The US Army National Guard's Mobile Training Simulators Location and Routing Problem

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Abstract

For training National Guard units, the US Army National Guard will field 21 combat vehicle training simulators called *Mobile Trainers.* Each National Guard unit must train at a station which is not farther than a specified maximum travel distance from its armory. We address the problem of finding:

- the optimum locations for the *home bases* for the mobile trainers,
- the locations of *secondary training sites* to which the mobile trainers will travel to provide training,
- and the actual routes that the mobile trainers will take to cover all these secondary training sites.

The aim is to allocate each National Guard unit to a training site within the maximum travel distance from its armory while simultaneously minimizing the mobile trainer fleet mileage, and the total distance traveled by all units.

The problem is too large and complex to solve as a single model. We apply a heuristic decomposition strategy to break the overall problem into manageable stages, developing suitable substitute objective functions for each. This approach led to a solution in which the mobile trainer fleet mileage is 72,850 miles per year: about 70% smaller than the 231,000 miles per year in the original Army's procurement plan. Our solution has been implemented (with minor modifications) by the Army.

Key words: Large scale combinatorial optimization, hierarchical decomposition with substitute objective functions for each stage, p-median problem, set covering models, traveling salesman problems, multi-depot vehicle routing problems, cycle cover method.

1 Background

Since field training using live ammunition has become very expensive, the US Army has decided to shift, by the year 2000, a major fraction of the training of army units to high-fidelity networked tank and infantry fighting vehicle training simulators. These simulators are specially designed to allow the units to achieve proficiency in tactical military operations without using ammunition. The Close Combat Tactical Trainer (CCTT) - which we refer to as a *fixed trainer* because once it is set up at a location, it cannot be moved easily - is one such simulator. The Army will field fixed trainers for training both the full-time armed forces and the Army National Guard (hereafter referred to as "the National Guard").

The National Guard consists of people who have regular jobs outside the Army, but can be called for military duty if a need arises. Because members of the National Guard have full-time jobs outside the Army, their training can only take place on weekends. The word *armory* in this paper refers to a place (village, town, or city) where some forces of the National Guard reside. Travel time to reach a unit's training center directly detracts from the short time available for effective training during a weekend. For this reason the Army has agreed to insure that each unit will be assigned to a training center no more than 100 miles - the stipulated maximum travel distance (MTD) - from their armory.

The armories of the National Guard forces are widely scattered throughout the country. Highly dispersed units and the MTD requirement combine to present a difficult challenge for the Army in planning their training program. To meet this challenge, the Army has decided to purchase 21 mobile close combat tactical trainers (M-CCTTs) which are similar to the fixed trainers except that they are mounted on mobile trailers rather than fixed in buildings. Each of these *mobile trainers* will be stationed at a site called its *home base*, which will be its headquarters. The Army will install fixed trainers at seven army installations (determined prior to our study) for training active duty forces during weekdays. These fixed simulators have a much greater capacity for training than the mobile simulators, and are available for National Guard training during weekends. Such an arrangement meets the training needs of all National Guard units whose armories are within the *MTD* of at least one of the fixed trainer installations or mobile trainer home bases. To meet the training needs of all other National Guard units, additional sites, called *secondary training sites* must be selected. The mobile trainers will travel to these secondary training sites, and each of the remaining National Guard units will be assigned to one such site for training.

The basic National Guard unit is called a platoon, and has four fighting vehicles allocated to it. Both the fixed and mobile trainers are platoon-level simulators, i.e., they train one platoon at a time using four fighting vehicles. The mobile trainers are produced in two versions: an *infantry version* to train units equipped with M2/3 Bradley Fighting Vehicles, and an *armor version* that will train units using the Abrams M1 Main Battle Tank. These are two separate systems and are not interchangeable. Of the 21 mobile trainers, 10 are of the infantry version, and 11 are of the armor version. Each armor platoon consists of 16 soldiers with four soldiers per vehicle. Each infantry platoon consists of 12 soldiers in four vehicles, plus 24 foot soldiers.

All of the soldiers in a platoon usually reside close to each other, so they can be considered to belong to a single armory. Each armory has exactly three platoons to be trained, which together form a unit called a Company. For training, each platoon travels together from their armory to their assigned training center, usually in a military bus. The bus has enough space for 48 people. If all three platoons in an armory train at the same training center during the same weekend, they travel together in one or more buses. Their trip starts at their armory on a Friday evening after work, and ends at the same place the following Sunday evening.

Since a platoon travels together by bus, the mileage of the National Guard soldiers to training centers is measured in units called *platoon-bus miles*. To make sure that the actual bus mileage remains small, the Army has agreed to the policy that all three platoons from an armory will always be allocated to the same training center during the same weekends of the year, so that they can travel together to the training center and back using the smallest possible number of buses.

