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How to Build a Mathematical Model?

Katta G. Murty Lecture slides

How To Build a Mathematical Model?

1. Identify All Decision Variables

Controllable parameters whose values can be controlled by decision maker, which affect functioning of system. Denote them by $x_1, \ldots, x_n$.

$$x = \quad \text{Decision vector} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \quad (x_1, \ldots, x_n)^T$$

2. Identify Objective Function and All Constraints on Decision Variables

$$g_i(x) = b_i \quad \text{An Equality Constraint}$$

$$\left.\begin{array}{l} g_i(x) \geq b_i \\ g_i(x) \leq b_i \end{array}\right\} \quad \text{Inequality Constraints}$$

Constraint Functions, Right Hand Side (RHS) Constants.

$x_j \geq b_j$ or $x_j \leq b_j$	Lower or Upper bound constraints on individual variables.
$x_j \geq 0$	Lower bound constraint called *Non-negativity restriction.*

Objective function called $\begin{cases} \textit{COST FUNCTION} & \text{if to be min.} \\ \textit{PROFIT FUNCTION} & \text{if to be max.} \end{cases}$

Some Definitions

LINEAR FUNCTION: One of form $c_1x_1 + \ldots + c_nx_n$

where $c = (c_1, \ldots, c_n)$ is *coefficient vector* of variables in it.

Example: $x \in R^4$. $3x_2 - 7x_4$ is a linear function with coefficient vector $(0, 3, 0, -7)$.

AFFINE FUNCTION: A linear function + a constant, i.e., One of form $c_0 + cx$.

FEASIBLE SOLUTION: A vector x that satisfies all the constraints.

OPTIMUM SOLUTION: A feasible solution that gives the best value for

objective function among all feasible solutions.

LINEAR PROGRAM: Optimization problem in which objective
function and all constraint functions are linear.

Steps in Modeling A Linear Program

1. LIST ALL DECISION VARIABLES: Each decision variable is the level at

   which an *ACTIVITY* is carried out.

2. VERIFY LINEARITY ASSUMPTIONS: *Proportionality Assumption*

   and *Additivity Assumption*. Must hold for

   objective function and all constraint functions.

3. VERIFY ALL VARIABLES ARE CONTINUOUS VARIABLES:

4. CONSTRUCT OBJECTIVE FUNCTION:

5. IDENTIFY ALL CONSTRAINTS & BOUNDS ON INDIVIDUAL VARIABLES: Each

   constraint is *Material Balance Equation* or *Inequality*

   of an *ITEM*.