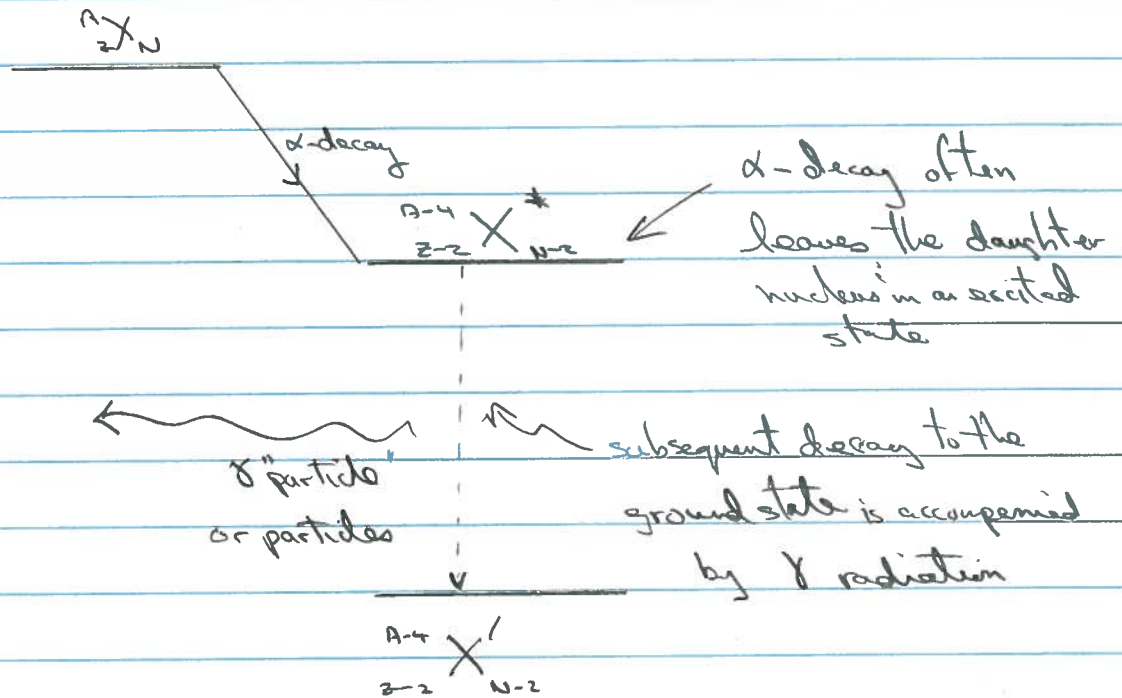
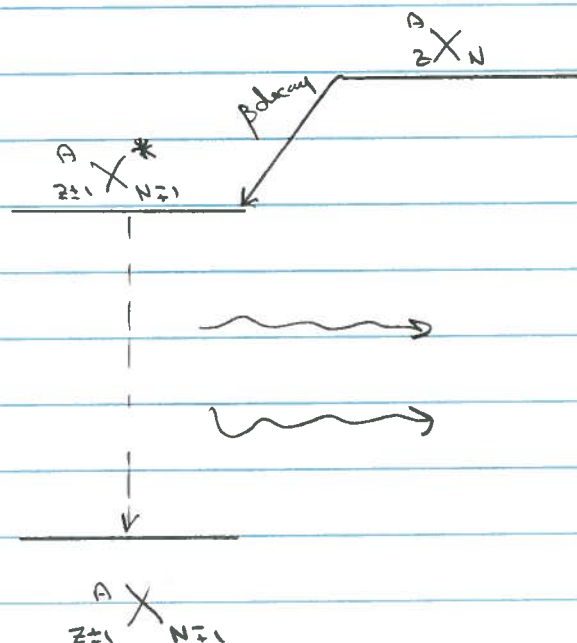


Chapter 10 Gamma Decay

So far, we studied α -decay



Like wise, for β -decay



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These γ decay energies are typically $0.1 \rightarrow 10 \text{ MeV}$, with half lives $> 10^{-19} \text{ s}$, usually much longer 10^{-12} is much more common.

γ spectroscopy yields the most precise knowledge that we have about nuclear structure, as spin, parity, ΔE are all measurable.

Borrowing from our knowledge of atomic physics, the nuclear wave function, expressed as a product of single-nucleon wavefunctions, is

$$\Psi_N = \prod_{a=1}^A \langle l_a, s_a, t_a \rangle \Psi_a(l_a, s_a, t_a)$$

$\uparrow$
 $t_a = n \text{ or } p$

and transitions are described by Fermi's Golden rule #2

$$\lambda = \frac{2\pi}{\hbar} \left| \langle \Psi'_N \Psi_f | O_{em} | \Psi_N \rangle \right|^2 \frac{dn_f}{dE_f}$$

$\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$

daughter
nuclear
wavefunction photon
wavefunction
(well known) parent
nuclear
wavefunction

decay rate
measured or
calculated

em Transition
operator
(well-known)

γ 's & O_{em} are well known, therefore measurements of λ provide detailed knowledge of the nuclear wavefunctions.

Theorists attempt to model the γ 's. The degree to which they agree with experiments tells us how good the models are, and hence, gives us cause to believe that we understand something about nuclear structure.

A gamma-decay lifetime of 10^{-12} s or so, even 10^{-17} seconds is an eternity in the life of a nucleus.

Estimate the time for a nucleus to cross the nucleus.

$$\Delta t = \frac{2R}{v}$$

$$\frac{1}{2} m_n v^2 = E \quad \text{classical OK}$$

$$v = (2E/m_n)^{1/2}$$

$$\Delta t = \frac{2R}{(2E/m_n)^{1/2}} = R \sqrt{\frac{2m_n}{E}}$$

$$= \frac{R}{c} \sqrt{\frac{2mc^2}{E}}$$

$$= \frac{7 \text{ fm}}{3 \times 10^{23} \text{ fm/s}} \sqrt{\frac{2000 \text{ MeV}}{7.5 \text{ MeV}}}$$

$$\approx 4 \times 10^{-22} \text{ s}$$

Why does γ decay take so long?

The excited nucleus, even under the favorable conditions for decay, crosses the nucleus 1000s of times before settling down! [Thinking classically, of course]

There are several reasons.

a) For $0.1 \text{ MeV} < E_\gamma < 10 \text{ MeV}$

$$\text{wavelength } \lambda_p = \frac{h}{p} = \frac{2\pi\hbar}{p} = \frac{2\pi\hbar c}{cp} = \frac{2\pi\hbar c}{E_\gamma}$$

$$100 \text{ fm} \leq \lambda_p \leq 10^4 \text{ fm}$$

$$\lambda_p \gg R_N, \text{ little overlap}$$

b) γ carries off at least 1 unit of angular momentum $\Rightarrow$ there is some degree of reorganization in the nucleus, $\psi_{X'} \neq \psi_X$

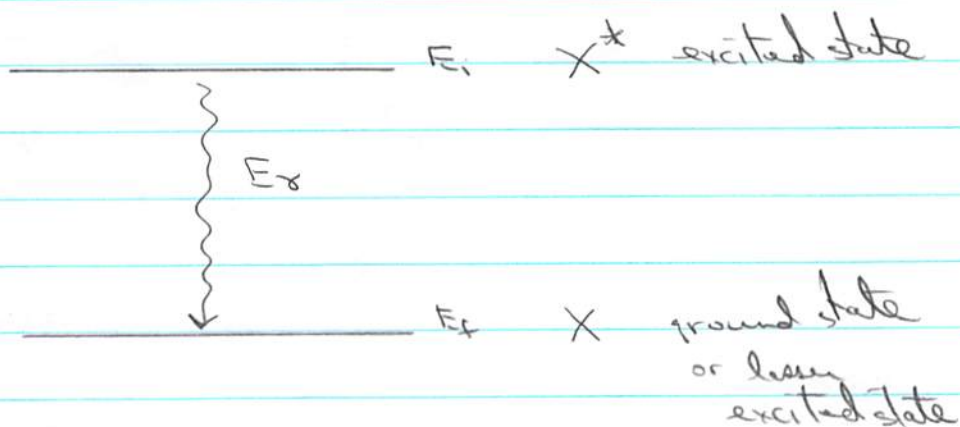
c) The electromagnetic force is relatively weak compared to the strong force

$$\text{e.g. } \frac{\text{Nuclear pin force}}{\text{e } \gamma \text{ force}} \approx 100$$

em forces are 100 times weaker

10.1 The energetics of γ -decay

$$Q = E_i - E_f = E_\gamma + T_x$$



How do we relate E_γ to Q ?

