7.7 A company manufactures products A to G using two types of machines P_1, P_2; and three raw materials R_1, R_2, R_3. Relevant data is given below.

Item	Item input (in units) to make one unit of							Max. available per day
	A	B	C	D	E	F	G	
R_1 (in units)	0.1	0.3	0.2	0.1	0.2	0.1	0.2	500
R_2	0.2	0.1	0.4	0.2	0.2	0.3	0.4	750
R_3	0.2	0.1	0.1	0.2	0.1	0.2	0.3	350
P_1 time (mc. hrs.)	0.02	0.03	0.01	0.04	0.01	0.02	0.04	60
P_2 time	0.04		0.02	0.02	0.06	0.03	0.05	80
Constraint on daily output	$\geqq 200$	$\leqq 800$			$\leqq 400$			
Profit (\$/unit)	10	12	8	15	18	10	19	

Formulate the product mix problem to maximize total daily profit as an LP, and solve it using one of the available software packages to obtain primal and dual optimum solutions. Also, answer each of the following questions about this original problem.

(i) Are the marginal values of the various items well defined in this problem? If so, what are they?

(ii) Is it worth increasing the supply of R_1 beyond the present 500 units/day? The current supplier for R_1 is unable to supply any more than the current amount. The procurement manager has identified a new supplier for R_1, but that supplier's price is \$15/unit higher than the current suppliers'. Should additional supplies of R_1 be ordered from this new supplier?

(iii) The production manager has identified an arrangement by which 20 hours/day of either P_1- or P_2-time can be made available at a cost of \$150/day. Is it worth accepting this arrangement?

(iv) The sales manager would like to know the relative contributions of the various products in the company's total profit. What are they?

(v) The sales manager believes that product C is priced too low for a good image. This manager claims that if the selling price of C were increased by \$2/unit, the demand for it would be 600 units/day. What is the effect of this change? The production manager claims that the manufacturing process for G can be changed so that its need for P_1-time goes down by 50% without affecting quality, demand or selling price. What will be the effect of this change on the optimum product mix and total profit?

(vi) The production manager believes that by changing specifications, it should be possible to make product B with 33.3% less of R_1, this would have no effect on the saleability of this product. What will be the effect of this change on the optimum product mix and total profit?

(vii) The company's research division has formulated a new product, H, which they believe can yield a profit of \$8-10/unit made. The input requirements to make one unit of this product will be

Item	R_1	R_2	R_3	P_1-time	P_2-time
Input	0.1	0.2	0.1	0.02	0.02

Is this product worth further consideration?

(viii) The sales manager feels that the selling price/unit of product F can be increased by \$2 without affecting the demand for it. Would this lead to any changes in the optimum production plan? What is the effect of this change on the total profit?

(D. C. S. Shearn [1984])

AMPL Model file

```
set P;

set Item;

param IO {Item, P};

param C {j in P};

param RHS {Item};

param l {j in P};

param u {j in P};

var x {j in P} >= 0;


maximize profit: sum {j in P} C[j]*x[j];

subject to MBEQ {i in Item}: sum {j in P} IO[i,j]*x[j] <= RHS[i];

limit {j in P}: l[j] <= x[j] <= u[j];
```

AMPL Data file

```
set P := A B C D E F G;

set Item := R1 R2 R3 P1 P2;

param IO default 0 :=

[R1,A] 0.1 [R1,B] 0.2 [R1,C] 0.2 [R1,D] 0.1
```

```
[R1,E] 0.2 [R1,F] 0.1 [R1,G] 0.2
[R2,A] 0.2 [R2,B] 0.1 [R2,C] 0.4 [R2,D] 0.2
[R2,E] 0.2 [R2,F] 0.3 [R2,G] 0.4
[R3,A] 0.2 [R3,B] 0.1 [R3,C] 0.1 [R3,D] 0.2
[R3,E] 0.1 [R3,F] 0.2 [R3,G] 0.3
[P1,A] 0.02 [P1,B] 0.03 [P1,C] 0.01 [P1,D] 0.04
[P1,E] 0.01 [P1,F] 0.02 [P1,G] 0.04
[P2,A] 0.04 [P2,C] 0.02 [P2,D] 0.02 [P2,E] 0.06
[P2,F] 0.03 [P2,G] 0.05;
param l default 0 :=
A 200;
param u default Infinity :=
B 800 E 400;
param RHS :=
R1 500 R2 750 R3 350 P1 60 P2 80;
param C:=
A 10 B 12 C 8 D 15 E 18 F 10 G 19;
```