Drink Me!

An Introduction to The Periodic Tables of Physics

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Based on the You-Tube presentations:

The Periodic Tables of Physics (parts 1-9)
[December 2008]

and

The Periodic Table of Electromagnetic Elements (parts 1-6)
[February 2009]

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Abstract

The periodic tables of physics are built using a small set of rules that describe the mathematical relationships between physical elements, and provide a graphical representation of those relationships. The periodic tables of physics are a powerful tool for interpreting, understanding and formulating mathematical expressions of physical relationships. The periodic tables of physics enforce self-consistency and completeness when they are applied to the analysis of physical problems.

The periodic table of mechanical elements is based on the basic definitions and fundamental theorems of classical mechanics. It shows the relationships between the elements of classical mechanics.

The periodic table of electromagnetic elements is based on the integral form of Maxwell's equations. It shows the relationships between the elements of classical electromagnetics, and clearly shows that a mechanical foundation underlies classical electromagnetics.

The periodic table of thermodynamic elements is based on a fundamental proposition that follows logically from the construction of the periodic tables of mechanical and electromagnetic elements. The periodic table of thermodynamic elements suggests relationships between the elements of thermodynamics. It also suggests that a mechanical foundation underlies thermodynamics.

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Preface

A new theory is like a rough rock torn from the ground. It's value cannot be determined until it has been polished.

Foreword

Students of physics are foot-soldiers in the battle for understanding. As understanding is refined, details of the battlefield are redrawn. While the battlefield may be redrawn, the landmarks must remain.

Mechanical Elements

Power	Energy	Action	Moment of Inertia
	Force	Momentum	Moment
	Spring Constant	Impedance	Mass
	Pressure		Linear Density
			Areal Density
			Volumetric Density

Diagram 1: The Periodic Table of Mechanical Elements

Introduction

Force ($\vec{\boldsymbol{F}}$) is normally defined as the time rate of change of momentum ($\vec{\boldsymbol{P}}$):

$$\vec{\boldsymbol{F}} = \frac{\partial \vec{\boldsymbol{P}}}{\partial t} \quad ,$$

and momentum is defined as the product of mass (m) and velocity ($\vec{\boldsymbol{v}}$):

$$\vec{\boldsymbol{P}} = m\vec{\boldsymbol{v}} \quad .$$

Force is found in Hooke's law:

$$\vec{\boldsymbol{F}} = \int k\, \overrightarrow{d\boldsymbol{x}} \quad ,$$

and force is also found in the work-energy theorem:

$$W = \int \vec{\boldsymbol{F}} \cdot \overrightarrow{d\boldsymbol{x}} \quad .$$

In fact, physics is filled with relationships involving force.

There is a natural tendency to think of force as having different manifestations such that there is one type of force arising from the stretching of a spring, and another type of force arising from a time rate of change of momentum. This case by case interpretation is incomplete.

As it turns out physical quantities, such as force, can only be described in terms of their relationship with other physical quantities. This may seem like circular reasoning, but it is not. There is a fundamentally simultaneous nature to the “fit” of physical quantities.

For example a moving body has both kinetic energy and momentum at the same time. From the work-energy theorem the work done on the body to impart the kinetic energy must have been the result of a force which, by definition, was the result of a time rate of change of momentum. The force-energy equation, and the force-momentum equation must be solved simultaneously.

Constructing the grid

Consider a two-dimensional grid of boxes where each box is related to it's immediate neighbors by a specific set of rules. The same set of rules applies to each and every box on the grid. Each box on the grid represents a particular mechanical element (power, energy, force, …).

Using a suitable set of rules all of the mechanical elements can be arranged on the grid in accordance with the fundamental definitions and theorems of mechanics in an unambiguous fashion. This arrangement constitutes the periodic table of mechanical elements. The periodic quality of the mechanical elements will become clear in the course of the presentation.

Rules of the grid

This diagram describes how any element, represented by ★, is related to it's immediate neighbors.

$\int \star \, d(x/t)$	$\int \star \, dx$	$\int \star \, d(xt)$
$\frac{\partial \star}{\partial t}$	Any Element ★	$\int \star \, dt$
$\frac{\partial \star}{\partial (xt)}$	$\frac{\partial \star}{\partial x}$	$\frac{\partial \star}{\partial (x/t)}$

Diagram 2: How an element is related to it's immediate neighbors

This is the complete set of rules by which the grid is constructed.

