}$$

$$\approx 1 \text{ fm for a } 20 \text{ MeV } p \text{ or } n$$

Large l compound reactions



estimating maximum l

$$\hbar l_{\max} = p_{\alpha} R$$

↖ radius of the nucleus

$$l_{\max} = \frac{p_{\alpha} R}{\hbar} = \frac{\sqrt{2m_{\alpha} T_{\alpha}} R}{\hbar}$$

$$= \frac{\sqrt{2mc^2 T_{\alpha}} R}{\hbar c}$$

$$\approx 7 \text{ for } T_{\alpha} = 5 \text{ MeV}$$

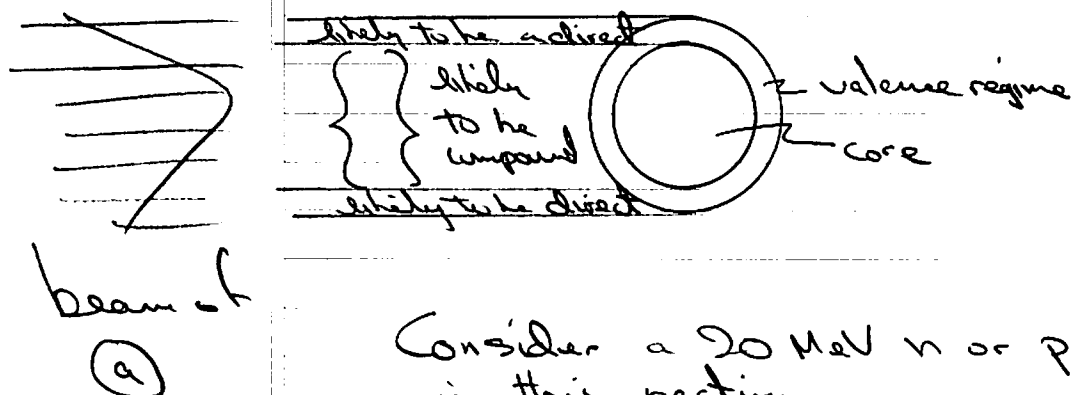
$X(\alpha, xn)Y$ reactions

$$x = 1, 2, 3, \dots$$

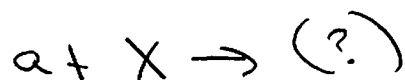
As $T_{\alpha} \uparrow$ x gets larger

See fig. 11.21
p. 419

11.11 Direct reactions



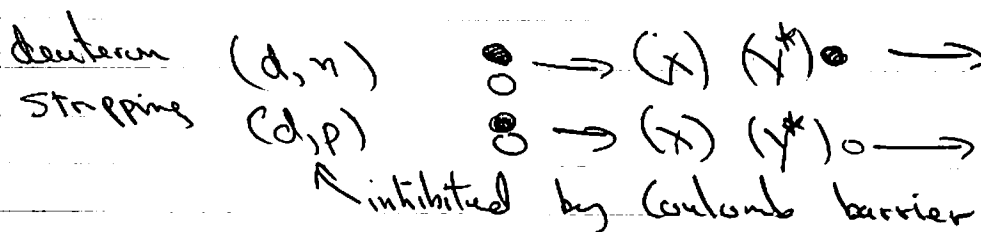
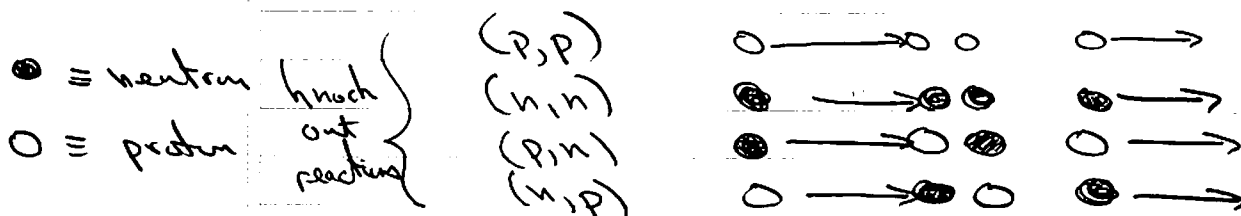
Consider a 20 MeV n or p as (a) in this reaction

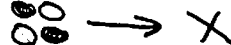


$X \cong 1 \text{ fm} \Rightarrow$ likely to interact with a single nucleon

isotropic (slow) { If this (a) is directed at the core the reaction is likely to be a compound one. (many collisions in the core) $10^{-16} - 10^{-19} \text{ s}$

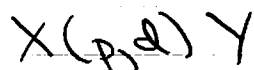
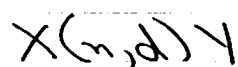
forward peaked (fast) { If (a) is directed at the valence region, there tends to be a direct single event reaction with the valence nucleons. $\sim 10^{-22} \text{ s}$



α -stripping

less likely
3 nucleons $\rightarrow$
valence shells

pick up reactions

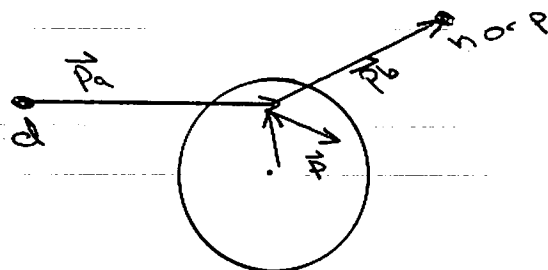


and so on.

All these reactions yield interesting info
on the shell structure of the excited states
of Y^*

Angular momentum in a deuteron stripping reaction

(d,n) or (d,p) reaction



$\vec{p}$ is the momentum
of the recoil
nucleus ($X +$ captured
nucleon)

$$\vec{p}_d = \vec{p} + \vec{p}_b$$

$$\vec{p}_d - \vec{p}_b = \vec{p}$$

$$p^2 = p_d^2 + p_b^2 - 2p_d p_b \cos\theta$$

using
angular
momentum
to isolate

$$1 - \cos \theta =$$

$$1 - \cos(\frac{\theta}{2} + \frac{\theta}{2}) =$$

$$1 - [\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}]$$

$$= 1 - [1 - 2\sin^2 \frac{\theta}{2}]$$

$$= 2\sin^2 \frac{\theta}{2}$$

$$\begin{aligned} p^2 &= p_a^2 + p_b^2 - 2p_a p_b + 2p_a p_b - 2p_a p_b \cos \theta \\ &= (p_a - p_b)^2 + 2p_a p_b (1 - \cos \theta) \\ &= (p_a - p_b)^2 + 4p_a p_b \sin^2 \frac{\theta}{2} \end{aligned}$$

$$\text{if } p_a \approx p_b$$

$$p = 2\sqrt{p_a p_b} \sin \frac{\theta}{2}$$

$$\frac{pk}{R} = 2\sqrt{p_a p_b} \sin \frac{\theta}{2}$$

$$l = \frac{R}{\hbar} 2\sqrt{p_a p_b} \sin \frac{\theta}{2}$$

e.g. $^{90}\text{Zr}(d,p)^{91}\text{Zr}$
 $Q = 5 \text{ MeV}$

for $T_a = 5 \text{ MeV}$

$p_a, p_b \approx 140 \text{ MeV}/c$

$$l \approx 8 \sin \frac{\theta}{2}$$

$$l$$

$$0$$

$$1$$

$$2$$

$$3$$

$$\ominus$$

$$0^\circ$$

$$14^\circ$$

$$29^\circ$$

$$44^\circ$$

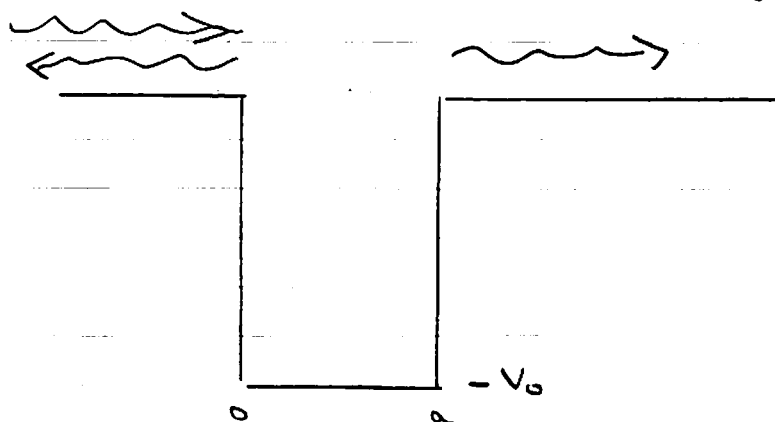
} expect to find the outgoing nucleus at this angle