We must account for recoil,

Do a relativistic analysis. It's easier!
relativistic factor

$$\text{CoE: } M^*c^2 = Mc^2\gamma + E_\gamma$$

$$\text{or } M^*c^2 - E_\gamma = Mc^2\gamma \quad (1)$$

$$\text{CoM: } E_\gamma = Mc^2\beta\gamma \quad (2)$$

relativistic factors

$$(1)^2 - (2)^2 \Rightarrow$$

$$(M^*c^2)^2 - 2M^*c^2E_\gamma + \cancel{E_\gamma^2} - \cancel{E_\gamma^2} = (Mc^2)^2(\gamma^2 - \gamma^2\beta^2)$$

$\underbrace{\gamma^2 - \gamma^2\beta^2}_{=1}$

$$(M_c^* c^2)^2 - (M c^2)^2 = 2 M c^2 E_\gamma$$

Since $Q = M_c^* c^2 - M c^2$

$$Q (2 M c^2 + Q) = 2 (M c^2 + Q) E_\gamma$$

$$E_\gamma = \frac{Q (2 (M c^2 + Q) - Q)}{2 (M c^2 + Q)}$$

$$E_\gamma = Q \left[1 - \frac{Q}{2 (M c^2 + Q)} \right] \quad (3)$$

$Q \ll M c^2$, so

$$(4) \quad E_\gamma = Q \left[1 - \frac{1}{2} \frac{Q}{M c^2} + \frac{1}{2} \left(\frac{Q}{M c^2} \right)^2 - \frac{1}{2} \left(\frac{Q}{M c^2} \right)^3 + O\left(\left(\frac{Q}{M c^2}\right)^4\right) \right]$$

Krone's non relativistic expression

$$(5) \quad E_\gamma = M c^2 \left[\left(1 + \frac{2Q}{M c^2} \right)^{1/2} - 1 \right]$$

$$(6) \quad \rightarrow Q \left[1 - \frac{Q}{2 M c^2} + \frac{1}{2} \left(\frac{Q}{M c^2} \right)^2 - \frac{5}{8} \left(\frac{Q}{M c^2} \right)^3 + O\left(\left(\frac{Q}{M c^2}\right)^4\right) \right]$$

④ - ③ gives us the relativistic correction

$$\Delta(E_{\gamma}^{\text{rel}} - E_{\gamma}^{\text{non-rel}}) = Q \left(\left(\frac{Q}{Mc^2} \right)^3 \right) \underbrace{\left(\frac{5}{8} - \frac{1}{2} \right)}_{\frac{1}{8}}$$

$$\frac{\Delta(E_{\gamma}^{\text{rel}} - E_{\gamma}^{\text{non-rel}})}{Q} = \frac{1}{8} \left(\frac{Q}{Mc^2} \right)^3$$

$$\approx \frac{1}{8} \frac{1}{(931.502)^3} \frac{Q^3}{\text{A}^3}$$

$$\approx 1.5 \times 10^{-10} \frac{Q^3 [\text{MeV}]}{\text{A}^3}$$

In the very worst case ($Q = 10 \text{ MeV}$)

$$\frac{\Delta(E_{\gamma}^{\text{rel}} - E_{\gamma}^{\text{non-rel}})}{Q} \approx \frac{1.5 \times 10^{-7}}{\text{A}^3}$$

This is a very small correction, and we can safely ignore it from now on.

(18)

Generally, just the 1st order correction is employed

$$E_r \approx Q \left[1 - \frac{1}{2} \frac{Q}{Mc^2} \right] \quad (7)$$

This also identifies the nuclear recoil

$$Q = E_r + T_r$$

From (7) to first order

$$T_r = \frac{1}{2} \frac{Q^2}{(Mc^2)} \quad (8)$$

These energies are typically 100 eV, strong enough to break chemical bonds and disrupt crystal structure.

The exact expression for T_r , from (3) is

$$T_r = \frac{Q^2}{2(Mc^2 + Q)} \quad (9)$$

10.2 Classical Electro Magnetic radiation

Previously, in the course

we discussed "static" moments in term of gravity that follows the same $1/r$ potential form.

By analogy with the development in that lecture

$$V_{em}(\vec{x}) = \frac{Z_e}{4\pi\epsilon_0} \int_{V_N} d\vec{x}' \frac{\rho_p(\vec{x}')}{|\vec{x} - \vec{x}'|} \quad (1)$$

$\nwarrow$ charge of the nucleus $\nearrow$ proton density
 $\uparrow$
 volume of the nucleus

By convention $\int_{V_N} d\vec{x}' \rho_p(\vec{x}') = 1$ (true charge only) (2)

Expanding as we did before, $(|\vec{x}| \gg |\vec{x}'|)$, we get

$$(3) \quad V_{em}(\vec{x}) = \frac{Z_e}{4\pi\epsilon_0} \left\{ \frac{1}{|\vec{x}|} + \frac{1}{r^2} \hat{z} \cdot \vec{Q}_1 + \frac{1}{r^3} Q_2 + \dots \right\}$$

and so on

(4) where $\vec{Q}_1 = \int d\vec{x}' \vec{x}' \rho_p(\vec{x}') \quad (\text{dipole moment})$

(5) $Q_2 = \frac{1}{2} \int d\vec{x}' (3z'^2 - r'^2) \rho_p(\vec{x}') \quad (\text{quadrupole moment})$

There are higher moments as well, the octupole (Q_3) hexadecapole (Q_4) and so on.

The potential takes the form

$$(6) \quad V_{em} = \frac{Ze}{4\pi\epsilon_0} \frac{1}{|\vec{r}|} \sum_{n=0}^{\infty} \frac{1}{|\vec{r}|^n} Q_n$$

This is called the monopole term

The Q_n 's only depend on the shape of the charge distribution, not $\vec{r}$

In classical em theory, the higher n 's diminish in contribution to V_{em} as $|\vec{r}| \rightarrow \infty$

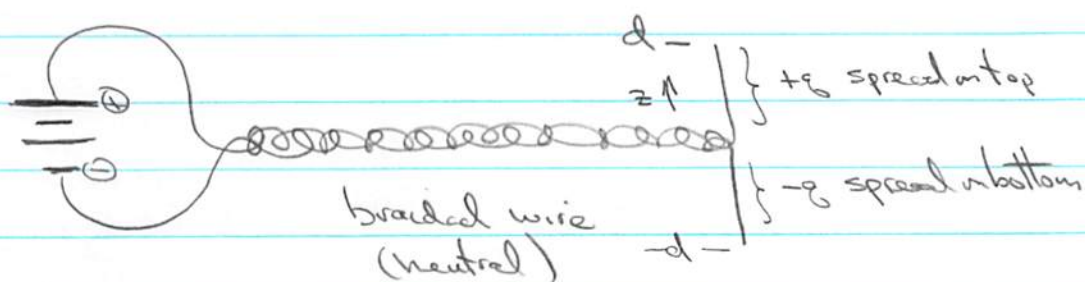
In quantum theory the moments are associated with transitions, that get weaker and weaker as n increases, and are associated with higher degrees of orbital angular momentum associated with the gamma ray emitted.

More generally, if there are charges of two signs

$$V_{em} = \frac{e}{4\pi} \frac{1}{|\vec{r}|} \sum_{n=0}^{\infty} \frac{g_n}{|\vec{r}|^n}$$

where $g_0 =$ net total charge

A simple example of a mixed charge dipole



$$f(\vec{x}) d\vec{x}' = \underbrace{2\pi g d g S(g)}_{\substack{\text{infinitely} \\ \text{thin wire}}} \underbrace{\frac{1}{d} \frac{z}{|z|}}_{\substack{\text{sign of} \\ \text{charge}}} q dz$$

$$\int_0^{2\pi} \int_0^d \int_0^d S(g) dg dz = 1$$

$$q_0 = \frac{q}{d} \int_{-d}^d dz \frac{z}{|z|} \stackrel{\text{odd function}}{=} 0$$

$q_0 = 0$ net charge = 0

$$q_1 = \frac{q}{d} \int_{-d}^d dz \frac{z^2}{|z|}$$

$$= \frac{q}{d} \int_{-d}^d dz |z|$$

$$= \frac{2q}{d} \int_0^d dz z \quad \swarrow \text{even function}$$

$$\underline{q_1 = qd}$$

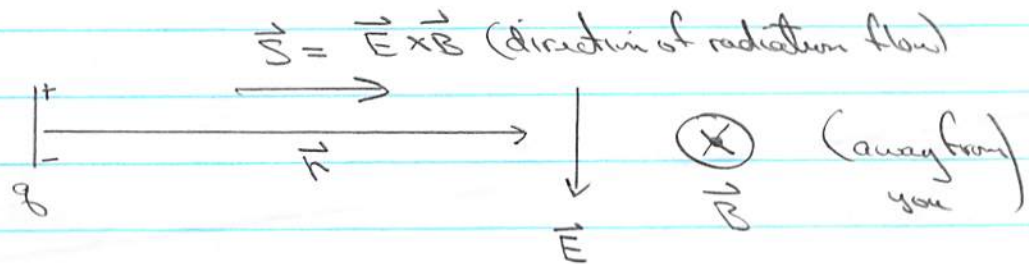
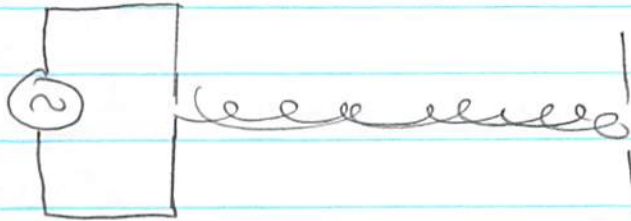
$$\underline{q_2 = 0} \quad \text{in this case}$$

time varying dipoles

electric
dipole

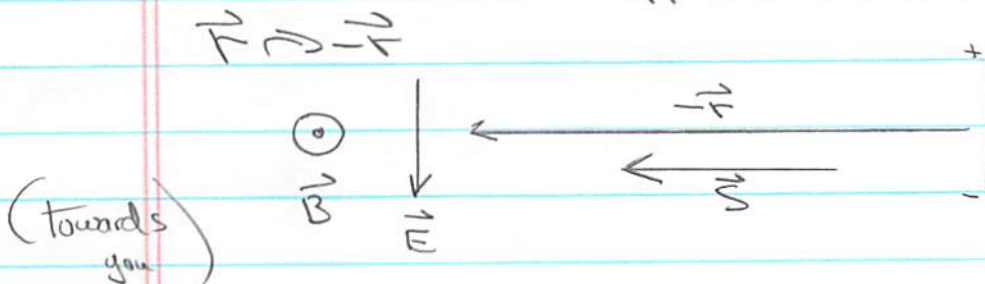
$$q_1 = qd \quad \text{static dipole}$$

$$\text{Set } q_1 = qd \cos \omega t$$



(a test charge moves
in the opposite
direction)

What happens under a parity transformation?