During each weekend a mobile trainer can train three platoons (i.e., one armory). A mobile trainer is expected to be available for use for about 42 weekends per year for National Guard training. Since each mobile trainer can train three platoons per weekend, each such trainer can deliver $42 \times 3 = 126$ National Guard weekend platoon-training sessions per year.

Armories are classified into two classes: *enhanced armories* and *nonenhanced armories*. Platoons in enhanced armories have higher priority for training and are required to train for four weekend sessions per year. Those in nonenhanced armories have lower priority, and are required to train for only one or two weekends per year, as specified by the Army for each unit. In all the National Guard has 181 infantry armories (68 enhanced, 113 nonenhanced) and 187 armor armories (48 enhanced, and 139 nonenhanced). Consecutive training sessions of each armory must be at least six weeks apart.

In addition to the weekend training sessions considered in this study, National Guard soldiers also participate in a two-week-long military exercise, once per year, held at the home bases of mobile trainers. Thus any site chosen as a home base must have adequate training support resources and a large enough garrison to house the visiting soldiers. This condition limits the location of home bases to 29 sites in the continental United States. The National Guard has 368 armories, all suitable for selection as secondary training sites in addition to any of the 29 sites eligible to be home bases but not selected as such.

2 The Problems

Our study was concerned with developing an optimum detailed plan for the weekend training of National Guard forces. We were asked to develop an optimum solution for the problem because an initial manual analysis by the Army using a map and concentric circles on transparent acetate did not yield a satisfactory solution.

First, we identified all armories within the *MTD* of at least one of the seven fixed simulator installations. We allocated the units in all such armories to their nearest fixed simulator installation for weekend training, and removed them from further consideration. However, only 37 out of 368 armories were allocated by this process, broken down as follows.

Type	Armories allocated*		Remaining armories	
	Armor	Infantry	Armor	Infantry
Enhanced	4	12	44	56
Nonenhanced	15	6	124	107

* within *MTD* of a fixed simulator installation

This left 331 armories to be allocated to secondary training sites. More specifically, the problem involves the following decisions:

1. **Home Base Selection:** Select home bases for the 21 mobile trainers from among the 29 suitable sites.

2. **Secondary Training Site Selection:** Select the necessary secondary training sites so that each armory is within *MTD* of at least one of the home bases or secondary training sites. Then allocate each unit for training to a home base or secondary site within *MTD* of its armory.

3. **Routing of Mobile Trainers:** Develop routes for the mobile trainers to cover all the secondary training sites. Each trainer begins its route at a home base, visits a subset of the secondary training sites in some order, staying at each site as long as necessary to train all units allocated there, and returns to its home base at the end. A mobile trainer may have to make multiple visits to some secondary training sites in order to satisfy the minimum time gap constraint between consecutive training sessions of a unit.

4. **Scheduling of Training Sessions:** Develop a time schedule over the year for each unit to receive the number of training sessions it is supposed to receive, taking into account the capacity of the mobile trainers.

All of these decisions must be made to optimize simultaneously the following objective functions, in decreasing order of importance.

(a) **OBJ1: Mobile Trainer Fleet Mileage:** Minimize the total annual mileage of the mobile trainer fleet. Each mobile trainer uses five or six tractor-trailer vans, and the cost per van-mile has been estimated to be over $10. Hence, this is the most important objective function.

(b) **OBJ2: Platoon-Bus Mileage:** Minimize the annual platoon-bus miles required for units to travel to training centers. Since the cost per platoon-bus mile is much smaller than the cost per mobile trainer mile, this objective function is of secondary importance.

(c) **OBJ3: Number of Secondary Training Sites:** Minimize the number of secondary training sites selected. Each secondary site must undergo a facility upgrade which involves building a stable concrete pad for the mobile trainer, and installing electrical equipment and security fences. The cost of upgrading has been estimated to be $10,000/facility. However, since this is a one-time fixed cost, this objective function is third in importance.

The overall problem is a large scale multi-objective problem involving many combinatorial decisions. Because the armor and infantry versions of the problem are independent of one another, we solved them as separate problems using the same basic mathematical models, but with data appropriate for each. The most important data for our problem are the driving distances (for mobile trailer vans, and military buses used by platoons) between pairs of locations, such as armories and sites suitable to serve as home bases. This distance data was obtained from a trucking industry road mileage guide.

3 The Models

The entire training assignment problem is too large and complex to solve as a single model. Hence we used a heuristic hierarchical decomposition strategy to break the overall problem into several manageable pieces, to be solved in stages.

