O. R.

Katta G. Murty Lecture slides

Operations Research is the branch of science dealing with techniques for *optimizing* the performance of systems.

System is any organization, large or small.

Performance Measure Any criterion measuring the way system is working, that is considered important. For example, if system is a company, some important performance measures are: total yearly profit, market share, etc.

Optimizing A Performance Measure Performance measures are of two types:

PROFIT MEASURE: is one, for which the higher its value, the more desirable it is.

COST MEASURE: is one, for which the lower its value, the more desirable it is.

Optimizing a performance measure means maximizing it if it is a profit measure, or minimizing it if it is a cost measure.

Examples of Systems & Relevent Performance Measures

1. A manufacturing company:

2. A Dam on a river for irrigation and hydro-electric power generation:

3. A Farmer with 50 acres of Farmland

Optimization involves Maximization or Minimization as desired.

To optimize implies Decision making to determine best operating conditions of system that yield best values for performance measures.

Main function of O. R. ⟶ Tool for decision making.

Optimization is main pillar of O. R.
Everyone is interested in Optimization!

O. R. Techniques:

Multicriteria Decision-making : To chose the best among a small number of alternatives, when there are many important characteristics to consider.

Linear Programming: To optimize a single linear objective function subject to linear constraints. A technique with many applications.

Network Programming: Optimization problems that can be described over a graph structure. Many applications in transportation, distribution, etc.

Dynamic Programming: Technique for solving multi-period or multi-stage decision problems.

CPM for Project Scheduling: Techniques for controlling, scheduling various jobs in a project, which have to obey a precedence relationship.

Integer and Combinatorial Optimization: Techniques for solving mathematical models involving discrete decision variables.

Nonlinear Programming: Techniques for solving mathematical models involving nonlinear functions.

Mathematical Programming: Includes linear, network, integer, combinatorial, and nonlinear programming.

Multi-objective Programming: Techniques for obtaining good solutions to optimization models involving two or more objective functions that need to be optimized simultaneously. **Goal Programming** is one technique for solving multi-objective programs.

Inventory Management: Techniques for maintaining inventories to meet demands for goods, while at same time minimizing inventory holding costs.

Queing Problems: To optimize performance of systems involving waiting lines.

Simulation: Techniques for modeling the operations of a system through computer simulation.

Algorithms

Many of the problems in O. R. are solved using procedures called algorithms.

An Algorithm for a Problem: is a systematic step by step procedure for solving the problem.

History: Word comes from title of Latin translation

Algoritmi de Numero Indorum

meaning *Al-Khwarzmi Concerning the Hindu Art of Reckoning* written in 825 AD by Arabic Scholar Muhammad Ibn Musa Al-Khwarizmi who lived in Iraq, Iran (mostly Bagdad), based on earlier Indian and Arabic treatizes.

Algorithms seem to have originated in work of ancient Indian mathematicians on rules for solving linear and quadratic equations.

An Example: Problem: Input: Given Positive Integer n. Find all primes $\leq n$.

Eratosthenes Sieve (3rd century BC)

Step 1: Write all integers 1 to n. Strike off 1 and set a pointer at 2, the first prime.

General Step: Suppose pointer now on r. Counting from r, strike off every rth number to right.

If all numbers to right of r now struck off, all numbers not struck off are all primes $\leq n$, terminate.

Otherwise move pointer right to next number not struck off , and repeat this general step with pointer in its new position.

EXERCISE: $n = 15$.

Classification of Optimization Problems

Optimization Problem: To select best among available alternatives

Category 1: Number of available alternatives very small: Usually solved by a *Scoring Method*.

Category 2: No. of alternatives finite but large: Integer & combinatorial optimization problems belong to this category.

Category 3: Number of alternatives ∞: Continuous variable problems (linear and nonlinear optimization problems) belong to this category.

For category 2, 3 problems, we need to construct a Mathematical Model for the problem, and solve it using an efficient algorithm.