Force on the grid

This diagram shows a 4x4 piece of the grid containing a box labeled force.

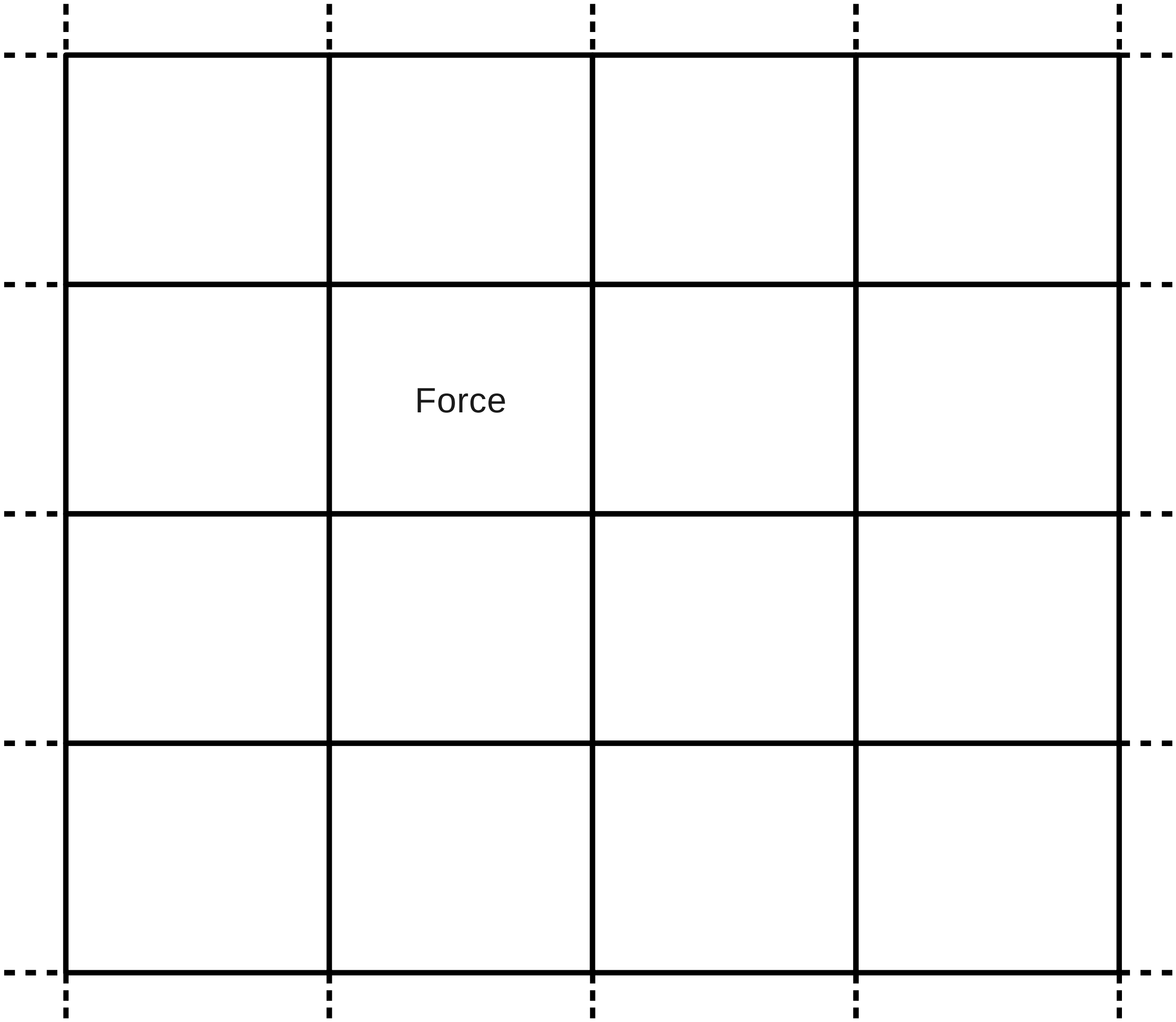


Diagram 3: Force placed on the grid

The grid must extend infinitely in all directions. This is a logical consequence of the fact that every box is related to eight immediate neighbors.