So, under a parity change

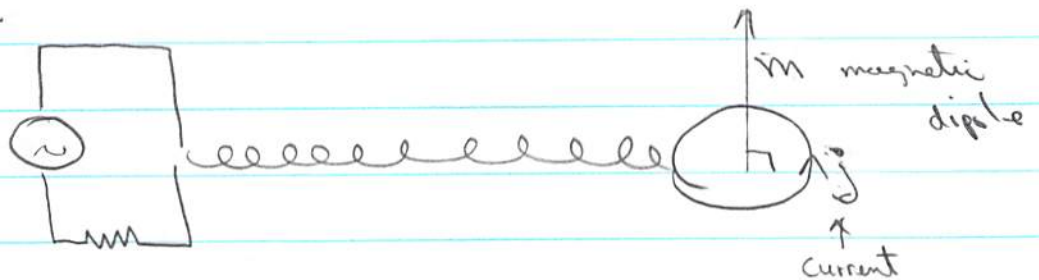
$$\vec{E}(-\vec{r}) \rightarrow \vec{E}(\vec{r})$$

$$\pi = +1$$

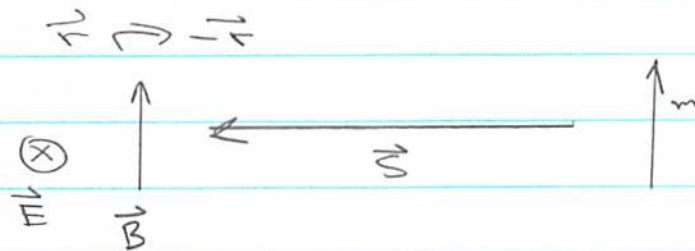
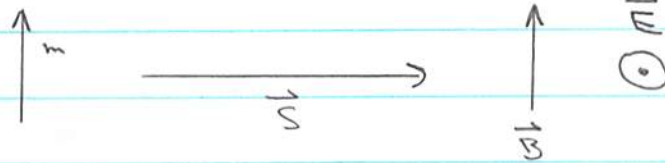
$$\vec{B}(-\vec{r}) \rightarrow -\vec{B}(\vec{r})$$

$$\pi = -1$$

magnetic
dipole



If $m = m_0 \cos \omega t$



So, for magnetic dipoles

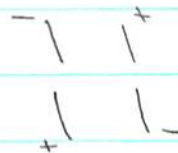
$$\vec{r} \rightarrow -\vec{r} \Rightarrow$$

$$\vec{E}(-\vec{r}) = -\vec{E}(\vec{r}) \quad \pi = -1$$

$$\vec{B}(-\vec{r}) = \vec{B}(\vec{r}) \quad \pi = +1$$

quadrupoles

electric



magnetic



Power radiated by classical dipoles

electric dipole $P_E = \frac{1}{12\pi\epsilon_0} \frac{\omega_0^4 (qd)^2}{c^3}$

magnetic dipole $P_m = \frac{1}{12\pi\epsilon_0} \frac{\omega_0^4 (\mu_0)^2}{c^5}$

$\mu_0 = iA$ ← area of the current loop

Classical multipole radiation

$L \equiv$ multipolarity 1 for dipoles
 2 for quadrupoles
 3 for octupoles & so on

1) Angular distribution (of power radiated) is proportional to

$P_{2L}(\cos\theta)$
 $\nearrow$ regular Legendre Polynomial
 $\nwarrow$ polar angle

Common examples

dipole $L=2$ $P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$

quadrupole $L=4$ $P_4(\cos\theta) = \frac{1}{8}(35\cos^4\theta - 30\cos^2\theta + 3)$

2) $\pi(EL) = (-1)^L$ e.g. $\pi(E1) = -1$ dipole
 $\nearrow$ electric $\nearrow$ multipolarity
 $\pi(E2) = +1$ quadrupole

$\pi(ML) = (-1)^{L+1}$ e.g. $\pi(M1) = +1$
 $\nearrow$ magnetic
 $\pi(M2) = -1$

For an arbitrary EoM 2^L -pole

$$P(\sigma L) \propto \frac{2(L+1)c}{\epsilon_0 L [(2L+1)!!]} \left(\frac{\omega}{c}\right)^{2L+2} [m(\sigma L)]^2$$

↑

$\sigma = EoM$

and more sophisticated
General Treatment of Classical Multipole fields

From advanced $\Sigma \frac{1}{r}$

If $\rho(\vec{r}, t) = \rho(\vec{r}) e^{i\omega t}$
 $\vec{j}(\vec{r}, t) = \vec{j}(\vec{r}) e^{i\omega t}$

charge density (i.e. protons)
 current density (moving
 protons)

$\vec{m}(\vec{r}, t) = \vec{m}(\vec{r})$

intrinsic magnetic
 moments (spins)

The radiation fields are characterized by

electric
 multipoles

$Q_{lm} = \int d\vec{r} |\vec{r}|^l Y_{lm}^* \rho(\vec{r})$

$\frac{-i\omega}{(l+1)c^2} \int d\vec{r} |\vec{r}|^l Y_{lm}^* \vec{\nabla} \cdot (\vec{r} \times \vec{m}(\vec{r}))$

magnetic
 multi-poles

$M_{lm} = -\frac{1}{(l+1)} \int d\vec{r} |\vec{r}|^l Y_{lm}^* \vec{\nabla} \cdot (\vec{r} \times \vec{j})$

$- \int d\vec{r} |\vec{r}|^l Y_{lm}^* \vec{\nabla} \cdot \vec{m}(\vec{r})$

characteristics of classical radiation fields

The power radiated per unit solid angle is

$$\frac{dP}{d\Omega}(\ell, m, [\vec{E}, \vec{M}]) = \frac{2(\ell+1)c}{\epsilon_0 \lambda [(2\ell+1)!!]^2} \left(\frac{\omega}{c}\right)^{2\ell+2} \left| \frac{Q_{\ell m}}{M_{\ell m}} \right|^2$$

$\uparrow$
 electric
or magnetic

$$\left\{ \begin{aligned} &\frac{1}{2}(\ell-m)(\ell+m+1) |Y_{\ell, m+1}|^2 \\ &+ \frac{1}{2}(\ell+m)(\ell-m+1) |Y_{\ell, m-1}|^2 \\ &+ m^2 |Y_{\ell, m}|^2 \end{aligned} \right\}$$

ℓ		m	
1 (dipole)	0	± 1	± 2
	$\frac{3}{8\pi} \sin^2 \theta$	$\frac{3}{16\pi} (1 + \cos^2 \theta)$	$\sin^2 \theta$
		$\uparrow$	
2 (quadrupole)	$\frac{15}{96\pi} \sin^2 \theta \cos^2 \theta$	$\frac{5}{16\pi} (1 - 3\cos^2 \theta + 4\cos^4 \theta)$	
		$\frac{5}{16\pi} (1 - \cos^4 \theta)$	

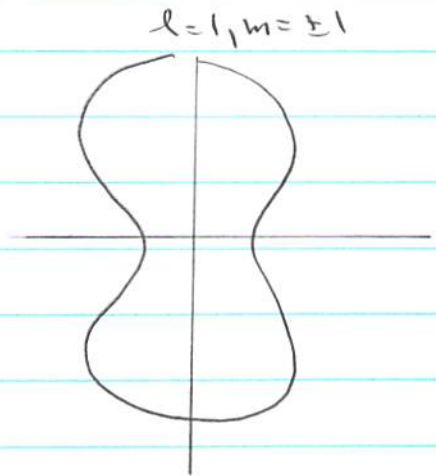
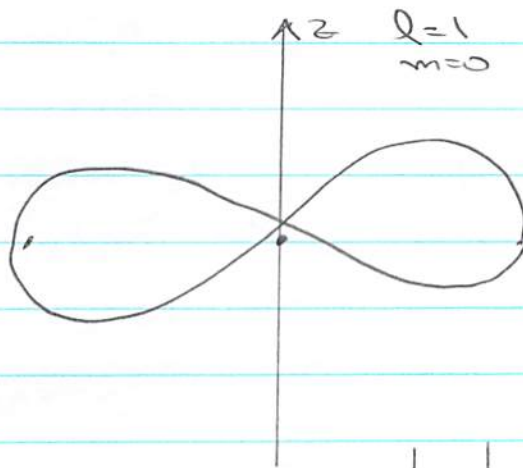
Define $|X_{\ell m}|^2 = \{ \}$

If all m 's contribute equally $\Rightarrow$
radiation is isotropically symmetric

$$\sum_{m=-\ell}^{\ell} |X_{\ell m}|^2 = \frac{2\ell+1}{4\pi} \quad \left(\text{use this to set Krone's (10.9)} \right)$$