List important characteristics to consider in following Category 1 Optimization Problems

1. Boy meets Girl: Boy has 3 girl friends. To decide with whom he will get engaged.

2. Supermarket Location: Company setting up new supermarket. 6 sites under consideration.

3. Fire Station Location: City setting up a new fire station. 5 sites under consideration.

4. Choosing Supplier: Company needs to select a supplier. 7 suppliers under consideration.

5. Engine Block Selection: Car company needs to decide whether to use cast iron engine block, or aluminium engine block for new model.

Scoring Method for Category 1 Problems

1. Measurement: For each alternative, measure each characteristic. Measurement could be quantitative, or it could be a score, say between 0 to 100 (for example, the higher the score the more desirable the alternative).

NOTE: All measurements should be expressed either as costs for all characteristics (Smaller the better); or as profits for all characteristics (larger the better). Cost can be converted into profit and vice versa, by multiplying by -1.

2. Convert to Common Unit: Convert measurements of all characteristics into one common unit using reasonable assumptions.

3. Determine Weights for All Characteristics: The weight indicates the importance of the characteristic, the higher

the weight, the more important the characteristic.

4. Compute Combined Score of each Alternative: It is the sum over the characteristics of the measurement multiplied by weight.

5. Select the Best Alternative: It is the one with highest (*lowest*) combined score if all measurements are profits (*costs*).

Example: A person has decided to buy a new car. Seven cars are investigated. The criteria are price, comfort, gas mileage, and looks. Data is given below. Determine which car the person should buy.

Characteristic	car							Weight
	1	2	3	4	5	6	7	
Price ($ 1000 units)	15	13.5	12.5	13	12	12	11	5
Comfort	E	E	A	A	A	W	W	4
MPG	20	17	22	24	18	25	28	3
Looks	D	D	D	O	D	D	O	3

E = Excellent, A = Average, W = Weak, D = Distinctive, O = Ordinary

Weight is importance of characteristic, the higher, the more important

HW Problems:

1.1 From *Mahabharata*, the great Indian epic dated earlier than 5000 BC. It is about the marriage of a beautiful princess Satyabhama.

Trait	Score on trait of				Weight
	Krishna	Sisupala	Jarasandha	Rukmi	
1. Easygoing nature, friendly disposition	80	70	60	50	10
2. Being a lively and animated companion	90	95	70	65	8
3. Sharing her concern about destruction of nature	40	30	20	45	10
4. Willingness to limit family size to two children	50	30	60	25	10
5. Archery skill	60	70	80	70	7
6. Skill in negotiating deals with opposing parties	80	75	70	60	6
7. Concern for people, particularly those of other tribes etc.	60	45	45	45	5
8. Willingness to let females to join in wars	60	40	40	40	8

Satya was very progressive. Most of her girl friends looked forward to having lots of children and a large family, she considered that not suitable as a goal for women. She was not opposed to having ≤ 2 children, but she felt very strongly that women should develop a passionate interest in something more worthwhile than bringing up a lot of children. Even in those days she was concerned about population growth and consequent destruction of nature. She used to go hiking in the forest on the outskirts of her father's capital city, and she particularly admired a rare flowering bush called *Parijata* in that forest. Every morning it used to blossom forth with what appeared to be a million flowers with very bright orange stems and a heavenly fragrance. To her great grief that Parijata bush was devastated in a recent spate of house building as the city expanded, and she was very concerned that it may have become extinct.

She became an expert in horse riding, and driving chariots, quite unusual for a woman in those days. She learned to launch arrows using a bow with deadly accuracy, and could compete with the best archers in her kingdom. Fighting little wars was almost daily work for kings in those days, and she made up her mind that after marriage she would join her husband in any wars that he may have to fight.

In those days in India, marriages for princesses used to be organized through a function called *swayamvara* (which literally means "self-chosen"). All the eligible princes would be invited to a gala party. There would be sumptuous meals followed by dancing where the princess dances and chats with each visiting prince. There would be contests in archery etc. where the various suitors display their skills. During this entire process the princess is gathering information about each suitor and weighing her choices. When her decision is finalized, she would come out with a garland of flowers with which she would adorn the prince of her choice, and then the wedding would be celebrated.