These can be used to find spin and parity assignments of the excited states of ^{91}Zr

See Fig 11.23

11.12 Resonance reactions

Recall the result we had in 311
on scattering from a finite well
(assignment 4 in 311)



Set $V_0 = 40 \text{ MeV}$ $\left\{ \text{typical values} \right\}$
 $a = 7 \text{ fm}$
 $m = \text{neutron}$

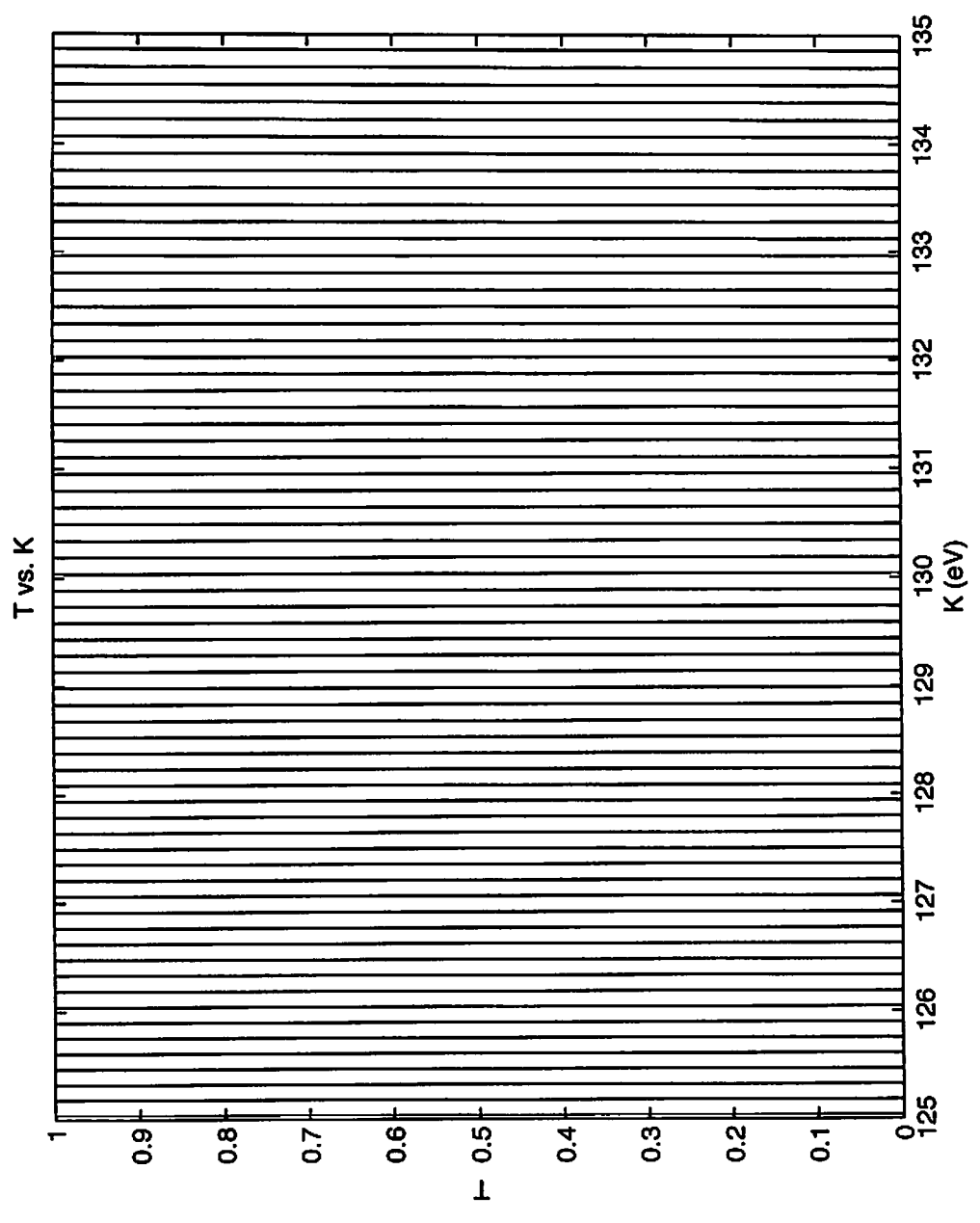
These show
Transmission
(T) vs
K in eV

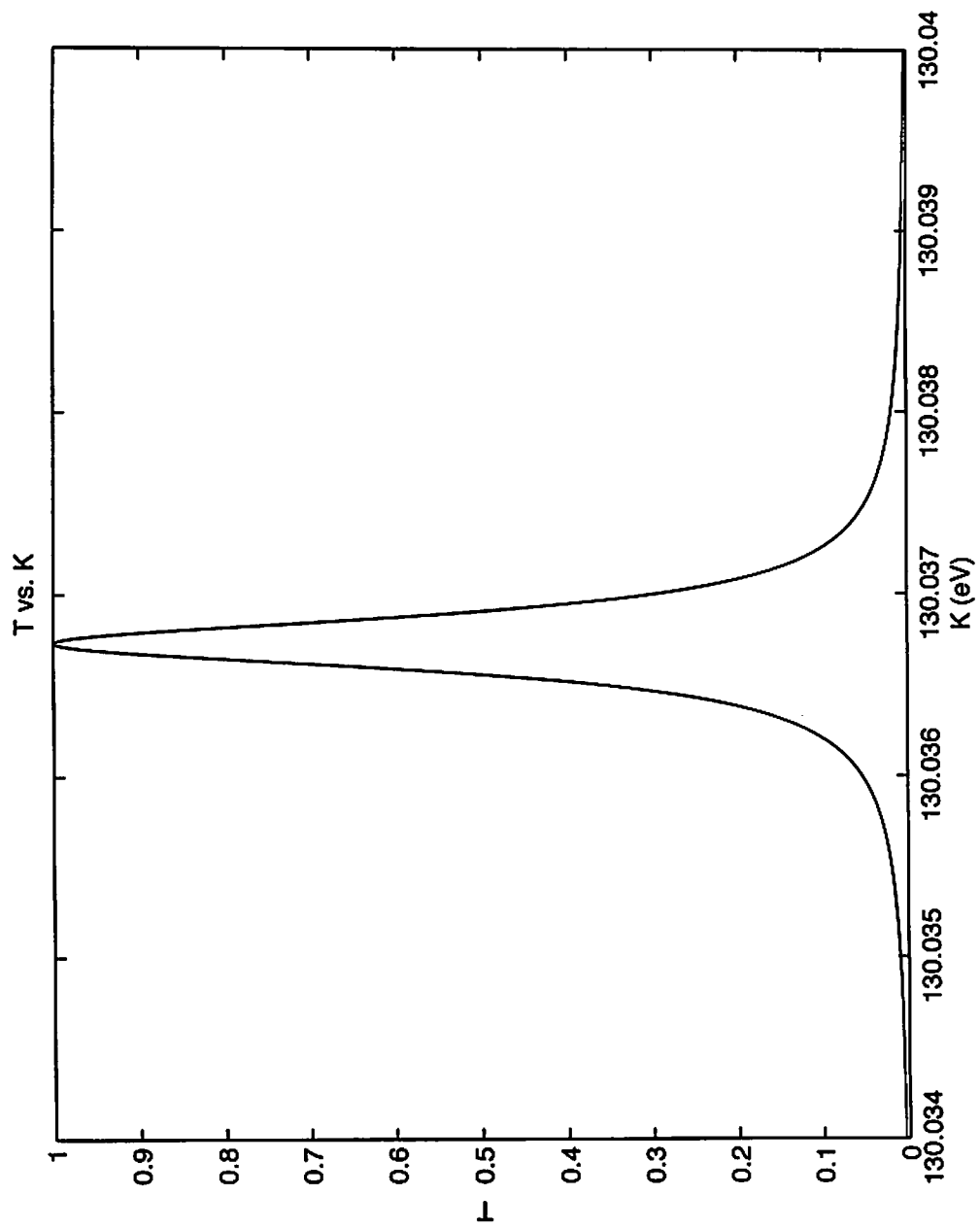
See Fig 1, note the densely packed resonance peaks. Note how many there are!