Even if the original problem has only a single objective function to be optimized, that single objective function may involve all the variables in the problem. In the hierarchical approach, at each stage we only deal with one part of the problem. The constraints in a stage are those that must be satisfied by the variables relevant to the part under consideration, but we do not have an objective function for this part alone. Therefore, we set up an objective function for each stage, in terms of the relevant variables in it, so as to satisfy the property that optimizing the objective function at this stage will move the final solution as close to the overall optimum as possible. We will call this the *substitute* or *surrogate objective function* for the part of the problem under consideration in this stage.

The construction of the substitute objective function for each stage plays a critical role in the quality of the final solution obtained by this hierarchical approach. We now provide a brief description of each stage in the hierarchical approach and the flow of information from each stage to the next.

Stage 1: HB (Home Base) Selection:

The first stage uses as input the distance data, the number of platoon-training sessions needed by each armory, and the number of platoon-training sessions that a mobile trainer can deliver per year. We determine the best home-base site (among the 29 potential sites specified by the Army) for each of the p mobile trainers ($p = 11$ for the armor version, and 10 for the infantry version). We then allocate each armory which is within *MTD* of at least one home base to its nearest home base for training. We let U denote the set of all armories whose distance from each home base is greater than the

MTD, and therefore not yet allocated to a training center.

Stage 2: *STS (Secondary Training Site)* Selection:

Using as input the home base sites determined in Stage 1, as well as distance data and the set U of unallocated armories, we determine in this stage a set of *STS*s such that each armory in U is within *MTD* of at least one *STS*. We then allocate each remaining armory in U to the *STS* nearest to it for training.

Stage 3: Routing the Mobile Trainers:

Using as input the home base sites and the *STS*s determined in Stages 1 and 2, along with the distance data, this stage finds minimum-distance routes for the mobile trainers stationed at home bases to visit all the *STS*s to provide training for the platoons from armories allocated to those *STS*s. This stage also finds yearly training schedules for all the units so as to satisfy the required time gap between consecutive sessions and the mobile trainer capacities.

3.1 Formulation of Each of the Models

Stage 1: HB Selection, A p-Median Type Model

The p-median model is concerned with finding optimum locations for exactly p facilities for the provision of a service to a set of customers that involves travel (by customers, or by the provider of the service, or both) between each customer and a 'nearby' facility, so as to minimize a measure of the total travel. The concept of the p-median of a graph has been defined by Hakimi [3, 4], and its relation to other location models has been shown by Revelle, Marks, and Liebman [8]. Our HB selection problem can be modeled as a p-median problem with minor modifications: the armories are the customers, and the mobile trainers are the facilities.

Let u denote the total number of armories that need to be allocated to training centers, and let the index i ($i = 1 \ldots, u$) denote these armories. For $i = 1, \ldots, u$, let b_i denote the number of platoon-training sessions needed by armory i per year (b_i depends on the number of platoons stationed at armory i, and their enhanced or nonenhanced status). The index j ($j = 1, \ldots, 29$) denotes the sites that can be selected as home bases for mobile trainers. The number of weekend platoon-training sessions that a mobile trainer can deliver per year is M ($M = 126$ as discussed above), and p is the total number of mobile trainers to be stationed ($p = 11$ for the armor version, and $p = 10$ for the infantry version). Let $d_{\ell m}$ denote the driving distance (for mobile trainer vans and army buses, in miles) between places (armories, or sites suitable for being home bases) ℓ and m. We define the decision variables

$$x_{ij} = \begin{cases} 1 & \text{if armory } i \text{ assigned to a mobile trainer at site } j;\ i = 1, \ldots, u; \\ & j = 1, \ldots, 29 \\ 0 & \text{otherwise} \end{cases}$$

$$y_j = \text{ number of mobile trainers based at site } j,\ y_j \in \{0, 1, 2, \ldots\}$$

Let D_{ij} ($i = 1, \ldots, u$; $j = 1, \ldots, 29$) denote the platoon-bus miles from armory i if it is allocated to a mobile trainer stationed at site j for training ($D_{ij} = b_i d_{ij}$). All three of the objective functions (OBJ1, OBJ2, OBJ3) are improved if each platoon is assigned to a mobile trainer stationed at a home base that is as close to the platoon's armory as possible. Thus minimizing $\sum_{i=1}^{u} \sum_{j=1}^{29} D_{ij} x_{ij}$ has the effect of assigning each platoon to a home base to minimize the total travel between all such pairs. This objective function is therefore an appropriate substitute objective function for the optimal choice of home base sites. This leads to the following p-median type model for home base selection.

$$\text{Minimize} \quad \sum_{i=1}^{u} \sum_{j=1}^{29} D_{ij} x_{ij}$$

$$\begin{aligned} \text{Subject to} \quad \sum_{j=1}^{29} x_{ij} &= 1, \quad i = 1, \ldots, u \\ \sum_{i=1}^{u} b_i x_{ij} &\leq M y_j, \quad j = 1, \ldots, 29 \end{aligned}$$

$$\begin{aligned}\sum_{j=1}^{29} y_j &\leq p \\ x_{ij} &= 0 \text{ or } 1 \text{ for all } i, j \\ y_j &\geq 0 \quad \text{and integer for all } j\end{aligned}$$