Momentum and force

Force ($\vec{F}$) is defined as the time rate of change of momentum ($\vec{P}$):

$$\vec{F} = \frac{\partial \vec{P}}{\partial t} .$$

According to the rules by which the grid is constructed the force box is located to the left of the momentum box.

$\vec{F}=\frac{\partial \vec{P}}{\partial t}$	$\vec{P}$	

Diagram 4: The relationship between the force box and the momentum box

This means the momentum box must be placed on the grid to the right of the force box.

Momentum on the grid

This diagram shows a 4x4 piece of the grid with the momentum box placed correctly.

	Force	Momentum	

Diagram 5: Momentum placed on the grid

Energy and force

The work-energy theorem states that work (W) is equivalent to energy (E), and work is the integral of force in the direction of motion:

$$E = W = \int \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{x}} \ .$$

According to the rules by which the grid is constructed the energy box is located above the force box.

	$E = \int \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{x}}$	
	$\vec{\boldsymbol{F}}$	

Diagram 6: The relationship between the force box and the energy box

This means the energy box must be placed on the grid above the force box.

Energy on the grid

This diagram shows a 4x4 piece of the grid with the energy box placed correctly.

	Energy		
	Force	Momentum	

Diagram 7: Energy placed on the grid

Power and energy

Power ($\mathcal{P}$) is defined as the time rate of change of energy:

$$\mathcal{P} = \frac{\partial E}{\partial t} \ .$$

According to the rules by which the grid is constructed the power box is located to the left of the energy box.

$\mathcal{P} = \frac{\partial E}{\partial t}$	E	

Diagram 8: The relationship between the power box and the energy box

Power on the grid

This diagram shows a 4x4 piece of the grid with the power box placed correctly.

Power	Energy		
	Force	Momentum	

Diagram 9: Power placed on the grid

Mass and momentum

Momentum ($\vec{P}$) is defined as the product of mass (m) and velocity ($\vec{v}$):

$$\vec{P} = m \cdot \vec{v} \quad .$$

This can be rearranged to yield mass as the ratio of momentum over velocity:

$$m = \frac{\vec{P}}{\vec{v}} \quad .$$

The directional components cancel, because the momentum and velocity vectors share the same direction. This leaves only the ratio of their magnitudes. Considering the instantaneous case yields mass as the velocity rate of change of momentum:

$$m = \frac{\partial \vec{P}}{\partial \vec{v}} \quad .$$

According to the rules by which the grid is constructed the mass box is located diagonally down and to the right of the momentum box.

	$\vec{P}$	
		$m=\frac{\partial \vec{P}}{\partial \vec{v}}$

Diagram 10: The relationship between the momentum box and the mass box

Mass on the grid

This diagram shows a 4x4 piece of the grid with the mass box placed correctly.

Power	Energy		
	Force	Momentum	
			Mass

Diagram 11: Mass placed on the grid

Linear density and mass

Mass (m) can be calculated as the integral of linear density (μ) with respect to distance:

$$m = \int \mu \, d\, x \quad .$$

According to the rules by which the grid is constructed the mass box is located above the linear density box.

	$m = \int \mu \, d\, x$	
	μ	

Diagram 12: The relationship between the linear density box and the mass box

This means the linear density box must be placed on the grid below the mass box.

Linear density on the grid

This diagram shows a 4x4 piece of the grid with the linear density box placed correctly.

Power	Energy		
	Force	Momentum	
			Mass
			Linear Density

Diagram 13: Linear density placed on the grid

Areal density and mass

Mass (m) can be calculated as the double integral of areal density (σ) with respect to area:

$$m = \iint \sigma \, d\,x \, d\,x \quad .$$

According to the rules by which the grid is constructed the mass box is located two boxes above the areal density box.

	$m=\iint \sigma \, d\,x \, d\,x$	
	$\mu=\int \sigma \, d\,x$	
	σ	

Diagram 14: The relationship between the areal density box and the mass box

This means the areal density box (σ) must be placed on the grid two boxes below the mass box.

Areal density on the grid

This diagram shows a 5x4 piece of the grid with the areal density box placed correctly.