Intensity radiation patterns

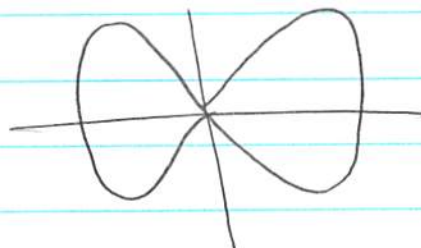
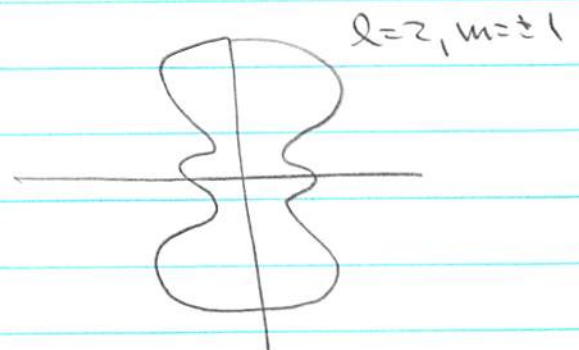
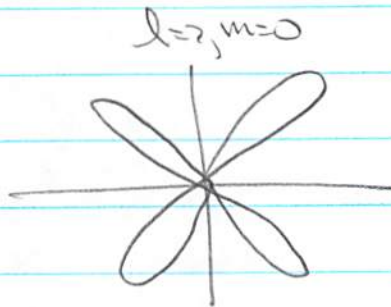
dipole



radio dipole array

AM $\lambda = 100\text{m} \rightarrow 1\text{km}$
 $520\text{kHz} \rightarrow 1610\text{kHz}$
 $600\text{m} \rightarrow 1200\text{m}$

quadrupole



(210)

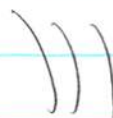
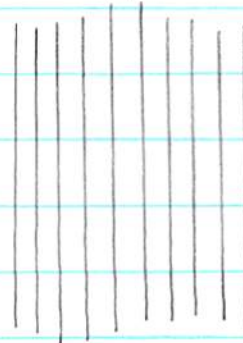
These angular distributions are measured to determine the "l" multi-polarity, but not either the M or E nature.

What about parity? Use polarization experiments.

e.g. electric dipole

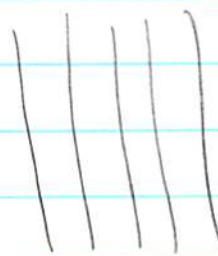
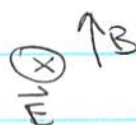


grating



passes radiation

e.g. magnetic dipole



blocks radiation

The leap to γ -transitions: Quantum Mechanics

$$\frac{d\Lambda(l, m, [E_n^E])}{dz} = \underbrace{\frac{1}{\hbar \omega}}_{\text{energy per photon}} \underbrace{\frac{dP(l, m, [E_n^E])}{dz}}_{\text{the classical expression, with}}$$

$$P_{lm}^{i \rightarrow f} = e \sum_{n=1}^Z \int d\vec{x}_n |\vec{x}_n|^l Y_{lm}^* \vec{q}_f \cdot \vec{q}_i$$

Sum over all the protons

+ intrinsic magnetization part that includes the neutrons

$$M_{lm}^{i \rightarrow f} = -\frac{1}{(2\pi)} \frac{e\hbar}{m_p} \sum_{n=1}^Z \int d\vec{x}_n |\vec{x}_n|^l Y_{lm}^* \vec{q}_f \cdot (\vec{q}_f \vec{L}_n \vec{q}_i)$$

angular momentum operator $\vec{x} \vec{L}_n = -i\hbar \vec{x}_n \times \vec{\nabla}_n$

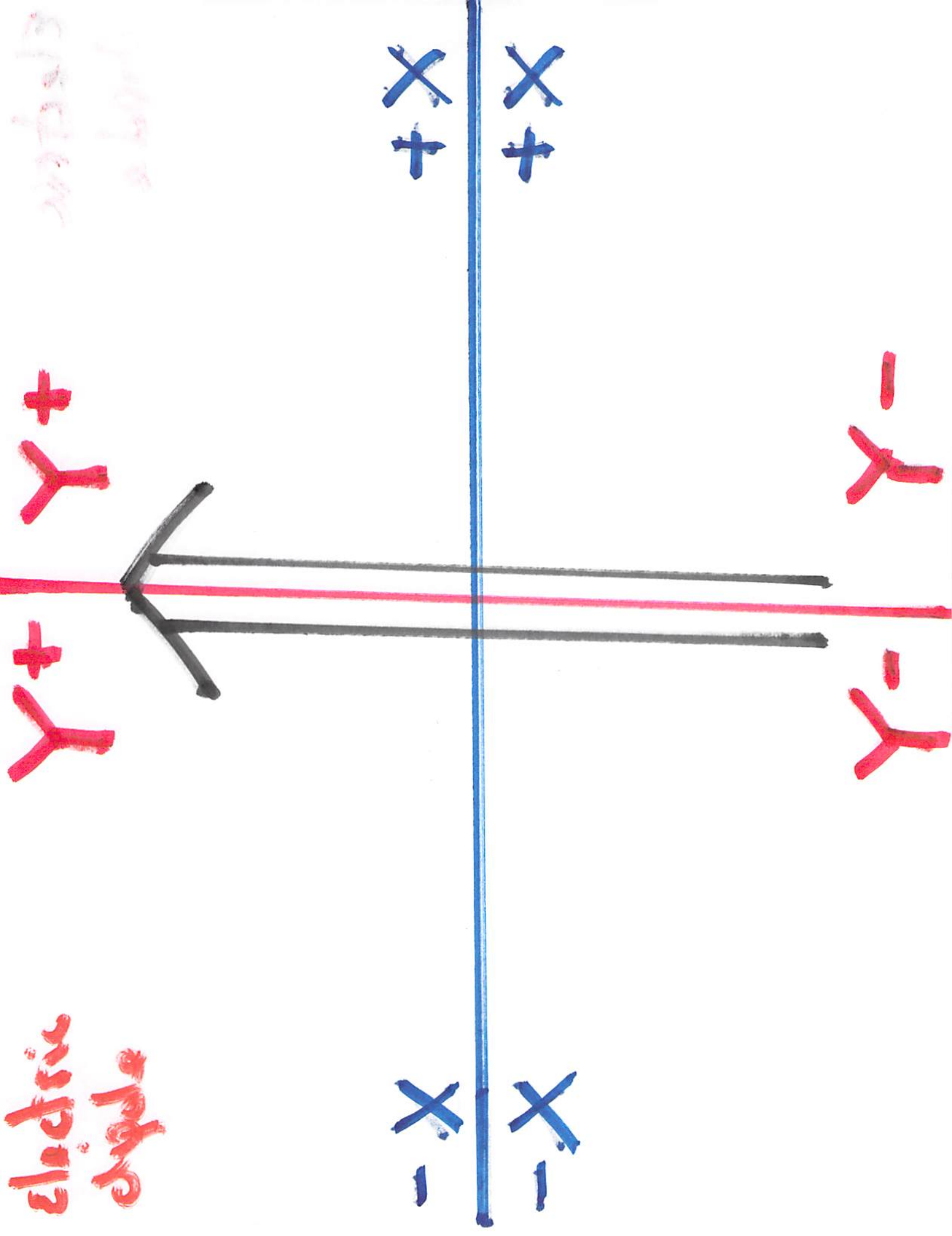
We really do not know what the q 's look like.
So, we can expect only "order of magnitude" agreement.

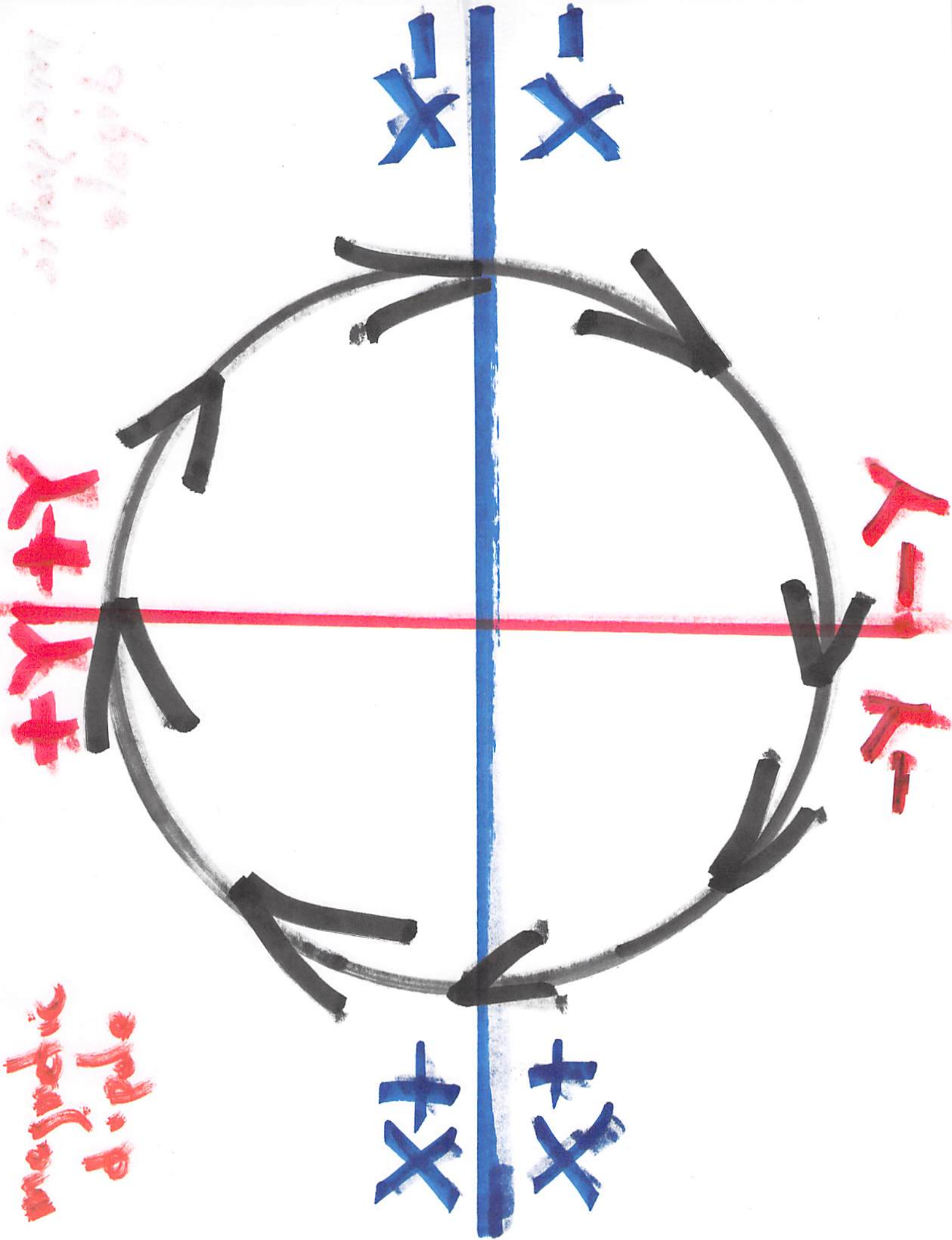
However, these equations work very well

for atomic physics.