Let $\bar{x} = (\bar{x}_{ij}), \bar{y} = (\bar{y}_j)$ denote an optimum solution of this model. This solution corresponds to the *set of home base sites* $J = \{j : \bar{y}_j \geq 1\}$, and the number of mobile trainers to station at each home base ($\bar{y}_j$ for $j \in J$). Finding the p-median simultaneously partitions the set of customers into p clusters, where each cluster represents the customers served by one of the facilities. In the optimum solution of the above model, for each home base $j \in J$, the cluster is $\{i : \bar{x}_{ij} = 1\}$.

However, even if $\bar{x}_{ij} = 1$, we may not be able to allocate armory i to a mobile trainer stationed at home base j for training, because of the *MTD* restriction which is not taken into account in the above model. Therefore after J is found by solving the above model, each armory which is within *MTD* of at least one home base site $j \in J$ is assigned to its nearest home base for training. Let U denote the set of all armories whose distance from each home base site $j \in J$ is greater than the *MTD*. These are the armories not yet allocated to a training center.

Stage 2: *STS* Selection, A Set Covering Model

The application of set covering models to facility location problems was first discussed in Toregas, Swain, Revelle, and Bergman [9]. Using the same approach, we model our *STS* selection as a set covering problem. From Stage 1 we know J, the set of home bases; and U, the set of armories not yet allocated to a training center. Each armory in U must to be allocated to an *STS* for training. Let S be the set of 368 armories and the eight sites eligible to be home bases but not selected as such. S is the set of all places eligible to be selected as an *STS*. For each unallocated armory $i \in U$, let S_i be the set of all places in S which are within *MTD* of armory i. The *MTD* restriction implies that at least one place in the set S_i must be selected as an *STS*. For each $t \in S$ define the binary decision variable

$$\xi_t = \begin{cases} 1 & \text{if } t \text{ is selected as an } STS \\ 0 & \text{otherwise} \end{cases}$$

For each potential STS $t \in S$, its *weight* g_t is defined as the distance between t and its nearest home base $j \in J$, i.e., $g_t = \min\{d_{tj} : j \in J\}$; g_t is a measure of the mobile trainer mileage that will be incurred if t is selected as an STS. The most important objective function in our problem, OBJ1 will be improved if STSs are selected to minimize $\sum_{t \in S} g_t \xi_t$. Therefore $\sum_{t \in S} g_t \xi_t$ is an appropriate substitute objective function for this stage. This leads to the following set covering model for STS selection.

$$\begin{aligned} \text{Minimize} \quad & \sum_{t \in S} g_t \xi_t & & \\ \text{Subject to} \quad & \sum_{t \in S_i} \xi_t & \geq & \; 1, \quad \text{for each } i \in U \\ & \xi_t & = & \; 0 \text{ or } 1 \text{ for all } t \in S \end{aligned}$$

Let $\bar{\xi} = (\bar{\xi}_t)$ denote an optimum solution of this model. This solution corresponds to the set $T = \{t : \bar{\xi}_t = 1\}$ of STSs. Once T is determined, each armory in $i \in U$ is assigned for training to the STS in T nearest to it.

Stage 3: Routing the Mobile Trainers

Stage 1 finds the set of home bases where the mobile trainers are stationed (J), and Stage 2 selects the set of STSs (T). In Stage 3 we develop routes for the mobile trainers to visit all the STSs. Each route begins at a home base, visits a subset of STSs in T, and ends at the home base where it originated. In traversing this route, the mobile trainer stationed at the home base node stops at each STS node on the route as long as necessary to give one training session to each unit allocated to that STS.

The Army has stipulated that in each mobile trainer route (originating from and returning to its home base) only one training session can be scheduled for any unit allocated to an *STS* on this route. If the unit has to receive more than one training session, the Army's policy requires that the mobile trainer make additional trips to the *STS* to which that unit is allocated. This policy is adopted to enable the units to schedule their training sessions according to their convenience, and to make sure that the repeat training sessions are evenly spread out over the year.

Let $G = (\mathcal{N}, \mathcal{A})$ be the graph with set of nodes $\mathcal{N} = J \cup T$, and set of edges $\mathcal{A} = \{(i,j) : i \neq j; i,j \in J \cup T$ and at least one of i or $j \notin J\}$. Then each route for a mobile trainer is a simple cycle in G that contains exactly one home base node in J. Since each *STS* must be visited by a mobile trainer, the routing problem is that of finding a set of routes $\{\mathcal{C}_1, \ldots, \mathcal{C}_k\}$ in G having minimum total distance, satisfying the following: (i) each route $\mathcal{C}_\ell$ is a simple cycle in G containing exactly one home base node, (ii) each *STS* node is contained in exactly one of the routes in the set, and (iii) other conditions imposed by mobile trainer capacities, etc.

