

3.6 Nuclear Excited States / Nuclear Structure

Chart p. 319

Chapter 4 The Force between nucleons

Preamble:

Imagine trying to understand chemistry before knowing QM?

Phenomenology, measurement
(empirical) relationships

Then the SE was discovered $-\frac{\hbar^2}{2m} \nabla^2 \dots$

To understand atomic structure more qualitatively

- a) Solve the SE for the H atom, treating p, e^- as fundamental point-charge particles
 ✓ wh's overlap very little
- b) Build up more complex atoms. Everything is mostly a perturbation, because the Coulomb force is relatively weak. $\alpha = 1/137$

c) the interactions between atoms is mostly a "derivative" of the Coulomb force.



covalent bonds

ionic bonds

hydrogen bonds

Van der Waals

share electrons

steal "

share H-bond

dipole-dipole

(permanent or induced)

It's tempting to attempt the same in nuclear physics

Simplest bound system

The deuteron $1p \frac{1}{2} 1n$



- They are not point particles (they have internal structure)
- force is strong, perturbation methods don't work, it's overlap greatly
-

a) internal structure



proton



$$\begin{array}{l}
 u \quad s=1/2, \quad q=\frac{2}{3}e \\
 d \quad s=1/2, \quad q=-\frac{1}{3}e
 \end{array}
 \left. \vphantom{\begin{array}{l} u \\ d \end{array}} \right\} \quad n \quad s=1/2 \quad q=0$$

$$p \quad s=1/2 \quad q=1e$$

masses:	p	938.272 013 (23) MeV/c ²	} most of the mass is gluons
	n	939.565 56 (81) MeV/c ²	
	u	1.5 - 3.3 MeV/c ²	
	d	3.5 - 6.0 MeV/c ²	

Quarks are very tightly bound
(free quarks have not been observed)

Hence covalent, ionic bonds between nucleons
do not exist
all the forces are "derivative"

1) The "derivative" nuclear-nuclear force

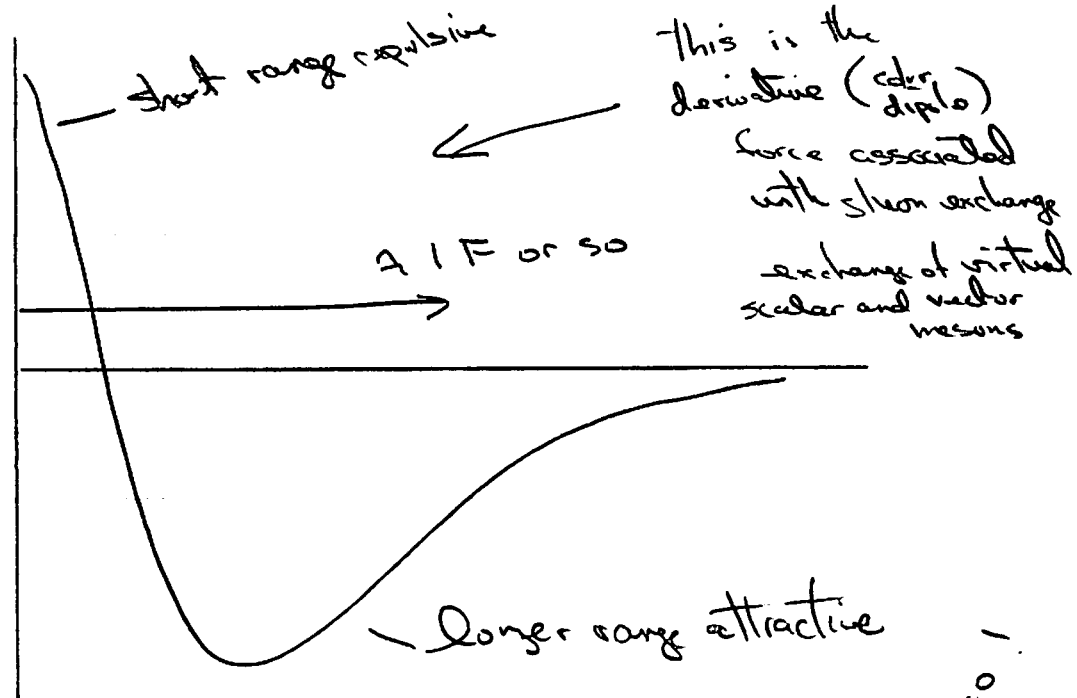
Also called the "meson exchange model"