So, why did we do all this hard work?

- 1) From angular distribution and polarization measurements, we find, unambiguously, the l (multipolarity) and π of the transition.
- 2) Then, measurements yield unambiguous empirical values for $M_{\text{em}} \mp Q_{\text{em}}$.





10.3 Transition to Quantum Mechanics

$$\frac{d\lambda_{em}}{d\Omega} \propto \left| \begin{matrix} Q_{em} \\ M_{em} \end{matrix} \right|^2 |X(\theta, \phi)|^2 \quad (1)$$

transition rate into solid angle $d\Omega$
 electric multipole
 magnetic multipole
 angular distribution part

$$\lambda_i = \int d\Omega \sum_{m=-l}^l \frac{d\lambda_{em}}{d\Omega} \quad (2)$$

Q_{em}, M_{em} while yielding highly accurate results for atomic physics, are difficult to evaluate

$$\begin{pmatrix} Q_{em} \\ M_{em} \end{pmatrix} = \int \psi_f^\dagger \hat{O}_{em} \psi_i d\vec{x}$$

nuclear wavefunctions are unknown

The Weisskopf estimates

Assume q, μ to be constant up to the "sharp" nuclear radius R , and zero afterwards

From the notes for March 18 (p. 10), we may perform the radial integral

$$\frac{\int dr r^2 r^L}{\int dr r^2} \text{ for normalization} = \frac{3}{L+3} R^L$$

Then $\lambda(E\ell) = \frac{8\pi(\ell+1)}{2[(2\ell+1)!!]^2} \underbrace{\left\{ \frac{e^2}{4\pi\epsilon_0 \hbar c} \right\}^2}_{\alpha = \frac{1}{137.036}} \underbrace{\left(\frac{ER}{\hbar c} \right)^2}_{\text{unitless}} \underbrace{\left(\frac{3}{L+3} \right)}_{\sim} \frac{c}{R}$

for electric transitions

note $\frac{R}{c}$ = time it takes light to cross a nucleus
(0.4-2.0) $\times 10^{-23}$ s

for magnetic transitions usually replaced by "10"

$$\lambda(M\ell) = \frac{8\pi(\ell+1)}{2[(2\ell+1)!!]^2} \underbrace{\left(\mu_p \frac{\ell+1}{\hbar} \right)^2}_{\text{usually replaced by "10"}} \underbrace{\left(\frac{\hbar c}{m_p c^2 R} \right)^2}_{\alpha} \underbrace{\left(\frac{e^2}{4\pi\epsilon_0 \hbar c} \right)}_{\alpha} \underbrace{\left(\frac{ER}{\hbar c} \right)^2}_{\sim} \times \left(\frac{3}{L+2} \right)^2 \left(\frac{c}{R} \right)$$

Putting in numerical values.

electric transitions	$\lambda(E1) = 1.0 \times 10^{14} A^{2/3} E^3$	$\Delta\pi?$ ✓
	$\lambda(E2) = 7.3 \times 10^7 A^{4/3} E^5$	x
	$\lambda(E3) = 34 A^2 E^7$	✓
	$\lambda(E4) = 1.1 \times 10^{-5} A^{8/3} E^9$	x

magnetic transitions	$\lambda(M1) = 5.6 \times 10^{13} E^3$	x
	$\lambda(M2) = 3.5 \times 10^7 A^{2/3} E^5$	✓
	$\lambda(M3) = 16 A^{4/3} E^7$	x
	$\lambda(M4) = 4.5 \times 10^{-6} A^2 E^9$	✓

conclusions 1) $\frac{\lambda(E1)}{\lambda(M2)} \gtrsim 2 A^{2/3}$ 2 $\rightarrow \gtrsim 100$

for a given I , E electric always dominates

However, parity sorts out the type of transition for a given polarity.

e.g.
$$\frac{\lambda(E(l+1))}{\lambda(Ml)} = 10^{-6} A^{1/3} E^2$$

For large A, E $E2$ can compete with $M1$
 $E3$ " " $M2$
etc.

$$\frac{\lambda(\sigma(l+1))}{\lambda(\sigma l)} \approx 10^{-8} \underbrace{A^{2/3} E^2}_{\leq 10^6}$$

As $l \uparrow$ $\lambda \downarrow$

10.4 Angular Momentum & Parity Selection Rules

Nuclear total spin, $\vec{I}$ and multipolarity $\vec{L}$

$$\begin{array}{ccc} \vec{I}_f & = & \vec{L} + \vec{I}_i \\ \uparrow & & \uparrow \\ \text{final} & & \text{initial} \end{array}$$

$$\pi_f = (-1)^L \pi_i \quad (\text{electric transitions})$$

$$\pi_f = (-1)^{L+1} \pi_i \quad (\text{magnetic transitions})$$

rules of vectorial addition

$$|I_i - I_f| \leq l \leq |I_i + I_f|$$

$\Delta\pi = \text{no} \Rightarrow \text{even } E, \text{odd } M, \Delta\pi = \text{yes} \Rightarrow \text{odd } E, \text{even } M$

note $l \geq 1 \therefore I_i = I_f = 0$ transitions
are not possible by γ de-excitation
(this happens by internal conversion, we'll
discuss that later)

Examples.

$$\frac{3}{2}^+ \pi \rightarrow \frac{5}{2}^+ \pi \quad \Delta\pi = \text{no}$$

$$|I_i - I_f| = 1 \quad I_i + I_f = 4$$

$$\therefore \begin{array}{cccccc} l = & 1 & 2 & 3 & 4 \\ \sigma & M & E & M & E \end{array}$$

$$\frac{3}{2}^+ \pi \rightarrow \frac{5}{2}^- \pi \quad \Delta\pi = \text{yes}$$

$$\begin{array}{cccccc} l = & 1 & 2 & 3 & 4 \\ \sigma & E & M & E & M \end{array}$$

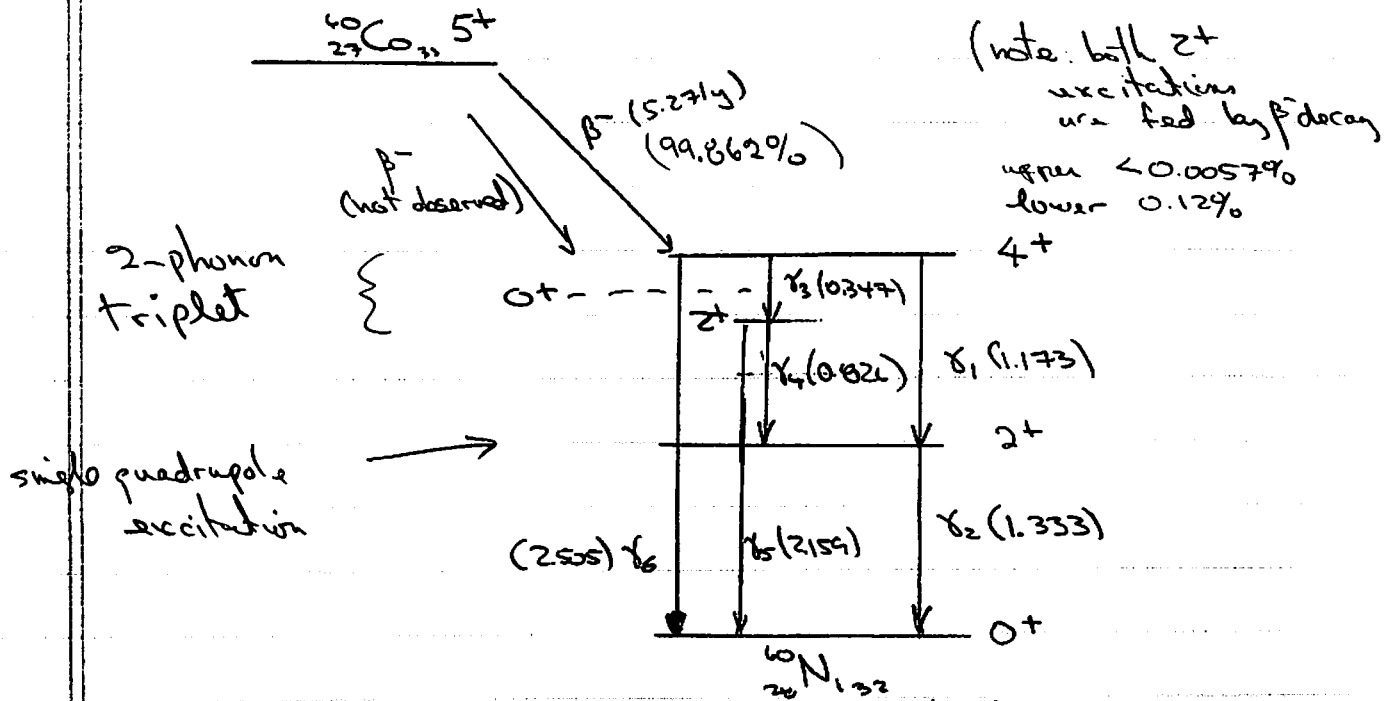
If $I_i = 0$ (or $I_f = 0$) analysis simpler