The vehicle routing problem (VRP) is a generic name given to a class of problems involving the visiting of *customers* by *vehicles* stationed at *depots*. Clearly our problem is a VRP in which the depots are the home bases, the vehicles are the mobile trainers, and the customers are the *STS*s. Since the number of home bases is 10 or 11 in our problem, our routing problems are multiple-depot vehicle routing problems (MDVRP).

There is vast literature on the VRP; see Golden and Assad [2], Laporte [5], and Laporte and Nobert [6] for surveys on the subject. However, most of the literature is focused on the single-depot VRP. Due to the high complexity of the problem, no technique for the exact solution of the MDVRP, with real-life constraints, has been proposed in the literature. Therefore, we developed a heuristic procedure for mobile trainer routing that takes advantage of the special features of the problem.

Let $\bar{G} = (\bar{\mathcal{N}} = T, \bar{\mathcal{A}} = \{(i,j) : i \neq j \in T\})$ be the subgraph of G induced by the set of *STS* nodes. A *cycle cover* for $\bar{G}$ is a set of pairwise node-disjoint simple cycles in $\bar{G}$ which together contain all the nodes in $\bar{G}$. The main idea of the heuristic procedure, which we call the *cycle cover heuristic*, is to first find a cycle cover for $\bar{G}$ of minimum total distance. This can be obtained efficiently by solving an assignment problem, that of finding a minimum cost perfect matching in the bipartite network $H = (\bar{\mathcal{N}}, \bar{\mathcal{N}}; \{(r,s) : r \neq s \in \bar{\mathcal{N}}\})$ with the distance between STS r and STS s as the cost of edge (r,s). The optimum assignment, i.e., the minimum cost perfect matching in the bipartite network H, decomposes into a minimum distance cycle cover for $\bar{G}$. As an example, suppose the nodes in $\bar{\mathcal{N}}$ are numbered 1 to 10, and suppose the optimum assignment is: $\{(1,6),(2,7),(3,8),\ (4,1),(5,2),(6,4),\ (7,5),(8,10),(9,3),(10,9)\}$. This breaks down into the cycle cover consisting of three simple cycles in $\bar{G}$. We give these cycles using standard notation for representing cycles (Murty [7]): 1, (1, 6), 6, (6, 4), 4, (4, 1), 1; 2, (2, 7), 7, (7, 5), 5, (5, 2), 2; and 3, (3, 8), 8, (8, 10), (10, 9), 9, (9, 3), 3.

Then we insert the best possible depot (i.e., home base) into each simple cycle in the cycle cover, thus converting each simple cycle into a route containing exactly one home base node. If the cycle cover for $\bar{G}$ obtained above consists of only one or a few simple cycles, then only a few of the available home bases will be used in the solution obtained by this procedure, and it will not be a good solution. However, when the points in T are widely spread out over a large geographical area, and the cost matrix for the assignment problem is the Euclidean distance matrix between points in T, the minimum length cycle cover obtained by this procedure usually consists of simple cycles, each containing a small number of nodes (typically between two to four), and the nodes in T tend to get evenly distributed among the routes with various home bases on them.

We number the training sessions for each unit over the year serially. Every unit gets its first training session, and all enhanced units also get their second, third, and fourth. Some nonenhanced units get first and second training sessions. We now provide a detailed description of the cycle cover heuristic that we used.

1. Initially we consider routes for the mobile trainers for giving the first training session of the year for the various units allocated to STS. Set t, the number of the training session under consideration, equal to 1.

2. T is the remaining set of STS nodes to be included on mobile trainer routes.

If $T \neq \emptyset$ and there are no home bases with spare mobile trainer capacity, then training capacity is inadequate to meet the need, and either the total mobile trainer capacity should be increased, or training workload should be reduced to bring it in line with available training capacity (for example by reducing the number of multiple training sessions required of some units). With modified data, the problem can be solved again by resuming the work at the appropriate stage.

If $T \neq \emptyset$ and there are home bases with available mobile trainer capacity, define the $|T| \times |T|$ cost matrix $c = (c_{vw})$ as follows: for each $v, w \in T$

$$c_{vw} = \begin{cases} +\infty & \text{if } v = w \\ d_{vw} & \text{otherwise} \end{cases}$$

3. Solve the assignment problem with cost matrix c. An optimum assignment decomposes into a set of simple cycles of minimum total length covering all the STS nodes in T. Let $C_1, \ldots, C_r$ be the simple cycles in this cycle cover.