At Satya's swayamvaram there were four suitors, Krishna, Sisupala, Jarasandha, and Rukmi. The personality traits that she considered important in her future husband are listed in the left hand column in the above table. She scored each suitor on each trait on a scale of 0 to 100 (the higher the score, the more desirable the suitor is on that trait). In the rightmost column of the table we provide the weight for each trait which measures the relative importance she attached to that trait (again, the higher the weight, the more important she considered that trait to be).

Help Satya choose her fiance from among the four suitors.

1.2. To determine the airconditioning system to be installed in the city library being built. 5 feasible plans called A_1 to A_5 offered. Alternatives to be evaluated under three major impacts: economic, functional, and operational. Two monetary attributes and six non-monetary attributes emerged from these impacts. The ratings of each alternative WRT each attribute are given below (the higher the rating, the more the preference for that alternative by that attribute; in fact a rating of 1 denotes the worst alternative, and a rating of 10 denotes the best). The weights for the various attributes in combining the ratings into the composite index evaluating the alternative are also given in the following table. Determine which airconditioning plan should be selected for the library.

Attribute	Weight	Rating by attribute for alternative				
		A_1	A_2	A_3	A_4	A_5
1. Economic						
1.1 Installation cost	0.0455	4.2	2	3.5	4	3
1.2 Monthly operational cost	0.0911	6.4	5.2	4.7	5	5.5
2. Functional						
2.1 Performance	0.1297	9	5	5	8	7
2.2 Comfort (noise level etc.)	0.1297	3.5	7	6.5	4	5.5
3. Operational						
3.1 Maintainability	0.1749	4	4	8	9	6
3.2 Reliability	0.2216	9.5	7.5	8.5	9	9
3.3 Flexibility	0.0426	4	8	8	7	5
3.4 Safety	0.1647	7	5	6	8	9

1.3. For the fall campaign the a presidential candidate has to decide how to allocate advertising budget among the four media: TV, radio, newspapers and magazines, and billboards. The expenses have already been worked out by campaign manager, and choice narrowed to two levels, low (L), or high (H), in each media. If H level chosen for TV advertising, budget would only permit L level advertising in each of the other three media. On the other hand if

L level chosen for TV advertising, budget would permit him H level in two of the other three media and at the L level in the remaining.

Statistical advisers came up with estimates in the table given below for the reach of the various media.

Each person who is positively (negatively) influenced by the advertisements is expected to discuss and positively (negatively) influence an additional 2.5 (3.5) persons in the same age group through personal conversations. Only 25% of the people in the age group 20-30 years, 50% of the people in the age group 30-60 years, and 70% of the people in the age group 60 years and up, are expected to vote; the corresponding fractions are the weights for the three age groups in developing a combined score for each alternative. The overall score for any alternative is the weighted average over the different age groups of (the number of positively influenced people − the number of negatively influenced people) summed over all the media. Determine the best advertising strategy for the candidate.

		Estimated number of people (millions) in age group who are influenced					
Medium	Advertising level	20-30 years		30-60 years		60 and up	
		P	N	P	N	P	N
TV	L	5	1	12	3	5	2
	H	9	1.5	20	4	8	3
Radio	L	2.5	0.9	6	1	1.8	0.6
	H	5	0.4	12	1.8	4	0.8
Newspapers and magazines	L	1.6	0.2	4	0.4	1.5	0.3
	H	3	0.1	8	0.6	3	0.2
Billboards	L	0.7	0.2	2	0.2	0.5	0.1
	H	1.2	0.3	4	0.3	0.7	0.2

P = positively influenced, N = negatively influenced

1.4. **An American in Hong Kong:** An IOE student spent a year as an exchange student at HKUST in Hong Kong, during which time she developed very close friendships with several students there. She planned to treat all these friends to a sumptuous lunch at one of the many fine restaurants on HKUST campus after her final exams. there.