eg $0^+ \pi \rightarrow 4^+ \pi \quad l = 4$

$$\begin{array}{ll} \sigma = E & \text{if } \Delta\pi = \text{no} \\ = M & \text{if } \Delta\pi = \text{yes} \end{array}$$

eg. $2^+ \rightarrow 0^+$ E2 quadrupole

Real life Example. $^{60}_{22}\text{Co}$ decay



	E (MeV)	$2^+ \rightarrow$	(not seen) poss	rate (ps)	f_{max}
γ_1	1.13274(4)	E2	M3, E4, M5, E6	1.1(3) ps	0.99983(1)
γ_2	1.33250(5)	E2	(pure)	0.715(11) ps	≈ 1
γ_3	0.34693(7)	unk	$[E2, M3, E4, M5, E6]$	$7.6(5) \times 10^{-5}$	—
γ_4	0.82620(9)	M3 E2 D4Q	$[M1, E7, M3, E4]$	$2.6(8) \times 10^{-5}$	—
γ_5	2.15877(4)	E2	(pure)	$1.1(4) \times 10^{-5}$	—
γ_6	2.505	E4	(pure)	$20(4) \times 10^{-8}$	—

Likely explanation

2^+ is a single phonon excitation
 $4^+, 2^+$ part of a double phonon excitation
 0^+ not fed by β^- decay

Tests of Weisskopf's "theory"

Comparison		theory	exp
γ_1, γ_2	$E2(1.173)/E2(1.373)$	0.53	0.65(20) ✓
γ_3 modes	$M3/E2$	2.6×10^{-5}	
	$E4/E2$	5.1×10^{-13}	
	conclude all E2		
γ_5, γ_6	$E2(0.347)/E2(1.173)$	2.3×10^{-3}	7.6×10^{-5}
			factor of 30
	conclude γ_5, γ_6 mismatch		
γ_6, γ_1	$E4(2.555)/E2(1.173)$	6.2×10^{-10}	$2.0(4) \times 10^{-9}$
			OK
γ_3, γ_6	$E4(2.555)/E2(0.347)$	2.7×10^{-5}	$2.6(5) \times 10^{-4}$
			collective effect?
γ_4	$E2/M1$	2.09×10^{-4}	
	conclude all E2		
γ_4, γ_5	$E2(2.151)/M1(0.346)$	0.026	0.15(3)
			factor of 6

ZERO-PHONON GAMMA-RAY TRANSITION IN $^{60}\text{Ni}^\dagger$

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Using a Compton-suppression spectrometer with a central Ge(Li) detector, we have observed a 346.95-keV gamma-ray transition between the 4_1^+ and 2_1^+ states in ^{60}Ni . According to the simple vibration model, this so-called zero-phonon transition is strictly forbidden. A lower limit of 0.174 is placed on the ratio of the observed to the single-particle rates.

In many cases, the low-lying states of medium-weight even-even nuclei have been shown to exhibit a collective nature in that they possess the properties of quadrupole surface vibrations about a spherical equilibrium shape. In the quadrupole-vibrator model, small interparticle couplings remove the degeneracy of a two-phonon vibration to give a triplet of levels with spin and parity of 0^+ , 2^+ , and 4^+ at about twice the energy of the first excited 2^+ state. The 2^+ and 4^+ members of this triplet are seen in many nuclei, but only in a few cases so far is the 0^+ member observed. Even then, it is not always clear that it has the same collective character as the other two members. Another prediction of the simple vibrator model is that transitions between members of the two-phonon triplet are strictly forbidden.¹

Such a zero-phonon transition, if it existed, would have a relatively low intensity because of its low energy; and it would be difficult to observe because it would be hidden in the usually intense Compton distribution present in both NaI(Tl)- and Ge(Li)-detector spectra. To our knowledge, no such zero-phonon transitions have been observed.²

The recent development of the Compton suppression principle,³ incorporating a Ge(Li) detector surrounded by NaI(Tl) detectors operated in anticoincidence, better equips the spectroscopist to search for these low-intensity gamma rays if they exist. The chances of observation are best if there is a relatively large 4^+ -to- 2^+ or 2^+ -to- 0^+ energy separation within the two-phonon triplet, and if the state from which the zero-phonon gamma ray originates is strongly populated. Such a favorable case is the decay of ^{60}Co .

Experiment.—The approximately 50- μCi ^{60}Co source used in the investigation was at least five years old and was produced by the reaction $^{59}\text{Co}(n, \gamma)$ in the Livermore pool-type reactor of the Lawrence Radiation Laboratory. The Compton-suppression system used in this work is described in detail elsewhere.^{3,4} Briefly, however, it incorporates a 7-cm³, 12-mm-deep Ge(Li) de-

tector surrounded by two 9-in.-diam by $4\frac{1}{2}$ -in.-thick NaI(Tl) detectors which completely enclose the detector. The source was 30 cm away from the detector and the gamma rays were collimated by a $\frac{1}{2}$ -in.-diam hole through 4 in. of lead. The detector is oriented so that its electric field is perpendicular to the collimated gamma-ray beam. Thus, no energy error is introduced when double-escape peaks are used for energy calibration.⁵ The ratio achieved in this experiment for the 1.33-MeV peak height to the minimum of the Compton continuum was 120 to 1.

Results.—A 4050-min Compton-suppressed spectrum of ^{60}Co is shown in Fig. 1. Insets show the not completely suppressed double-escape peak of the 1332.5-keV γ ray and the annihilation-radiation peak due to pair interaction of the 1173- and 1333-keV γ rays in material near the detector. Also shown are a (346.95 ± 0.1) -keV transition and a single-escape peak at 821.51 keV, as well as a very weak transition at 826.18 ± 0.2 keV.

The double-escape peak at 310.51 keV and the annihilation peak provide an excellent conversion gain calibration for determination of the energy of the 346.95-keV gamma ray. The energy of the 826-keV gamma-ray transition to the 1332.52-keV level was measured recently by Rauch, Van Patter, and Hinrichsen⁶ as 826.4 ± 0.2 keV, which places the second excited 2^+ level at 2158.92 keV. Hence, according to their measurement, the energy difference between the 4^+ state and the second excited 2^+ state is 346.83 ± 0.2 keV. Our observed energy of 346.95 ± 0.1 keV is in excellent agreement with this value; therefore, this transition must be a zero-phonon transition between the 4^+ and 2^+ two-phonon states at 2505.75 and 2158.80 keV. The relatively small error assigned to our measurement of the very weak 826.18-keV gamma ray results from the nearness of the single-escape peak (only 14 channels away). From our measurement of the 346.95-keV gamma-ray energy and the well-known 1173.23-keV gamma-ray energy, we deduce an energy of 826.28 ± 0.1 keV for this transition

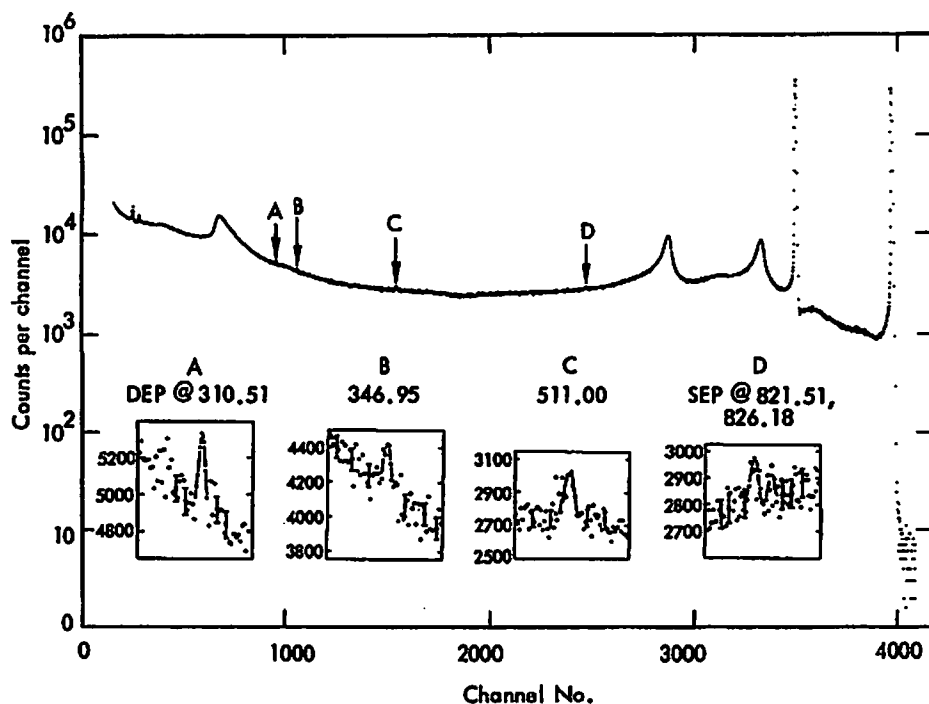


FIG. 1. A 4050-min Compton-suppressed spectrum of the decay of ^{60}Co with specified regions expanded.