Fig. 2 shows the structure of just one peak. Note how sharp and narrow it is

(note: pp ⑦ → ⑪ come from the previous lecture, hence the page numbering is off.

Fig. 1





11.12 Resonance Reactions

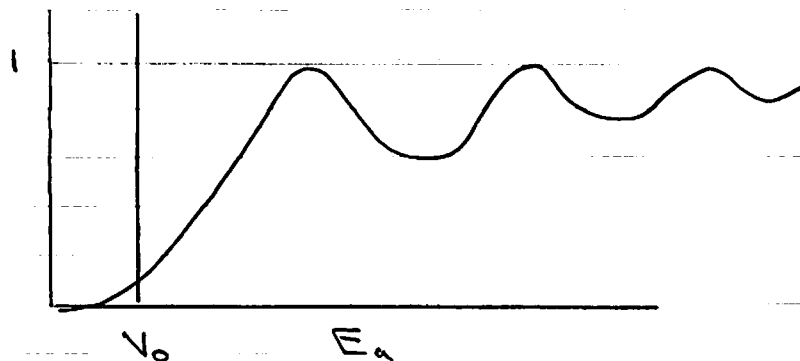
Cross sections in a 3D analysis are analogous to T and R in a 1D analysis.

The Transmission coefficient T is like the forward scattering cross section

$$\frac{d\sigma}{d\Omega}(\theta=0^\circ)$$

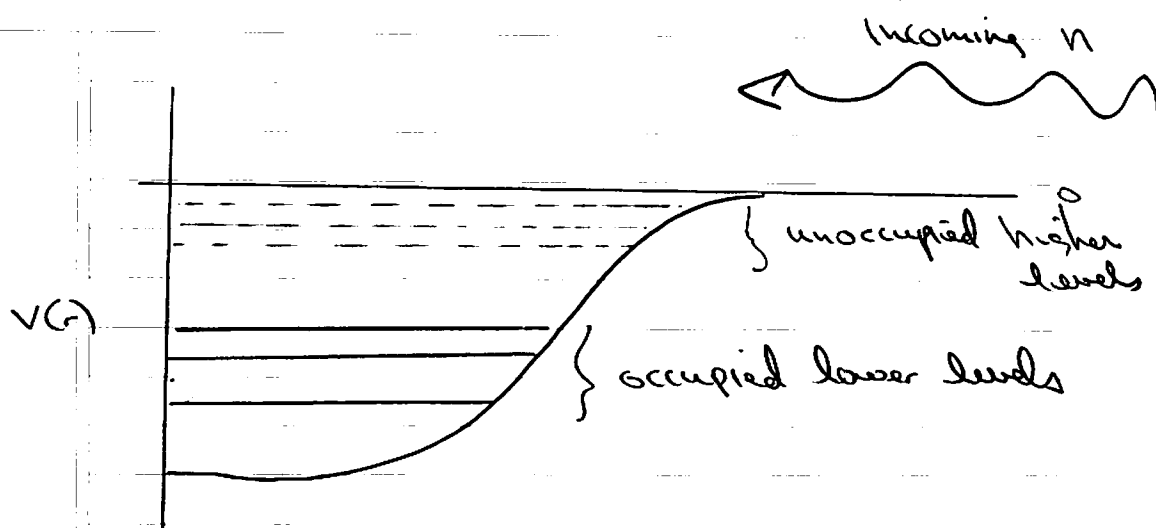
while the Reflection coefficient is like the backscattering cross section.

In 1D we saw that T had this type of behavior



The same sort of behavior is seen in 3D systems.

Imagine the reaction of a 3D nucleus to an incident neutron (no coulomb barrier)



The incoming unbound n can "ring" the unoccupied levels by having an energy that closely matches the frequency of one of the unbound states. This "ringing" results in a dramatic increase in cross section.

We can use our partial wave analytic technique to characterize this

$$\text{From (11.40)} \quad \sigma_T = \sum_{l=0}^{\infty} 2\pi \chi^2 (2l+1) (1 - \text{Re} \eta_l)$$

This is a low energy phenomenon, so, χ is huge. e.g. $n + {}^{59}\text{Co}$ $E_n = 130 \text{ eV}$

$$\begin{aligned} \chi &= \frac{\hbar c}{\sqrt{2mc^2 T}} = \frac{200 \text{ MeV fm}^2}{\sqrt{2000 \text{ MeV} \times 140 \times 10^6 \text{ MeV}}} \\ &= \frac{2000 \text{ fm}}{\sqrt{280}} \approx 400 \text{ fm} \end{aligned}$$

This is enormously larger than any nuclear dimension

In (11.40) we imagine that just one η_e is causing the resonance.

In (11.40) σ_x maximizes when $\text{Re } \eta_e = -1$

So, let's imagine that the process is elastic and set $\eta_e = -1$

From $\eta_e = -1 = e^{2i\delta_e}$

we see that $\delta_e = \frac{\pi}{2}$

The shape of the resonance can be obtained by expanding the phase shift in the vicinity of $\delta_e = \pi/2$

This is done as follows

Let $\cot \delta_e$ be a function of E
where $\delta_e(E=E_r) = \frac{\pi}{2}$

$\cot \delta_e(E) = \cot \delta_e(E_r) + (E-E_r) \frac{\partial \cot \delta_e}{\partial E} \bigg|_{E=E_r}$

+ higher order terms

$\frac{\partial \cot \delta_e}{\partial E} = -\frac{\partial \delta_e}{\partial E} - \frac{\cos 2\delta_e}{\sin^2 \delta_e} \frac{\partial \delta_e}{\partial E}$

Define $\Gamma = 2 \left(\frac{\partial \delta_e}{\partial E} \right)^{-1} \bigg|_{E=E_r}$

$$\cot \delta_2 = - \frac{E - E_R}{\Gamma/2} \quad (11.63)$$

Since

$$\sin \chi = \frac{1}{\sqrt{1 + \cot^2 \delta_2}}$$

$$\sin \delta_2 = \frac{\Gamma/2}{[(E - E_R)^2 + \Gamma^2/4]^{1/2}} \quad (11.64)$$