4. For each $k = 1, \ldots, r$, find the best home base node with available training capacity to insert into the cycle C_k and the best position to insert it, resulting in the smallest increase in its length. It is permissible to insert a home base node into more than one cycle, as long as the mobile trainer stationed at that home base has enough spare capacity to give one training session to platoons from all the armories allocated to STS nodes on this cycle. The resulting cycle after the insertion is one of the routes for the mobile trainer stationed at the home base node on the cycle; that mobile trainer covers all the STS nodes on this cycle.

   If any of the mobile trainers has reached its capacity for training, go to Step 5.

   If all the STS nodes are now covered, go to Step 7 if $t = 4$, or to Step 6 for routing the mobile trainers for the next training session of the year for units which are required to undergo it.

5. If any of the mobile trainers has reached its capacity for training, remove that trainer from further consideration. Also take out all STS nodes that are already

covered for the current training session t. Letting T denote the remaining set of STS nodes, and with the set of home bases that have available capacity, return to Step 2.

6. Now consider the problem of routing the mobile trainers for the next numbered training session of the year. Increment t by 1. Remove from consideration all of the STSs that do not have any unit allocated to them that require this tth training session. Consider a route constructed in Step 4 for the $(t-1)$th training session. Suppose the home base node on it is j, and the STS nodes on it are $k_1, \ldots, k_s$. If each of these STSs is currently under consideration, and if the mobile trainer stationed at j has the capacity to handle all the training workload on this route, then repeat this route as is for this tth training session, and take these STS nodes $k_1, \ldots, k_s$ from further consideration for this tth training session. Do the same with all the routes constructed for the $(t-1)$th training session.

   If there are no STS nodes left, go to Step 7 if $t = 4$; otherwise increment t by 1 and repeat this step.

   If some STS nodes are left, let T denote the remaining set of STS nodes, and with the set of home bases that have available capacity, go to Step 2 to construct routes for covering the STS nodes in T for this tth training session.

7. The routes constructed above for the mobile trainers to cover all the STSs for each training session could be used as they are. Or, to improve OBJ1, one could do the following for each t separately $(t = 1, \ldots, 4)$: for each home base node $j \in J$, collect all the STS nodes on routes for the tth training session that contain home base j. Let this be the set $T_{t,j}$. If this home base has only one mobile trainer stationed at it, find a minimum length traveling salesman tour covering all the nodes in $\{j\} \cup T_{t,j}$, and take that tour as the route for that mobile trainer for this training session. If there are two or more mobile trainers stationed at this home base, the allocation of STSs in $T_{t,j}$ to these mobile trainers is a single depot

VRP (a multiple salesmen traveling salesmen problem) with vehicle capacities, for which several nice heuristic or exact methods are available [2, 5, 6]. Once the set of *STS*s, say Δ, to be handled by one of these mobile trainers is determined, the route for that mobile trainer is a minimum length single traveling salesman tour covering all the nodes in $\{j\} \cup \Delta$.

In our problem each home base had exactly one mobile trainer stationed at it, so in Step 7 we had to solve just one single traveling salesman problem for each home base. Once the routes for all the mobile trainers for each training session have been determined, the training sessions can be manually scheduled over the year, one after the other, in some order. The Army found this manual procedure quite convenient to implement.

4 Computational Results

The p-median model for HB selection has 4872 binary and 29 integer variables in the armor version, and 4727 binary and 29 integer variables in the infantry version. The set covering model for *STS* selection has about 400 binary variables. The traveling salesman problems in Step 7 of the cycle cover heuristic (Stage 3) all involved 16 or less cities, and we used a well known mixed $0-1$ integer programming formulation for each of them (Tucker [10]). We solved each of these integer programming models using the GAMS [1] modeling language with the OSL solver on a Sun Sparc workstation. Each problem was solved to optimality in no more than 5 minutes of CPU time.

We now discuss the solutions we obtained.

Home Bases Selected, Implemented

In the optimum solution of the p-median models, 11 (10) home base sites were selected for the armor (infantry) version with a single mobile trainer stationed at each. Six home base sites are common to the armor and infantry versions, i.e., each of them will station both an armor and an infantry version of the mobile trainer.

The Training Directorate of the Army National Guard Bureau carefully reviewed these home base selections with respect to various practical and implementation aspects. They accepted 19 of 21 home base selections generated by the model, but changed the remaining two sites for practical and managerial convenience. They changed Edwards AFB, CA, selected as an armor home base site, to Camp Roberts, CA (about 220 miles from Edwards AFB) because the facilities at Camp Roberts are considered superior for training purposes. They also changed Catoosa, GA, selected as an infantry home base site, to Ft. McClellan, AL, for the same reasons. With these two modifications, the home base site selections were finalized and implemented by the Army in 1995 [12]. The names of the final home base sites are provided in tables given below.