—in excellent agreement with our measurement and in good agreement with Rauch, VanPatter, and Hinrichsen.⁶

The relative intensities of all of the transitions observed are shown in Table I. The intensity of the 1173.23-keV gamma ray was set equal to 10^6 . We were not able to identify clearly a gamma-ray transition at 467.4 keV; however, we were able to place an upper limit on this intensity. According to Rauch, VanPatter, and Hinrichsen⁶ it is the most intense gamma-ray branch from the 3^+ state at 2626.2 keV. Using this upper limit and the other intensities, we obtained the beta-branching and $\log ft$ values shown in Fig. 2. An upper limit for the beta feeding of the 2158.80-keV level was determined by assuming the minimum intensity for the 346.95-keV transition (86) and the maximum intensity for the 826.18-keV transition (102). The branching ratio and $\log ft$ value to the first excited state were taken from Camp, Langer, and Smith.⁷ The left-hand side of Fig. 2 shows our results, with deduced energies and intensities shown in parentheses. We made no effort to observe the 2158.8-keV ground-state transition. The right-hand portion of Fig. 2 shows relevant data from Rauch, VanPatter, and Hinrichsen.⁶ An initial consistency check on gamma-ray intensities is possible using the

1332.5-keV transition. Once the 1173.23-keV transition intensity is adopted at 10^6 units, an intensity can be calculated for the 1332.5-keV transitions. Our experimental value was only 0.09% higher than this calculated value.

Discussion.—The transition between the 4^+ and 2^+ members of the two-phonon triplet has been observed in ^{60}Ni . An upper limit⁸ of 3.5 psec on the half-life of the 2506-keV level places a lower limit on the ratio of the experimental (exptl) and single-particle (SP) rates of the 1173-keV transi-

Table I. Cobalt-60 gamma-ray energies and intensities.

E_γ (keV)	I_γ	(%)
346.95 ± 0.10	78	(16.4)
467.4 ^a	≤ 4	(470)
826.18 ± 0.20	55	(85)
1173.231 ± 0.036^b	10^6	
1332.518 ± 0.034^b	$10^{6.4}$	

^aThe energy of this transition is deduced from the difference in energy of 3^+ level at 2626.2 keV (see Ref. 6) and our value for the second 2^+ level at 2158.80 keV.

^bWeighted averages of values from G. Murray et al. Nucl. Phys. 63, 353 (1965), and D. White and D. Groves, Nucl. Phys. A91, 453 (1967).

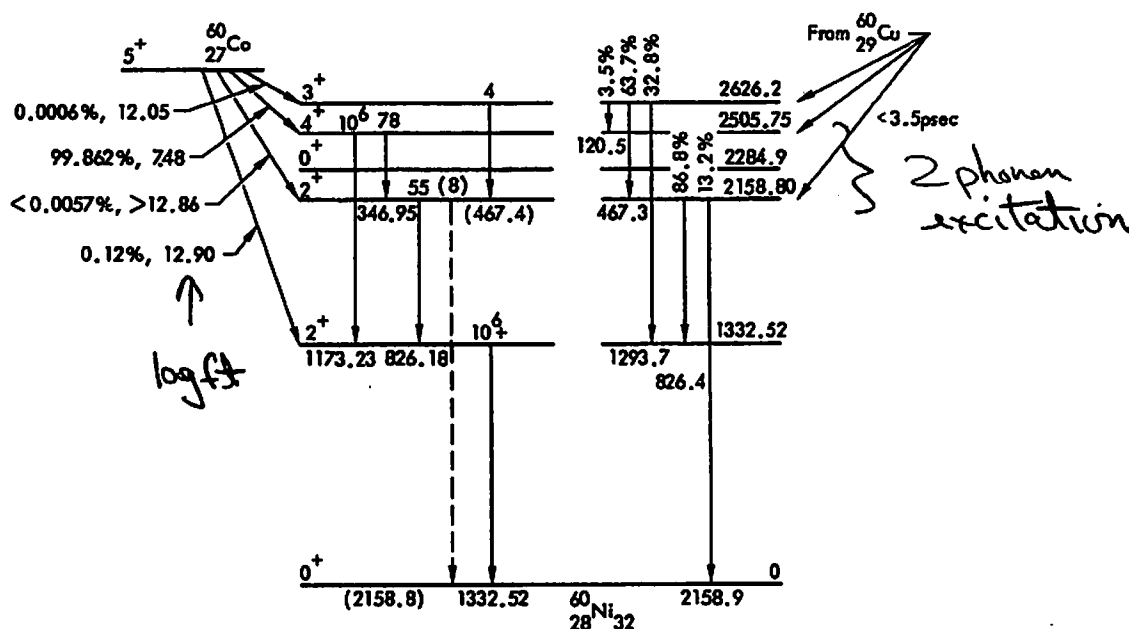


FIG. 2. The decay of ^{60}Co to the levels of ^{60}Ni with relevant data from ^{60}Cu . See text for discussion.

tion. That is,

$$\lambda_{1173}^{\text{expt}}/\lambda_{1173}^{\text{SP}} \geq 4.7, \quad (1)$$

where $\lambda^{\text{SP}} = 1.6 \times 10^8 A^{4/3} E_\gamma^5$ as given in Appendix IV of Lederer, Hollander, and Perlman⁹ for gamma rays with $E2$ characters.

Further, we have the following relationship which gives the ratio of experimental to SP rates for the 347-keV transition as

$$\frac{\lambda_{347}^{\text{expt}}}{\lambda_{347}^{\text{SP}}} = \frac{I_{347}}{I_{1173}} \frac{\lambda_{1173}^{\text{expt}}}{\lambda_{1173}^{\text{SP}}} \left(\frac{1173}{347} \right)^5, \quad (2)$$

where I_{347}/I_{1173} is the ratio of measured intensities for the 347- and 1173-keV gamma rays, and the exponent of 5 on the energy ratio results from an assumption of $E2$ character for both gamma rays.

When the result from Eq. (1) is used in Eq. (2) we find that the ratio $\lambda_{347}^{\text{expt}}/\lambda_{347}^{\text{SP}}$ must be greater than or equal to 0.174. Thus, not only does the (in vibrator language) zero-phonon transition exist, but its rate is not too much smaller than single-particle strength. Although such a model failure might be attributed to level mixing while simultaneously retaining the simple vibrational picture, it would be interesting to know whether the recent formulations of Baranger and Kumar¹ could incorporate this observation more

naturally.

[†]Work performed under the auspices of the U. S. Atomic Energy Commission.

[‡]Summer visitor to Lawrence Radiation Laboratory. Permanent address: Department of Physics, Tri-State College, Angola, Ind.

¹D. A. Bromley and J. Weneser, Comments Nucl. Particle Phys. **1**, 75 (1967). (In this reference are included the references to the theoretical work of Baranger and Kumar.)

²While Ref. 1 suggests that such a transition has been observed in work on $^{113}\text{Cd}(\alpha, \gamma)$, we were unable to substantiate this fact from the references given in Ref. 1.

³D. C. Camp, "Nuclear Spectroscopy via Ge(Li) Detectors in Compton Suppression and Pair Spectrometers," Proceedings of the International Conference on Radioactivity in Nuclear Spectroscopy, Vanderbilt University, Nashville, Tennessee, 11-15 August 1969 (to be published).

⁴D. C. Camp, in Semiconductor Nuclear Particle Detectors and Circuits (National Academy of Sciences, Washington, D. C., 1969), p. 693.

⁵R. Gunnink, R. A. Meyer, J. B. Niday, and R. P. Anderson, Nucl. Instr. Methods **65**, 26 (1968).

⁶F. Rauch, D. M. VanPatter, and P. F. Hinrichsen, Nucl. Phys. **A124**, 145 (1969).

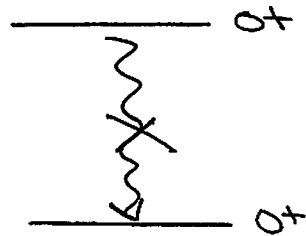
⁷D. C. Camp, L. M. Langer, and D. R. Smith, Phys. Rev. **123**, 241 (1961).

⁸Nucl. Data, Sec. B, **2**, No. 5 (1968).

⁹C. M. Lederer, J. M. Hollander, and I. Perlman, Table of Isotopes (John Wiley & Sons, Inc., New York, 1967), 6th ed. Note: Our quantity λ^{SP} is given the symbol T_{SP} in this reference.

We discussed previously that a

$0^+ \rightarrow 0^+$ transition can not occur.



This is because there is no $l=0$, em operator, for a γ in the final state

Physically, this is because free photons carry at least one unit of intrinsic spin.

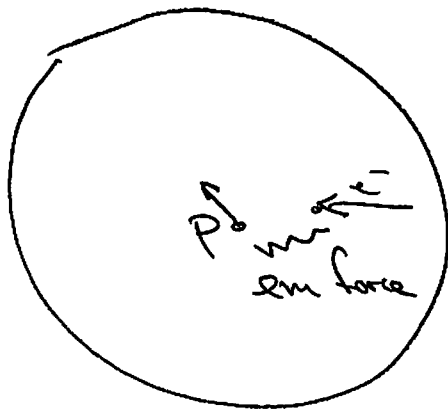
However, there is another process called internal conversion, that can facilitate the $0^+ \rightarrow 0^+$ transition.