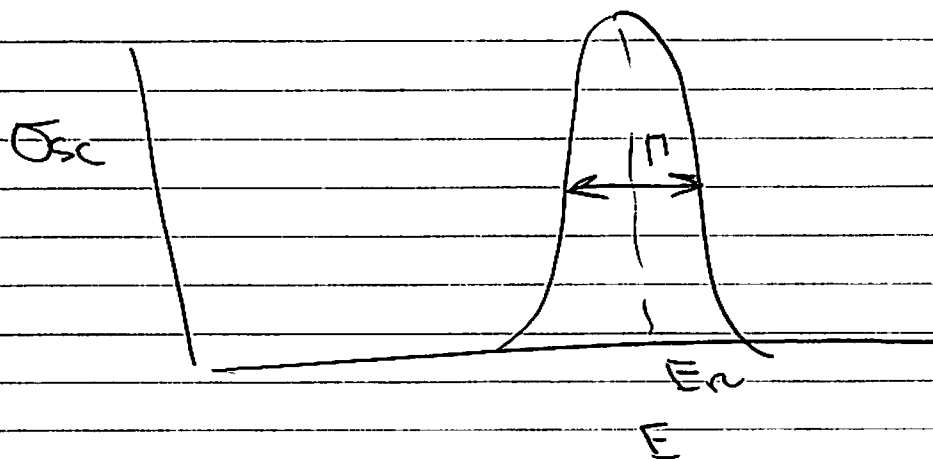
And from (11.45)

$$\sigma_{sc} = 4\pi \chi^2 (2l+1) \sin^2 \delta_2$$

$$\sigma_{sc} = \pi \chi^2 (2l+1) \frac{\Gamma^2}{(E - E_R)^2 + \Gamma^2/4} \quad (11.65)$$

Krone uses

$$\chi = \frac{1}{k}$$



note that

$$\sigma_{sc}(E=E_R) = 4\lambda^2(2l+1)$$

and this can be huge!

Generalizations

1) If $(a) \div (x)$ have intrinsic spins

$$(2l+1) \equiv g = \frac{2I+1}{(2S_a+1)(2S_x+1)} \quad (11.67)$$

note that $g = (2l+1)$ when $S_a = S_x = 0$

2) If the resonance has more than one way to decay (other than the elastic channel)

$$\Gamma = \sum_i \Gamma_i \quad \uparrow \text{partial widths}$$

$$\Gamma = \frac{\hbar}{\tau} \quad \uparrow \equiv \text{mean life of the resonance}$$

Show Figure 11.26 on p. 427

3) Generalizing to inelastic events

Let Γ_{ax} represent the elastic channel

$$a + X \rightarrow a + X$$

and Γ_{by} represent an elastic channel

$$\sigma_{sc} = \pi \lambda^2 g \frac{\Gamma_{ax}^2}{(E - E_R)^2 + \Gamma^2/4} \quad (11.69)$$

$$\sigma_{in} = \pi \lambda^2 g \frac{\Gamma_{ax} \Gamma_{by}}{(E - E_R)^2 + \Gamma^2/4} \quad (11.70)$$

$$\Gamma_{by} = \sum_i \Gamma_{by_i} \quad \text{if more than one way}$$

$$\Gamma = \Gamma_{ax} + \Gamma_{by}$$

(11.69) & (11.70) are known as the Breit-Wigner formulas for isolated resonances.

potential scattering and resonance

These resonances do not occur in isolation.

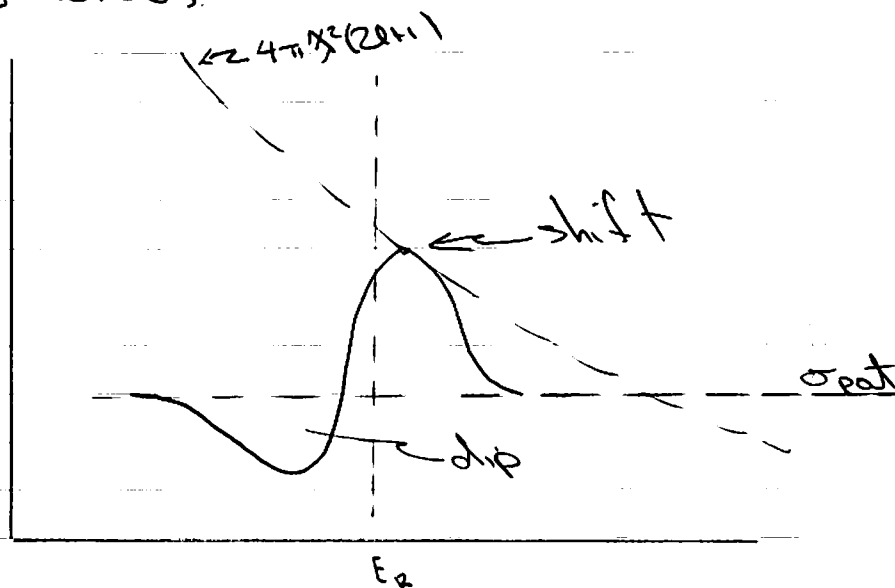
A neutron not only has an amplitude for resonance scattering, but also might have an amplitude associated with a direct interaction with one of the valence nucleons. This is called "potential" scattering or "shape-elastic" scattering. This amplitude competes (destructive interference) with resonant scattering.

It is modelled as

$$\eta_L = e^{2i(\delta_{0R} + \delta_{0P})}$$

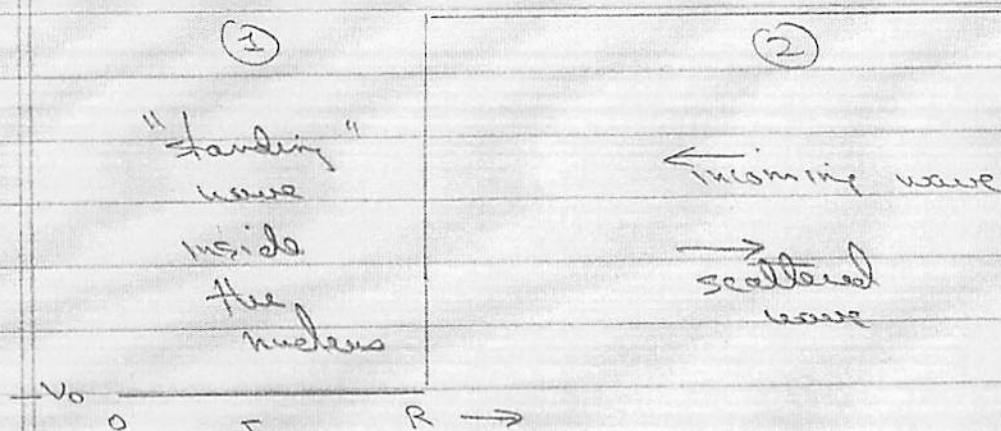
$\uparrow$ $\uparrow$
 resonant potential

The effect is to show the resonant peak as follows.



The "dip" and the "shift" are caused by interference

Some notes on Resonance Scattering



This is a 3D problem. Let look at elastic scattering

Consider the s-wave ($l=0$) solns to the SE:

$$u'' + k^2 u = 0 \quad (\text{recall } u = rR(r))$$

Region 1

$$u_1 = A \cos k'r \quad (1)$$

$$k'^2 = \frac{2m(E + V_0)}{\hbar^2}$$

only this soln
for 0 slope at $r=0$

Region 2 $u_2 =$

$$k^2 = \frac{2mE}{\hbar^2}$$

$$B \left[\underset{\substack{\uparrow \\ \text{incoming}}}{e^{-ikr}} + e^{i2\delta_0} \underset{\substack{\uparrow \\ \text{outgoing}}}{e^{ikr}} \right] \quad (2)$$