Forty armor armories are within MTD of at least one of the armor home bases, and 42 infantry armories are within MTD of at least one of the infantry home bases. These armories have been allocated to their nearest home base for training. This leaves a total of 128 armor armories, and 121 infantry armories still to be allocated to a training center.

STSs Selected

Forty two STSs were selected for the armor version, and 37 STSs were selected for the infantry version, in the optimum solutions of the set covering models.

An Illustration of the Routing Results

As an example, we show the routes obtained for the armor mobile trainer stationed at Camp Roberts, CA. Camp Roberts, CA, was the best home base to insert into six different cycles in the simple cycle cover of minimum total length for all the secondary training sites selected in the armor version. We represent each of these cycles by the order in which the STSs in it occur. These are:

Apple Valley, CA – Henderson, NV – Apple Valley, CA

El Centro, CA – Indio, CA – Vista, CA – El Centro, CA

Freedom, CA – Yerington, NV – Hollister, CA – Freedom, CA

Great Falls, MT – Helena, MT – Great Falls, MT

Dillon, MT – Hamilton, MT – Dillon, MT

Palmdale, CA – Pomona, CA – Palmdale, CA

The route for the mobile trainer stationed at Camp Roberts, CA to cover the secondary training sites (14 in all) in all of these cycles is the minimum distance traveling salesman tour covering these 14 sites and Camp Roberts. This tour is:

Camp Roberts, CA – Palmdale, CA – Pomona, CA – Vista, CA – El Centro, CA – Indio, CA – Apple Valley, CA – Henderson, NV – Hamilton, MT – Great Falls, MT – Helena, MT – Dillon, MT – Yerington, NV – Hollister, CA – Freedom, CA – Camp Roberts, CA.

The Training Director at the Army National Guard Bureau reviewed our secondary training site selections, and the routes for the mobile trainers to cover them, approved them, and incorporated them into the system fielding plan. The National Guard renegotiated the original logistics support contract to reflect changes implied by our solution.

Solution Summary

The selected home base sites, each stationing one mobile trainer, are shown in Table 1 for the armor version, and Table 2 for the infantry version. These tables also summarize the total route mileage of the mobile trainer stationed at each home base, its utilization percentage, and the total platoon-bus miles of all the platoons trained by that trainer.

Table 1: Armor Solution Summary

Home Base	Mobile Trainer		Platoon-bus
	miles	utilization	miles
Ft. Polk, LA	3751	100%	348
Camp Bowie, TX	5444	100%	1590
Ft. McClellan, AL	3888	100%	1316
Camp Dodge, IA	5876	100%	3493
Camp Shelby, MS	1740	88%	2112
Camp Perry, OH	2394	95%	1954
Camp Roberts, CA	5800	100%	789
Catoosa, GA	1372	100%	2410
Leesburg, SC	1480	95%	1605
Ft. Indianatowngap, PA	1297	100%	1489
Gowenfield, ID	3896	100%	3200
Total Miles	36938		20306

Table 2: Infantry Solution Summary

Home Base	Mobile Trainer		Platoon-bus
	miles	utilization	miles
Ft. Polk, LA	2600	95%	2454
Camp Bowie, TX	3926	67%	324
Camp Shelby, MS	8318	95%	1097
Camp Robert, CA	2090	95%	2040
Ft. McClellan, AL	1696	100%	3024
Ft. Bragg, NC	2816	86%	1316
Ft. Indianatowngap, PA	1454	100%	2825
Ft. Dix, NJ	1256	81%	1458
Leesburg, SC	1616	95%	3006
Yakima, WA	10140	95%	1294
Total Miles	35912		18838

The total yearly mobile trainer fleet mileage is 72, 850 miles. The total platoon-bus miles for the platoons to reach their training center is 39, 144 miles. All platoons travel less than the stipulated MTD of 100 miles to reach their training center. The utilization rate for 17 of the 21 mobile trainers exceeds 95%. Although such high utilization rates for mobile computing equipment may cause concern, the logistics support contract contains maintenance time of five weeks per year. All enhanced armories will receive the required four training sessions per year, and all the nonenhanced armories participate in one or two training sessions as specified by the Army. Thus our solution satisfies all the conditions spelled out by the Army National Guard Bureau's training strategy.