Internal conversion

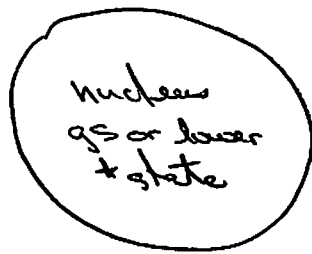
Classical picture



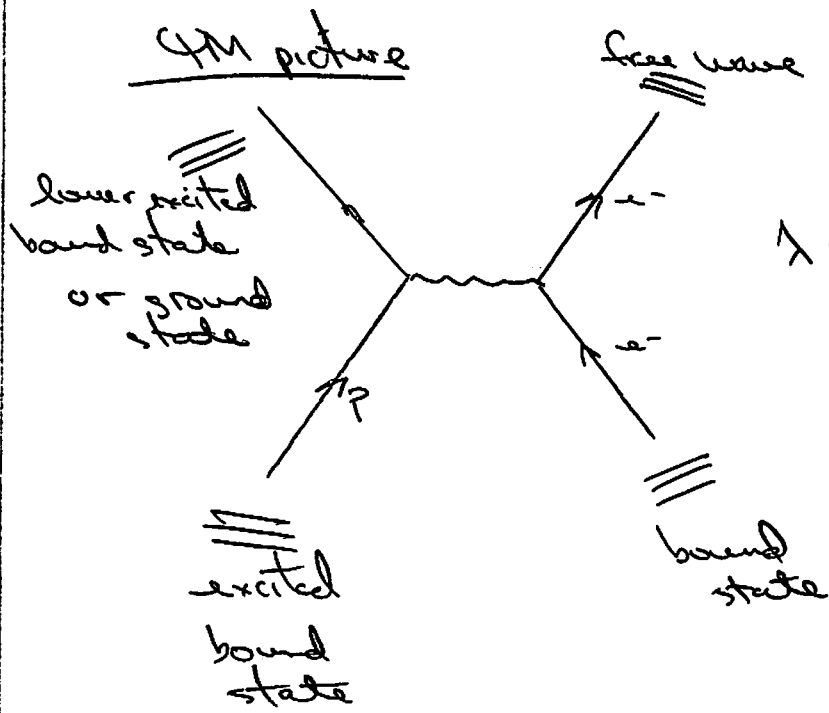
e^- electron enters the nucleus
(predominantly the K shell
atomic electron)



p gives $E + p$ to the e^-



e^- excited



$$\lambda = \frac{2\pi}{k} \left| \langle \psi_f | Q_{em} | \psi_i \rangle \right|^2$$

* ρ_f

← nuclear final state

← free e- wf

← band e- wf

← nuclear initial state

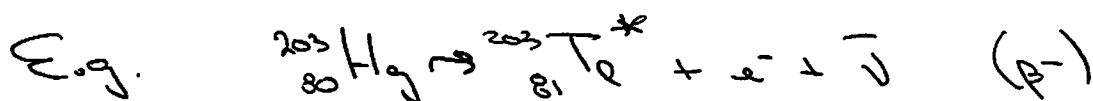
(talk about photoelectric effect)

In fact, internal conversion, while the only $0^+ \rightarrow 0^+$ de-excitation process possible, competes with regular γ transitions, sometimes swamping the γ mode, sometimes greatly suppressed with respect to it.

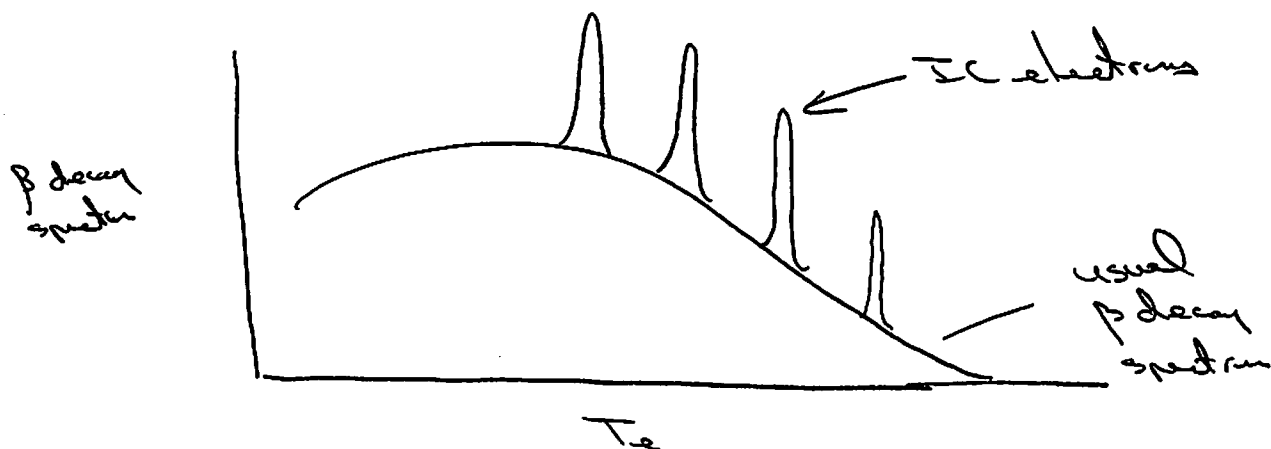
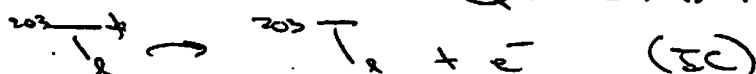
Energetics of Internal conversion

$$T_e = \Delta E - B$$

$\uparrow$
binding energy of
the atomic e^- ,
can not be ignored, and
is different for different shells
and subshells.



$$Q = 279.190$$



For the various
²⁰³Tl shell

$$\begin{aligned} B(K) &= 85.529 \text{ keV} \\ B(L_I) &= 15.347 \text{ " } \\ B(L_{II}) &= 14.690 \text{ " } \\ B(L_{III}) &= 12.657 \text{ " } \\ B(M_I) &= 3.704 \text{ " } \end{aligned}$$

$$\begin{aligned} T_e(K) &= 279.910 - B(K) = 193.661 \text{ keV} \\ T_e(L_I) &= 263.843 \\ T_e(L_{II}) &= 264.492 \\ T_e(L_{III}) &= 266.533 \\ T_e(M_I) &= 275.406 \end{aligned}$$

Note: This is X-Ray notation

$$\begin{aligned} K &\equiv 1s_{1/2} \\ L_I &\equiv 2s_{1/2} \\ L_{II} &\equiv 2p_{1/2} \\ L_{III} &\equiv 2p_{3/2} \\ M_I &\equiv 3s_{1/2} \end{aligned}$$

Contribution to decay rate

$$\lambda_x = \lambda_\gamma + \lambda_e \quad (1)$$

$\uparrow$ $\uparrow$ $\uparrow$
 total gamma Σ

Define $\alpha \equiv \frac{\lambda_e}{\lambda_\gamma}$

$$\lambda_x = \lambda_\gamma (1 + \alpha) \quad (2)$$

Separating out the shells

$$\lambda_x = \lambda_\gamma + \lambda_{eK} + \lambda_{eL} + \lambda_{eM} + \dots$$

$$\lambda_x = \lambda_\gamma (1 + \alpha_K + \alpha_L + \alpha_M + \dots)$$

Similarly

$$\begin{aligned} \lambda_x &= \lambda_\gamma + \lambda_{eK} + \lambda_{eL\pi} + \lambda_{eL\pi\pi} + \lambda_{eL\pi\pi\pi} + \lambda_{eL\pi\pi\pi\pi} \dots \\ &= \lambda_\gamma (1 + \alpha_K + \alpha_{L\pi} + \alpha_{L\pi\pi} + \alpha_{L\pi\pi\pi} + \alpha_{L\pi\pi\pi\pi} \dots) \end{aligned}$$

Note that in computing

$$\langle \psi_f \psi_i^{\text{far}} | \hat{O}_{\text{em}} | \psi_e^{\text{band}} \psi_i \rangle$$

↙ nuclear ↘

The nuclear part does not distinguish between an π or a γ transition

$$\therefore \lambda_{\gamma}(\sigma_2) \propto |M_{\text{el}}(\sigma_2)|^2$$

$$\lambda_e(\sigma_2) \propto |M_{\text{el}}(\sigma_2)|^2$$

The only difference is the constant of proportionality.

If we use the hydrogenic wavefunctions

$$\alpha(E2) \approx \frac{Z^3}{n^3} \left(\frac{1}{l+1} \right) \alpha^4 \left(\frac{2m_e c^2}{E_{\gamma}} \right)^{l+5/2}$$

$$\alpha(M2) \approx \frac{Z^3}{n^3} \alpha^4 \left(\frac{2m_e c^2}{E} \right)^{l+3/2}$$

← principle atomic quantum #

These calculations are only approximate but

They predict

$$\begin{array}{llll}
 \alpha_k & \propto Z^3 & & \checkmark \text{ verified} \\
 \alpha_k & \downarrow & \text{as } E_x \uparrow & \checkmark \\
 \alpha_k & \uparrow & \text{as } l \uparrow & \checkmark \\
 \alpha_k & \downarrow & \text{as } n \uparrow & \checkmark
 \end{array}$$

$\therefore$ as $E_x \downarrow$ $l \uparrow$ we expect $\alpha \uparrow$