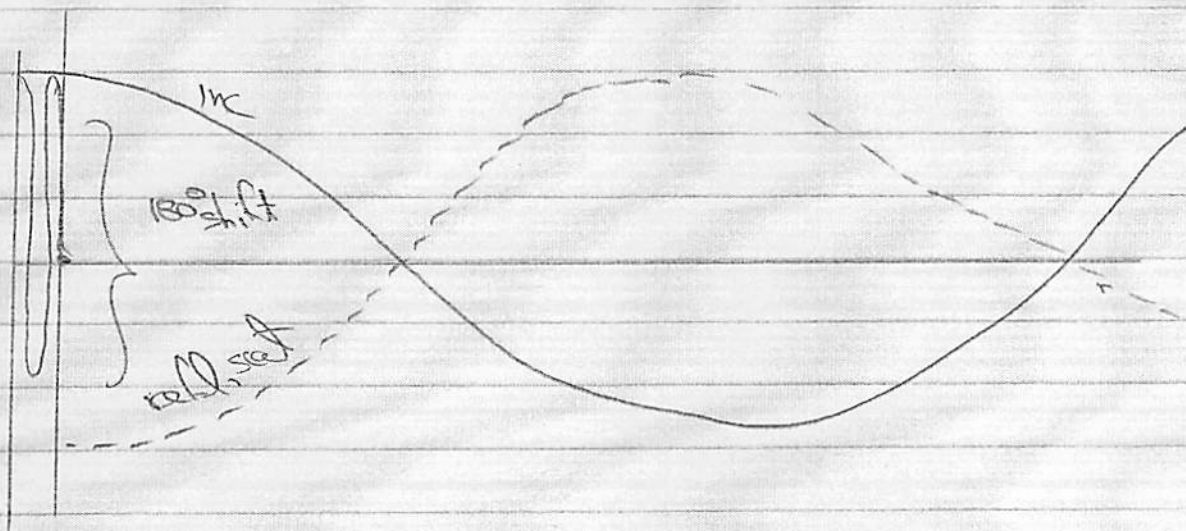
$$\eta_0 = e^{i2\delta_0} \Rightarrow |\eta_0|^2 = 1$$

elastic scattering

$$\therefore u_2 \approx B \left[e^{-ikr} + \underbrace{e^{i\pi}}_{=-1} e^{ikr} \right]$$

$\Rightarrow 180^\circ$ shift

Incoming & reflected waves



That was for $l=0$, what about the rest?

Far away from the nucleus, the centrifugal term

$$\left[\nabla^2 - \frac{l(l+1)}{r^2} + k^2 \right] \psi_l = 0$$

$\underbrace{\quad}_{\text{this vanishes}}$

So all $l=0$'s $\Rightarrow \frac{\pi}{2}$

From our previous expression, at resonance

$$\sigma_{sc} = \pi \lambda^2 \sum_{l=0}^{\infty} (2l+1) |1 - \eta_l|^2$$

elastic

maximum l $L\lambda = R$

→ $4\pi \lambda^2 \sum_{l=0}^L (2l+1)$ semi-classical approx

$$= 4\pi \lambda^2 (L+1)^2$$

$$= 4\pi (R + \lambda)^2$$

The behavior in the vicinity of the resonance is due to each partial wave

$$\sigma_{sc}^l = \pi \lambda^2 (2l+1) \frac{\Gamma_l^2}{(E - E_R)^2 + \frac{\Gamma_l^2}{4}}$$

$$\Gamma_l = \frac{2}{\left(\partial \sigma_l / \partial E \right) \Big|_{E=E_R}}$$

12 Neutron Physics

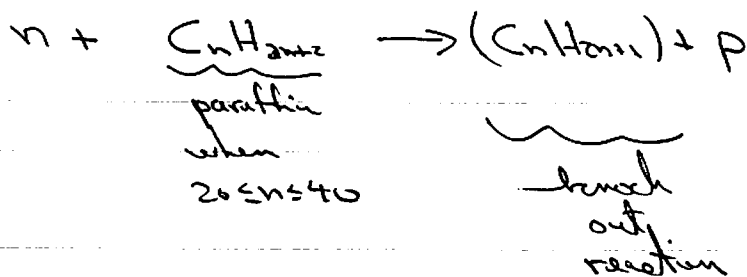
- neutron - the electrically neutral nucleon
- ideal probe of the nucleus at low energies (no Coulomb barrier)
 - (hard to make beams) (hard to detect)

First "discoveries" Bothe & Becker (1930)



undiscovered
penetrating
non-ionizing
radiation
(photon?)

Curie & Joliot



For the n to be a photon, it would have to have unrealistically high energy.

Chadwick (acknowledged as the discoverer) identified it as a neutral particle with about the same mass as a proton.

(note: Rutherford suggested the existence of the n as early as 1920)

free neutrons are unstable

$$n \rightarrow p + e^- + \bar{\nu} \quad \left\{ \begin{array}{l} \tau = 885.7(8) \text{ s} \\ \text{measurable} \\ \text{half-life} \approx \tau/2 \approx 10.6 \text{ m} \end{array} \right.$$

recall $\mu_n = \frac{e\hbar}{2m_p} \uparrow$
proton
mass

$$\begin{array}{ll} \text{magnetic moment} & -1.9130427(5) \mu_N \\ \text{mass} & = 939.565560(81) \text{ MeV}/c^2 \end{array}$$

$$(m_n - m_p)c^2 = [939.565560(81) - 938.272013(23)] \text{ MeV}/c^2$$

$$1.293547(91) \text{ MeV}/c^2$$

$$\text{dipole moment} < 2.9 \times 10^{-26} \text{ e-cm}$$

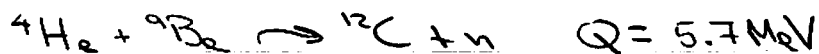
both the magnetic moment and dipole moments would be 0 if it were a point-like particle (like the electron)

12.1 Neutron sources

We can't accelerate neutrons, like we can protons, but we can slow them down. Conventions

name	energy (eV)
thermal	~ 0.025
epithermal	~ 1
slow	$\sim 1 \text{ keV}$
fast	$100 \text{ keV} \rightarrow 10 \text{ MeV}$

α -Beryllium sources (α, n) sources



⊖ mix of ^9Be and
a long-lived
 α -emitter

$\Rightarrow$ long-lived neutron source
(constant rate)

e.g. using ^{226}Ra $t_{1/2} = 1602 \text{ y}$

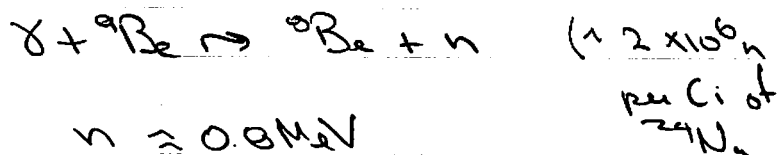
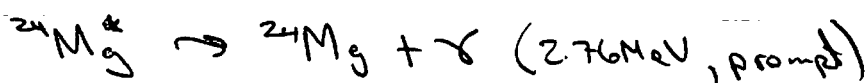
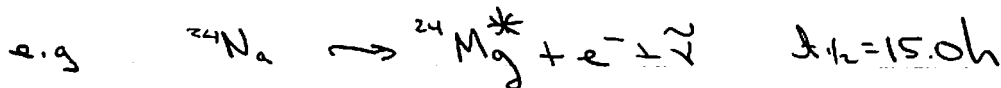
$\nearrow$
 α decay of ^{226}Ra and
its daughters ($\alpha = 5 \rightarrow 8 \text{ MeV}$)
neutrons with spectrum
upto 13 MeV
Most probable energy $\sim 5 \text{ MeV}$

other popular combinations

yield	{	^{210}Po	^{210}Po	^{241}Am
$2-3 \times 10^6$		$\uparrow$	$\uparrow$	$\uparrow$
n per Ci of α activity		130d	86y	450y

1 Ci =
 3.7×10^{10}
decays/s
(exact)

Photon neutron sources (γ, n) sources



$$n \approx 0.8 \text{ MeV}$$

other example

^{124}Sb (60.2 d)

Sb (antimony
 $Z=51$)

γ just exceeds ^9Be binding energy

$n \approx 2.4 \text{ MeV}$

spontaneous fission

$^{252}\text{Cf} \xrightarrow{\text{fission (3\%)}} (2.65 \text{ y}) \approx 4 \text{ neutrons/decay}$