The restaurants she is considering are: GFCR (Ground floor Chinese Restaurant), LG1AR (American style restaurant on LG1 floor), CS (Coffee Shop style restaurant), LG1C (Cafateria style restaurant on LG1 floor), LG7SR (Singapore style restaurant on LG7 floor), LG5CR (Cantonese style restaurant on LG5 floor); the top 6 restaurants on that campus.

She considers 5 criteria to be important for selecting the restaurant to go to. These are: CC (Comfort and Class), V (Variety of available dishes), FT (Food Taste), P (Price) and S (Speed of service).

Of these characteristics CC, V, FT are rated with scores: D (Distinctive), E (Excellent), A (Average), O (Ordinary), W (Weak) in decreasing order of merit [i.e., “D” is the best, and “W” is the least meritorious].

Price P is given in terms of expected charge (in HK $) per head for the food she is planning to order. Speed of service S is given in terms of expected no. of minutes for the food to be served. For both these characteristics, *the smaller the value, the better.*

Here is all the data on the restaurants. The *weight* measures the importance she attaches to the characteristics; the higher the weight, the more important the characteristic.

Characteristic	Rating for restaurant						Weight
	GFCR	LG1AR	LG1C	CS	LG5CR	LG7SR	
CC	D	D	E	A	W	O	5
V	D	E	E	O	O	E	4
FT	D	O	A	E	W	E	4
P (HK $)	108	123	83	85	65	75	3
S (mts.)	25	20	15	15	10	20	2

Determine where she should hold her lunch, showing all your work very carefully. Any assumptions you make should be stated clearly with justification.

Category 2 Problems: An Example : An Assignment Problem in Marketing

Company operates in 6 zones with different backgrounds (economic status, ethnicity of shoppers etc.). To appoint Marketing Director for each zone. 6 candidates of different backgrounds selected. Need to appoint one of them to each zone. Data below. OBJEVTIVE: Find assignment that maximizes total annual sales.

Table 1.1

	c_{ij} = annual sales volume in \$million if candidate i assigned to zone j					
Zone j =	1	2	3	4	5	6
Candidate $i = 1$	1	2	6	10	17	29
2	3	4	8	11	20	30
3	5	7	9	12	22	33
4	13	14	15	16	23	34
5	18	19	21	24	25	35
6	26	27	28	31	32	36

The Greedy Method and its Drawbacks

Most commonly used method, everyone uses it. Here are the steps:

Transform: Transform problem into one of making a series of small decisions one after the other.

After each small decision, fix it, and move on to the next, until whole solution obtained.

Greedy Selection: For each small decision, select the best alternative (by an appropriate greediness criterion) available for it at that time.

EXAMPLE: Marketing Problem:

WARNING: The solution obtained by greedy method may depend on:

greediness criterion used

order in which decisions are made.

In general, greedy method is a **heuristic method** not guaranteed to give optimum solution. If you use greedy method, pl. check carefully how good the solution obtained is, very carefully.

Category 3 Problems: An Example: A transportation problem:

To ship iron ore from mines to steel plants at minimum cost. Data below.

	c_{ij} (cents/ton)			Availability at
Plant	$j = 1$	2	3	mine (tons) daily
Mine $i = 1$	11	8	2	800
2	7	5	4	300
Requirement at plant (tons) daily	400	500	200	

HW problems.

1.5. Find greedy solution to problem of assigning jobs to machines with following cost matrices, to minimize total cost. In each case check if you are able to verify whether greedy sol. is a good one. Entry in matrix is cost ($) of performing job on machine. Each machine can do only one job, each job goes to one machine. A "." entry indicates job can't be handled by mc.