$$V_{nn}(r) = \left(\underset{\substack{\uparrow \\ \text{repulsive} \\ \text{part}}}{g_v e^{-\frac{m_v r}{\hbar c}}} - \underset{\substack{\uparrow \\ \text{attractive part}}}{g_s e^{-\frac{m_s r}{\hbar c}}} \right) / r$$

g_s "scalar" coupling constant

g_v "vector" coupling constant

m_s, m_v masses of virtual scalar & vector mesons



Note analogy $V_{Coulomb} = \left(\pm \frac{e^2}{4\pi\epsilon_0} \right) \frac{1}{r} e^{-\frac{m_\pi r}{\hbar c}}$ $m_\pi = 0$

2) only hadrons (bound states of quarks)

↳ baryons (e.g. n, p)

↳ mesons (e.g. $\pi^\pm, \pi^0$)

note π 's are
not mesons

Feel the strong force

3) The nuclear force is "almost" independent of n & p , i.e. $n-n, n-p, p-p$ the same

4) The nuclear force depends strongly on spin alignment

5) The nucleon-nucleon force is non-central
 $V(\vec{r}) \neq V(r)$

Hence $\vec{L}$ is not conserved

$$(\vec{I} = \vec{J} + \vec{S} \text{ is})$$

Characteristics of the nucleon-nucleon force

1) Form of the Potential

Phenomenologically speaking, the nucleon-nucleon force looks like

$$V_{nn}(\vec{r}) = \underbrace{\left(g_v e^{-\frac{m_v c}{\hbar} r} - g_s e^{-\frac{m_s c}{\hbar} r} \right)}_{\text{central component}} + \underbrace{V_{so}(\vec{r}) + V_{ss}(\vec{r})}_{\text{non-central component}}$$