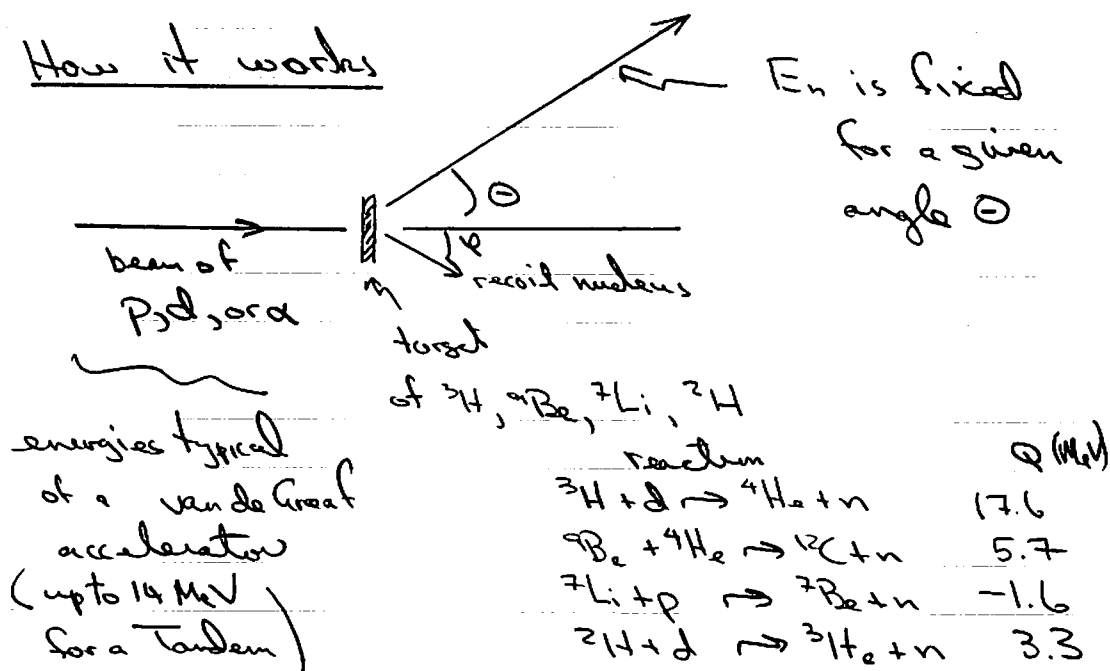
yield $4.3 \times 10^9/\text{s}$ per Ci of ^{252}Cf

n 's have continuous energy distribution
from about 1-3 MeV

nuclear reactions

the only source of monoenergetic neutrons

How it works



Show figs 12.1 & 12.2

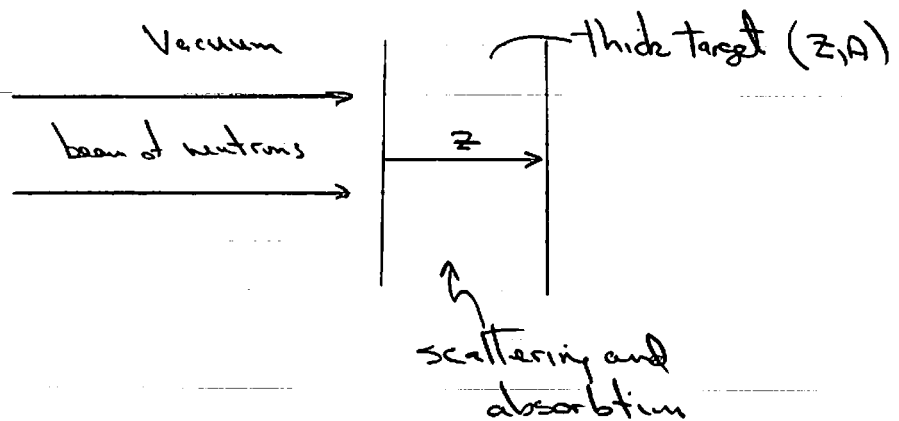
Reactor sources

(highest flux)

as high as 10^{14} n/cm²/s, energy spectrum
 $\rightarrow 5-7$ MeV
 1-2 MeV peak



12.2 Absorption and moderation of neutrons



scattering and absorption are characterized by a total cross section σ_t

Intensity as a function of depth

Beam intensity $I(z)$

$$dI = -I \sigma_t n dz \quad (12.1)$$

$\uparrow$ density of scatterers

$$I(z) = I_0 e^{-\sigma_t n z} \quad (12.2)$$

$\uparrow$ intensity at the surface

Some notes

- 1) n is the density of scatterers. For a pure elemental or molecular target

$$n = \frac{N_A \rho}{M}$$

$$N_A = \text{Avogadro's \#} \\ 6.02214179(30) \times 10^{23} / \text{mole}$$

ρ = mass density

M = molar mass (mass of 1 mole of material)

- 2) (12.2) assumes a monoenergetic beam.

To account for an incoming spectrum,

(12.2) can be modified to

$$I(z) = \int dE I_0(E) e^{-n\sigma_x(E)z} \quad (12.2')$$

- 3) (12.2) and (12.2') only model the decrease of the incoming beam, and not the generation of new neutrons from the target material through e.g. $(n, 2n)$ process

(12.2') can be modified with

$$I(z) = \int dE I_0(E) B(z, E) e^{-n\sigma_x(E)z} \quad (12.2'')$$

↑
build up factor

$B(z, E)$ can be $\gg 1$



$n\sigma$ is sometimes called " Σ " or " μ "
↓
macroscopic cross section
↓
attenuation coefficient

build-up factors are calculated using advanced transport techniques or "Monte Carlo" methods

(288)

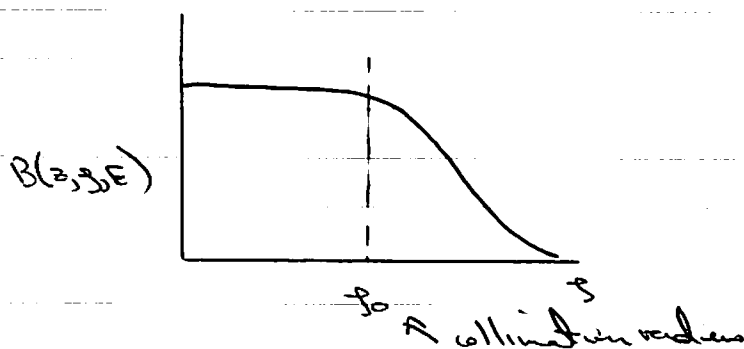
4) (12.2) also assumes that the incoming beam is infinitely broad. Usually it is collimated to a circular shape

Then, (12.2") becomes

$$I(z, r) = \int dE I_0(E, r) B(z, r, E) e^{-\mu_{\text{eff}}(E)z} \quad (12.2'')$$

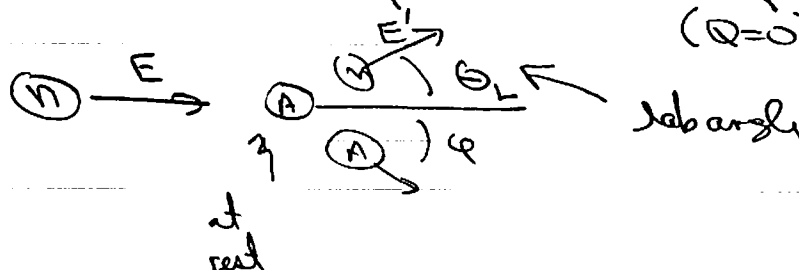
$B(z, r, E)$ "off-axis" build-up factor

typical shape for some E, z



moderation

Consider the following elastic scattering event ($Q=0$)



a relativistic
analysis is fine

$$\text{CoE: } \frac{p^2}{2m_n} = \frac{p'^2}{2m_n} + \frac{p_A^2}{2m_A}$$

$\uparrow$ inc n $\uparrow$ scd n

$$\text{or } \frac{m_A}{m_n} (p^2 - p'^2) = p_A^2$$

$$A(p^2 - p'^2) = p_A^2$$

(1)