Cost Reduction

Originally the Army signed a preliminary procurement contract with the prime contractor for constructing and operating the CCTT program which specified the estimate of 44 moves per year, averaging 250 miles per move, for a total of 11,000 miles per year for each mobile trainer, leading to a total mobile trainer fleet mileage of 231,000 miles per year [11]. Our solution calls for only 72, 850 mobile trainer miles per year, or about 70% fewer miles than the original contract. The infantry version of the mobile trainer contains six tractor-trailer vans in a set, and the armor version five, and the cost per van mile has been conservatively estimated to be over $10. At this rate, the annual savings produced by our procedure, in mobile-trainer fleet mileage cost, amounts to over $8.6 million. The mobile trainer system has an expected life of 20 years. Over the life of the system, at a discount rate of 3.5% per year, the cost savings in mobile trainer mileage alone amounts to over $123 million. Additional indirect savings will accrue due to reduced wear-and-tear on these mobile computing systems, and diminished probability of traffic accidents due to significant reduction in miles traveled.

As a side benefit, a one-time cost saving accrues from a reduction in the number of armory facility upgrades. At each secondary training site, the mobile trainer system requires a stable concrete pad, electrical conditioning equipment, and security fences. The cost of upgrading is estimated to be approximately $10,000/facility. Our solution requires 79 sites to undergo facility improvement to serve as secondary training sites, compared to hundreds in the original plan. This yields a one time savings of approximately $3 million in capital costs.

5 Conclusion

Each year, some changes are expected to occur in the list of armories that require training and in the number of training sessions that some armories have to receive that year. This may mean that a small number of additional secondary training sites have to be selected. So, operationally, it may be necessary to run a simplified version of the *STS* selection model every year, and rework the routes for the mobile trainers and schedules for the armories.

We have worked very closely with the Army National Guard and Reserve Command, the TRADOC System Manager for Combined Arms Training, and the Program Manager for Close Combat Tactical Trainer, over the course of four years to clarify the decision process for this problem. Our models have undergone numerous revisions as we have incorporated various conditions that have arisen as a result of the solution review process. In addition, modifications to the projected Army National Guard structure and personnel strength have necessitated additional analysis. Our sponsors have seen the value of modeling in giving them the ability to explore the effects of changing various parameters on the quality of the solution and, ultimately, the training strategy. While reducing costs is important, we and our sponsors fully understand that providing quality training is the primary objective because the performance of soldiers critically depends on the effectiveness of their training.

Acknowledgement: We are grateful to the referees and the editors for suggestions that helped improve the presentation of this paper substantially.

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Revision 3: Summary of revisions carried out

1. All the comments marked by Professor Brandeau with a pen in the previous version are incorporated.

2. In page 12, the notation used to represent simple cycles is explained as standard notation, and a reference provided for it.

3. The editorial suggestions made by the AE are incorporated.

4. Referee 1 suggested deleting a para in page 5. This was done.

5. Referee 1 asked to better justify claimed savings by giving references. Unfortunately these are not published references, but Army documents that are not accessible to the public. We provide references to these documents.

6. Referee 1 pointed out as a topic for future research that savings can be made by relaxing the Army policy mentioned in page 11. We know this. In our first solution submitted for Army review we ignored this policy, but were overruled.

7. Referee 2 asked whether the para about two-week exercises in page 3 can be omitted. We would like to leave it in because it explains the reasons why only 29 sites in the country are suitable for being home bases.

8. Referee 2 suggested omitting the Narula reference if we did not use that algorithm. This was done.

9. Referee 2 asked whether scheduling the armories for training over the weekends of the year is part of Stage 3. No, Stage 3 is only concerned with finding routes for mobile trainers to cover all the STSs.

The Army decided to do the scheduling manually, after the routes are finalized. The para just above Section 4 explains this.

10. Referee 2 asked whether the TSPs in Step 7 of the Stage 3 algorithm were solved using a variant of 3-opt etc. No, these were all solved exactly using the mixed $0-1$ integer programming formulation of TSP originally due to A. Tucker. We added the Tucker reference, and mentioned this fact in the first para in Section 4 (page 15).

11. Referee 2 asked how the Stage 3 algorithm handles the case of a home base with two or more mobile trainers. This did not happen in our problem, because each home base had exactly one mobile trainer stationed at it.

However, we revised Step 7 in the Stage 3 algorithm to mention what to do if some home bases happen to have more than one mobile trainer stationed at them.

12. Referee 2 wanted to know what to do if a home base has only a couple of *STS* allocated to it. As explained in pages 12, 11, the special features of the problem (Euclidean distances, and the widespread of the points over a large geographical area, leading to each cycle in the cycle covers having only a small number of nodes), and the capacity limits of mobile trainers, resulted in this phenomenon not occurring.

But if this occurs, the mobile trainer needs to visit any *STS* at most four times because four is the maximum number of training sessions any unit gets. After finishing its work the home base can be shut down for the year.

In a future study we can investigate what the optimal capacity is to provide at each home base. Then if a mobile trainer has finished its work at a home base, we can transfer it to another home base where additional training capacity is needed, for the rest of the year.

Our thanks to the referees and the editors for all their patient help.