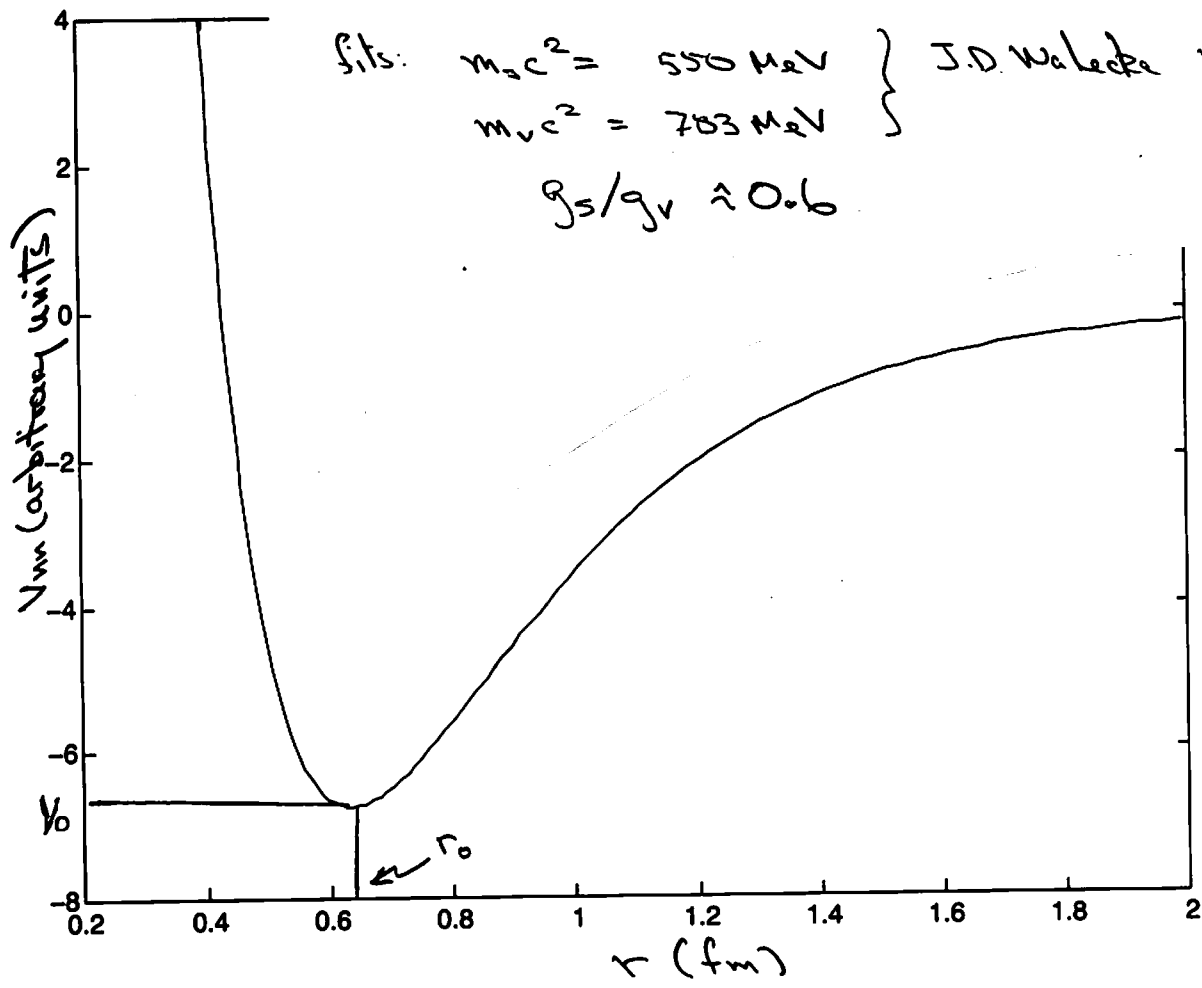
repulsive attractive

g_v, g_s : coupling constants
 m_v : mass of virtual "vector" meson
 m_s : " " " " "scalar" meson

non-central component due to spin-orbit (so) spin-spin (ss) interactions

Recall that V_{nn} is a "derivative" potential, like van der Waals' force in chemistry

Fits: $m_\pi c^2 = 550 \text{ MeV}$
 $m_\rho c^2 = 783 \text{ MeV}$ } J.D. Walecka Ann Phys 83 (1974) 491.
 $g_s/g_v \approx 0.6$



Q: How to fix the depth of this V_{nn} to give us the binding energy of the deuteron?

2) Only hadrons (bound states of quarks) feel this force

$\swarrow$ $\searrow$
 mesons baryons (e.g. n, p [fermions])
 (e.g. $\pi^\pm, \pi^0$ [bosons])

3) The nuclear force is "almost" independent of the nucleon type
 i.e. $n-n$, $n-p$, $p-p$ are the same

4) The nuclear force has very strong spin-orbit, spin-spin, orbit-orbit interactions

5) Hence, $\vec{L} \neq \vec{S}$ are not constants of the motion.
 However, $\vec{J} = \vec{L} + \vec{S}$ is.

(4.1) The deuteron

^2H (aka D) is made up of 1n & 1p
It is weakly bound, but stable, and has
a 0.015% isotopic abundance

The isotopes of H

$\overset{A}{X}$	abundance or $\lambda/2$	J^π
^1H	0.99985	$\frac{1}{2}^+$
^2H (D)	0.00015	1^+
^3H (T)	12.3y (β^-)	$\frac{1}{2}^+$

aside

The D is also the only bound di-nucleon

Why?

p-p
n-n

Coulomb forces

??

(dineutron)

Deuterium is extracted from water by electrolysis or centrifugal techniques producing heavy water, DOH or D_2O .

Heavy water is very similar chemically to H_2O , but the difference has an impact on biological systems. Heavy water is toxic 10-14 days of drinking nothing but heavy water is lethal

The D is the most ideal "laboratory" to study the nucleon-nucleon force.

There are no bound excited states of D

Binding energy of D.