Since $\frac{m_A}{m_n} \gg A$

$$\text{CoM } \vec{p} = \vec{p}' + \vec{p}_A$$

$$\text{or } (\vec{p} - \vec{p}') = \vec{p}_A$$

(2)

$$(2)^2 \Rightarrow 1 \Rightarrow A(p^2 - p'^2) = p^2 - 2pp' \cos \theta_L + p'^2 \quad (3)$$

(3) $/p^2$ and reorganize

$$(A+1) \left(\frac{p'}{p} \right)^2 = (A-1) + 2 \left(\frac{p'}{p} \right) \cos \theta_L$$

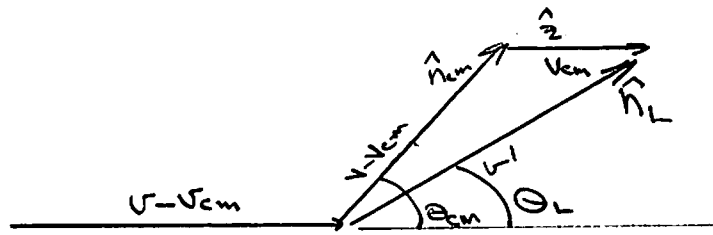
$$\frac{p'^2}{p^2} = \frac{E'}{E}$$

$$\frac{E'}{E} = \frac{1}{(A+1)} \left\{ (A-1) + 2 \left(\frac{p'}{p} \right) \cos \theta_L \right\} \quad (4)$$

Krane uses the CM angle rather than θ_L , so,
obtain the relationship

$$v = \frac{p}{m}$$

$$v' = \frac{p'}{m}$$

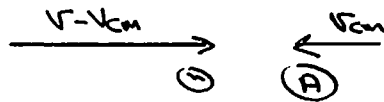


$$(v - v_{cm}) \hat{n}_{cm} + v_{cm} \hat{z} = v' \hat{n}_L \quad (5)$$

$$(5) / v \cdot \hat{z} \Rightarrow$$

$$\frac{p'}{p} \cos \theta_L = \left(1 - \frac{v_{cm}}{v}\right) \cos \theta + \frac{v_{cm}}{v} \quad (6)$$

↑
change



COM in CM frame

$$m_n(v - v_{cm}) = m_A v_{cm}$$

// by $m_n v$

$$1 - \frac{v_{cm}}{v} = A \frac{v_{cm}}{v}$$

$$\frac{v_{cm}}{v} = \frac{1}{A+1}$$

$$1 - \frac{v_{cm}}{v} = \frac{A}{A+1}$$

$$\therefore \frac{p'}{p} \cos \theta_L = \left(\frac{1}{A+1}\right) (A \cos \theta + 1) \quad (5)$$

$$(5) \rightarrow (6) \Rightarrow$$

$$\frac{E'}{E} = \frac{1}{(A+1)^2} \left\{ A^2 - 1 + 2A \cos \Theta + 2 \right\}$$

$$\frac{E'}{E} = \frac{A^2 + 2A \cos \Theta + 1}{(A+1)^2} \quad (12.3)$$

Note $\frac{E'}{E} = 1 \quad \Theta = 0$

$$\frac{E'}{E} = \left(\frac{A-1}{A+1} \right)^2 \quad \Theta = \pi$$

$$\frac{E'}{E} = \frac{1}{2} (1 + \cos \Theta) \quad A=1$$

derived in 311

$$\frac{E'}{E} \rightarrow 1 \quad \text{as } A \rightarrow \infty$$

isotropic, also derived in 311

$$\frac{E'}{E} = \frac{A^2 + 1 + 2A \cos \Theta_{cm}}{(A+1)^2} \quad (12.3)$$

↑

ratio of outgoing
to incoming
neutron
energy in
the lab frame

Θ_{cm} is the
scattering
angle in the
center of
mass

$$\frac{m_A}{m_n} \approx A \quad \text{non-relativistic analysis}$$

In (12.3) $\frac{E'}{E} = 1$ when $\Theta = 0$

$$(12.4) \quad \left(\frac{E'}{E}\right)_{\min} = \left(\frac{A-1}{A+1}\right)^2 \quad \Theta = 180^\circ$$

Note $\frac{E'}{E} = 0$ when $A = 1$ (proton)

Cumulative collision analysis

Cumulative average values

After one collision

$$\frac{E_1}{E_0} = e^{-\xi}$$

ξ is some
parameter
 > 0

After 2 collisions

$$\frac{E_2'}{E_0} = \underbrace{\frac{E_1'}{E_0}}_{e^{-\xi}} \underbrace{\frac{E_1'}{E_0}}_{e^{-\xi}}$$

$$= e^{-2\xi}$$

which assumes that each collision involves a fixed fraction of energy.

So, after n collisions

$$\frac{E_n'}{E_0} = (e^{-\xi})^n$$

$$\log \frac{E_n'}{E_0} = n \log(e^{-\xi})$$

$$= -n\xi$$

$$\log E_0 - \log E_n' = -n\xi$$

$$\log E_0 = \log E_n' - n\xi \quad (120)$$

Estimating ξ

$$\left(\frac{E_1'}{E_0} \right)_{av} = e^{-\xi}$$

$$\xi = \left(\log \frac{E_0}{E_1'} \right)_{av}$$

Using (12.3)

$$\xi = \frac{\int d\Omega \log \left[\frac{(A+1)^2}{A^2+1+2A \cos \theta} \right]}{\int d\Omega}$$

$$\xi = 1 + \frac{(A-1)^2}{2A} \log \left(\frac{A-1}{A+1} \right) \quad (12.8)$$

$$E_0 \approx 2 \text{ MeV}$$

<u>moderator</u>	<u>ξ</u>	<u># collisions to thermalization</u>
^1H	1	18
^2H	0.725	25
^4He	0.425	43
^{12}C	0.158	110
^{238}U	0.0084	2200

Maxwellian Distribution

As the neutrons slow down, the thermal energy of the moderator can no longer be ignored. Eventually the neutrons obtain a final energy distribution given by the Maxwell-Boltzmann distribution

$$f(E)dE = \frac{2}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} E^{1/2} e^{-E/kT} dE \quad (12.10)$$

where T is the absolute temperature and k is the Boltzmann constant

$$k = 8.6174 \times 10^{-5} \text{ eV/K}$$

(12.10) represents the probability of finding the neutron in a dE neighborhood of E .

It is easy to show that

$$\int_0^\infty f(E) dE = 1$$

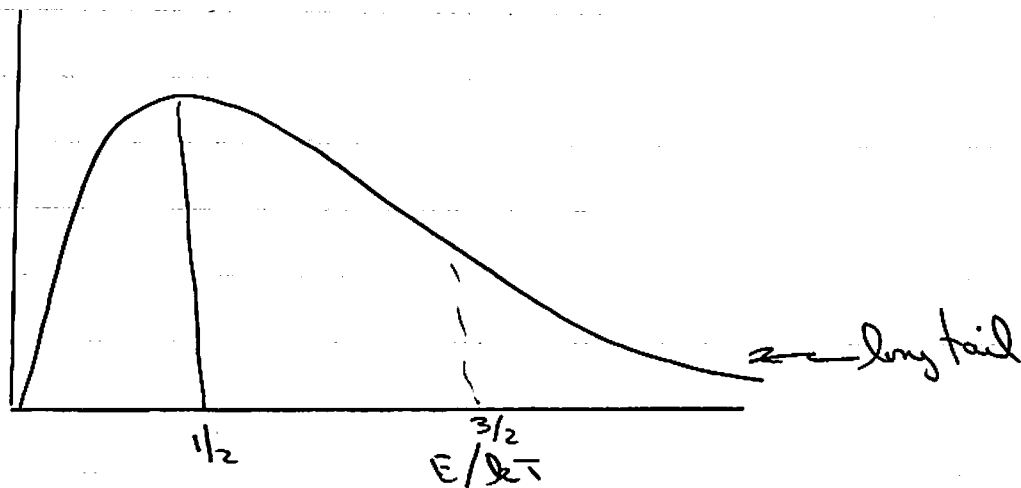
$$\langle E \rangle = \int_0^\infty E f(E) dE = \frac{3}{2} kT$$