(a)	Mc. 1	2	3	4	5
Job 1	15	22	13	4	19
2	.	21	13	4	15
3	16	20	15	7	18
4	6	11	22	.	25
5	3	7	4	9	10

(b)	Mc. 1	2	3	4	5	6
Job 1	10	11	10	5	6	4
2	5	26	14	18	15	10
3	6	.	18	17	15	8
4	2	14	16	16	24	.
5	4	15	19	10	8	14
6	10	22	22	15	28	24

(c)	Mc. 1	2	3	4	5	6	7
Job 1	.	6	17	9	8	10	3
2	8	33	45	15	20	.	10
3	5	19	30	16	14	22	8
4	7	22	35	25	27	26	12
5	2	10	24	18	31	14	11
6	4	12	.	17	18	18	12
7	8	18	21	18	18	18	14

1.6. Following tables give data for balanced transportation problems to minimize total shipping cost. Rows correspond to sources with a_i available to ship out. Cols. correspond to sinks with material requirements b_j tons. Entry in matrix c_{ij} = cost to ship from source i to sink j per ton. Find greedy solution. If possible check whether they are really optimal to problem.

(a)	Sink $j = 1$	2	3	4	5	6	a_i
Source $i = 1$	$c_{11} = 5$	3	7	3	8	5	4
2	5	6	12	5	7	11	3
3	2	8	3	4	8	2	3
4	9	6	10	5	10	9	7
b_j	3	3	6	2	1	2	

(b)	Sink $j = 1$	2	3	4	5	a_i
Source $i = 1$	$c_{11} = 13$	16	3	12	28	15
2	22	28	15	12	20	15
3	21	29	6	18	14	12
4	17	29	3	25	29	32
5	4	14	4	15	2	26
b_j	3	39	27	7	24	

(c)	Sink $j = 1$	2	3	4	a_i
Source $i = 1$	$c_{11} = 25$	36	20	10	45
2	47	40	30	20	70
3	20	33	15	14	50
b_j	60	40	15	50	

1.7. A salesman has the following itinery: Start in city 1; visit each other city exactly once in some order; return to city 1 at end.

The order in which cities are traveled (for example: 1-2-3-4-5-6-1 is the natural order) is called a **tour**. The cost of a tour is the sum of costs of traveling between consecutive pairs in it. Following are cost matrices giving costs for travel between various pairs of cities. Find the greedy sols. for minimum cost tour problem, beginning always with starting city 1.

(a)	to $j = 1$	2	3	4	5	6
from $i = 1$	x	15	10	14	13	20
2	15	x	16	18	18	8
3	10	16	x	28	25	24
4	14	18	28	x	3	17
5	13	18	25	3	x	13
6	20	8	24	17	13	x

(b)	to $j = 1$	2	3	4	5	6	
from $i = 1$	x	13	9	19	22	3	7
2	6	x	7	8	39	4	8
3	18	3	x	5	18	4	3
4	4	16	6	x	5	13	16
5	14	22	99	17	x	8	10
6	3	21	7	3	10	x	9
7	11	19	8	19	8	3	x

(c)	to $j = 1$	2	3	4	5	6
from $i = 1$	x	27	43	16	30	26
2	7	x	16	1	30	25
3	20	13	x	35	5	0
4	21	16	25	x	18	18
5	12	46	27	48	x	5
6	23	5	5	9	5	x

1.8. In problem 1.7 find greedy solutions starting at city 3 instead of city 1. Are same solutions produced here?

1.9. In problem 1.7 suppose the starting city is left to your choice. Find the best greedy solution under this situation.

1.10. We have a knapsack which can hold upto 16 KG by weight. Our aim is to load a subset of following objects in knapsack, to maximize total value of objects loaded subject to th weight limit given above. Objects can't be broken, they have to be loaded whole or left out. Find greedy sol. clearly specifying how you are applying greedy algorithm.

(a)	Object j	Weight w_j	Value v_j
	1	2	16
	2	15	20
	3	1	6
	4	13	13

Find true opt. sol. by enumeration & check how good your greedy sol. is.

(a)	Object j	Weight w_j	Value v_j
	1	2	16
	2	15	20
	3	1	6
	4	13	13

Same problem as above, but the knaosack's weight capacity is 35. Here, try different criteria to be greedy upon and compare results obtained.

(b)	Object j	Weight w_j	Value v_j
	1	3	21
	2	4	24
	3	3	12
	4	21	168
	5	15	135
	6	13	26
	7	16	192
	8	20	200
	9	40	800