$$B(1,2) = [m(^1\text{H}) + m(n) - m(\text{D})]c^2$$

$$= 2.22463(4) \text{ (MeV)}$$

Krauss

modern data $0.002388169(9) \text{ (u)}$

$$2.224565(6) \text{ [MeV]}$$

From $\text{H}(n,\gamma)\text{D}$ $B = 2.224569(2) \text{ [MeV]}$

from $\text{D}(\gamma,n)\text{H}$ $B = 2.224(2) \text{ [MeV]}$

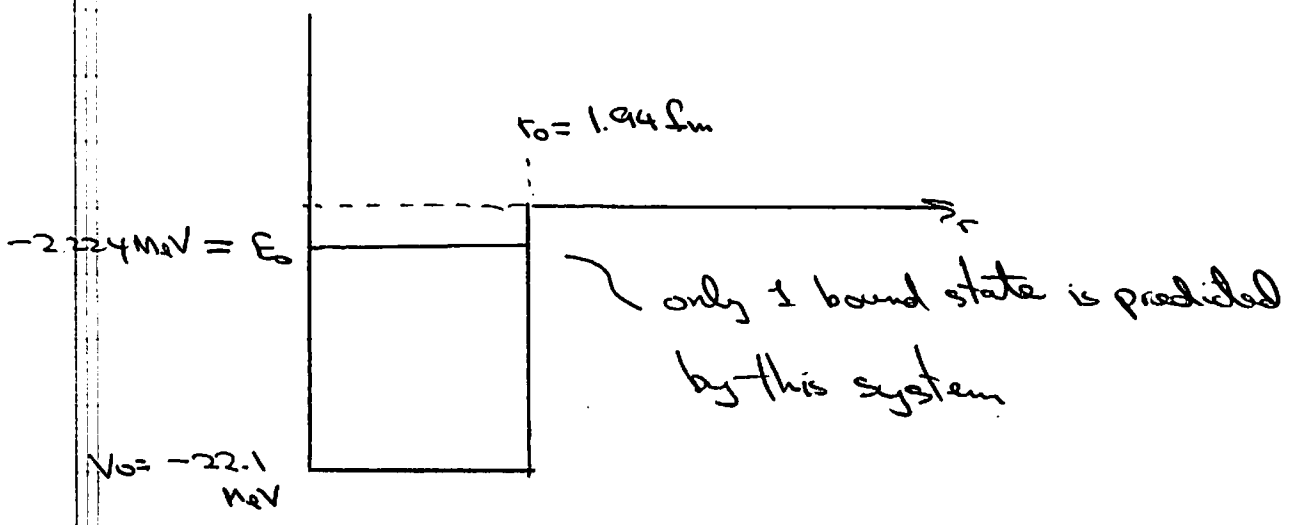
$$B(\text{D})/A \cong 1 \text{ MeV}$$

$$B(Z,A)/A \cong 8 \text{ MeV} \quad (\text{most of the periodic table})$$

D is very weakly bound.

Why does the deuteron have only 1 bound state?

Use spherical box as a model.
We solved this last term



Spin and parity

$$\vec{I} = \vec{S} + \vec{L} + \vec{L}$$

$I=1$ 4 possibilities

Orbital parity $= (-1)^l$

S_0 , only 2 possibilities

S	l	
$\uparrow\uparrow$	0	$\checkmark$
$\uparrow\downarrow$	1	$\times$
$\uparrow\uparrow$	1	$\times$
$\uparrow\uparrow$	2	$\checkmark$

$$S=1, l=0 \text{ or } 2$$

Which one?

We might suppose that $l \neq 0$ would cause the weakly bound system to fly apart. But we can't exclude it without experimental proof.

Two ways

Measure the magnetic dipole moment

If $l=0$, only S can contribute to μ

$$\vec{\mu} = \frac{g_{sn}\mu_B}{\hbar} \vec{S}_n + \frac{g_{sp}\mu_B}{\hbar} \vec{S}_p$$

$$g_{sn} = -3.826084(2)$$

$$g_{sp} = 5.58569(2)$$

$$\mu = \frac{\hbar}{2} \mu_B (g_{sn} + g_{sp}) = 0.879804(3) \xleftarrow{\text{theory}}$$

$$0.857438(1) \xrightarrow{\text{expt}}$$

$$\Delta = 0.022366(3)$$

Difference is

$\Delta \neq 0 \Rightarrow$ Can not be purely $l=0$!

So, assume $\psi = a_0 \psi(l=0) + a_2 \psi(l=2)$

$\uparrow$
 amplitude $\longrightarrow$

$$\mu = a_s^2 \mu(l=0) + a_d^2 \mu(l=2)$$

$$\Rightarrow \begin{aligned} a_s^2 &= 0.96 \\ a_d^2 &= 0.04 \end{aligned}$$

The deuteron is 96% s $\bar{4}\%$ d !
(d is not unbound)

How can you visualize this?