$$f \text{ maximizes at } \frac{1}{2} kT$$

$$\langle E^2 \rangle - \langle E \rangle^2 = \frac{3}{2} (kT)^2$$

} very wide distribution

246



6.3 Units for measuring radiation

radioactivity

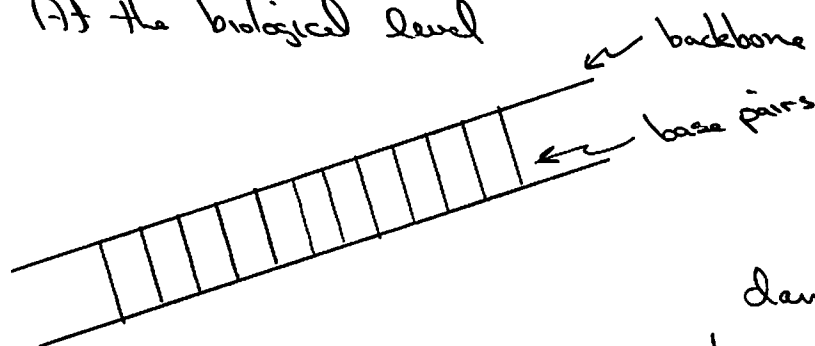
Ci : $1 \text{ Ci} = 3.7 \times 10^{10} \text{ decay/s (exact)}$
(Curie) named after Pierre & Marie Curie

Bq : $1 \text{ Bq} = 1 \text{ decay/s}$
(Becquerel) named after Henri Becquerel
who shared the Nobel Prize for
discovery of radioactivity with
the Curies.

But this is radioactivity.

The desire to measure radiation directly is driven
by the need to measure the effect of radiation on
people and materials.

At the biological level



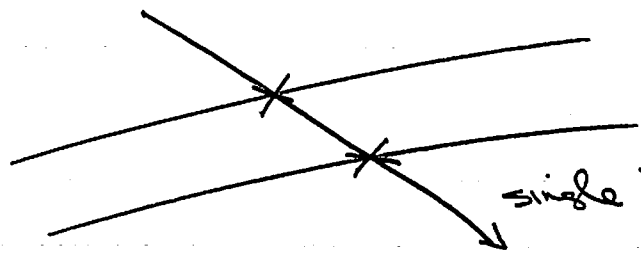
A single hit (called a lesion)
is quickly repaired (within a few min)
A double hit on the backbone
within 10 base pairs can be
lethal or can be repaired

3+ hits within 10 base pairs certainly lethal

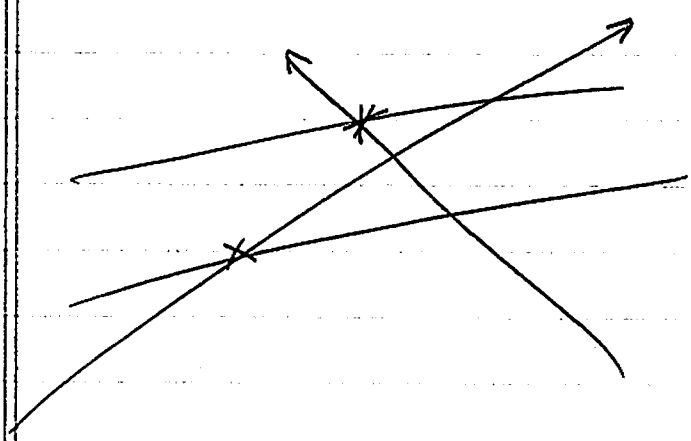
Damage to DNA
happens due to direct
ionization, indirect
ionization (from a
radical created by
ionization)

radiation ionization potential (QF)

γ, β	1
low energy p,n	2-5
high energy p,n	5-10
α	20



single "track" of highly ionizing radiation can be lethal



weakly ionizing radiation lethally caused by multiple tracks.

measuring "damage" $\Rightarrow$ measuring ionization

Exposure symbol X

$$X = \lim_{\Delta m \rightarrow 0} \frac{\Delta Q}{\Delta m}$$

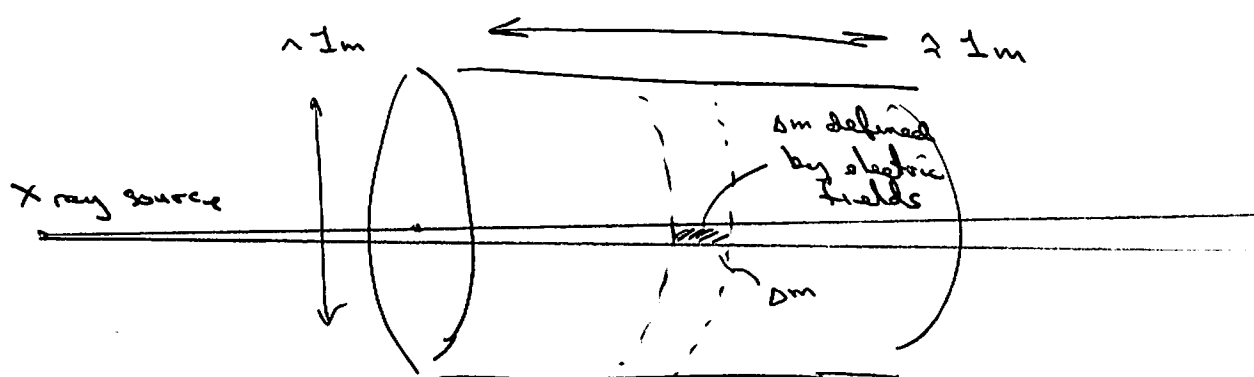
ΔQ is the absolute value of the total charge of ions of one sign produced in air when all the electrons (e^- & e^+) liberated by photons in air of mass Δm are completely stopped in dry air at NTP (1 atm pressure, 20°C).

unit: Roentgen
after

$$1R = 2.58 \times 10^{-4} \text{ C / kg}$$

Wilhelm Conrad Röntgen - produced & detected X rays

measured directly & absolutely in a free air chamber



these measurements
are $\ll 0.1\%$ accuracy

Dose Symbol "D"

Energy deposited per unit mass

$$\frac{dE}{dm}$$

units

rad

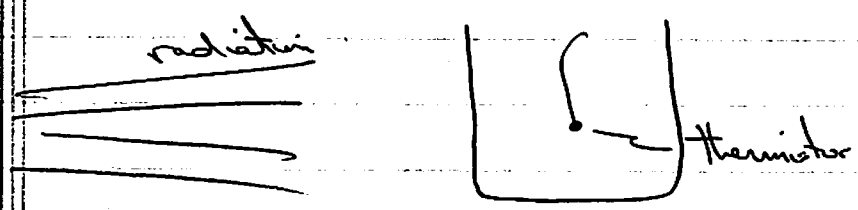
Gy

$$1 \text{ Gy} = 1 \text{ J/kg}$$

$$1 \text{ Gy} = 100 \text{ rad}$$

←
Louis Harold Gray
(dose measurement pioneer)

direct ^{absolute} measurement of dose to water



these measurements are good to
about 1%.

Biological effectiveness Symbol "DE"

$$DE = D \times QF$$

↑

quality factor

hormesis?

ab initio theories of radiation damage?