Power	Energy		
	Force	Momentum	
			Mass
			Linear Density
			Areal Density

Diagram 15: Areal density placed on the grid

Volumetric density and mass

Mass (m) can be calculated as the triple integral of volumetric density (ρ) with respect to volume:

$$m = \iiint \rho \; d\,x\,d\,x\,d\,x \; .$$

According to the rules by which the grid is constructed the mass box is located three boxes above the volumetric density box.

	$m = \iiint \rho \, d\,x\,d\,x\,d\,x$	
	$\mu = \iint \rho \, d\,x\,d\,x$	
	$\sigma = \int \rho \, d\,x$	
	ρ	

Diagram 16: The relationship between the volumetric density box and the mass box

This means the volumetric density box (ρ) must be placed on the grid three boxes below the mass box.

Volumetric density on the grid

This diagram shows a 6x4 piece of the grid with the volumetric density box placed correctly.

Power	Energy		
	Force	Momentum	
			Mass
			Linear Density
			Areal Density
			Volumetric Density

Diagram 17: Volumetric density placed on the grid

Moment and linear density

Moment (M) can be calculated as the double integral of linear density (μ) with respect to area:

$$M = \iint \mu \, d x d x \; .$$

According to the rules by which the grid is constructed the moment box (M) is located two boxes above the linear density box.

	$M=\iint \mu \, d x d x$	
	$m=\int \mu \, d x$	
	μ	

Diagram 18: The relationship between the moment box and the linear density box

Moment on the grid

This diagram shows a 4x4 piece of the grid with the moment box placed correctly.

Power	Energy		
	Force	Momentum	Moment
			Mass
			Linear Density

Diagram 19: Moment placed on the grid

Moment of inertia and linear density

Moment of inertia (I) can be calculated as the triple integral of linear density (μ) with respect to volume:

$$I = \iiint \mu \, dx \, dx \, dx \quad .$$

According to the rules by which the grid is constructed the moment of inertia box (I) is located three boxes above the linear density box.

	$I = \iiint \mu \, dx \, dx \, dx$	
	$M = \iint \mu \, dx \, dx$	
	$m = \int \mu \, dx$	
	μ	

Diagram 20: The relationship between the moment of inertia box and the linear density box

Moment of inertia on the grid

This diagram shows a 4x4 piece of the grid with the moment of inertia box placed correctly.

Power	Energy		Moment of Inertia
	Force	Momentum	Moment
			Mass
			Linear Density

Diagram 21: Moment of inertia placed on the grid

Spring constant and force

Hooke's law states that force (F) is the integral of spring constant (k) with respect to distance:

$$F = \int k\,d\,x \quad .$$

According to the rules by which the grid is constructed the force box is located above the spring constant box.

	$F = \int k\,d\,x$	
	k	

Diagram 22: The relationship between the spring constant box and the force box

Spring constant on the grid

This diagram shows a 4x4 piece of the grid with the spring constant box placed correctly.

Power	Energy		Moment of Inertia
	Force	Momentum	Moment
	Spring Constant		Mass
			Linear Density

Diagram 23: Spring constant placed on the grid

Pressure and energy

The energy (E) in a volume of gas can be calculated as the triple integral of pressure (P) with respect to volume:

$$E = \iiint p \, dx dx dx \quad .$$

According to the rules by which the grid is constructed the Energy box is located three boxes above the pressure box.

	$E = \iiint P \, dx \, dx \, dx$	
	$F = \iint P \, dx \, dx$	
	$k = \int P \, dx$	
	P	

Diagram 24: The relationship between the pressure box and the energy box

Pressure on the grid

This diagram shows a 4x4 piece of the grid with the pressure box placed correctly.

Power	Energy		Moment of Inertia
	Force	Momentum	Moment
	Spring Constant		Mass
	Pressure		Linear Density

Diagram 25: Pressure placed on the grid

Impedance and momentum

Impedance (Ω) is the rate at which momentum ($\vec{\boldsymbol{P}}$) changes as space is traversed:

$$\Omega = \frac{\partial \vec{\boldsymbol{P}}}{\partial \vec{\boldsymbol{x}}} \quad .$$

According to the rules by which the grid is constructed the impedance box (Ω) is located below the momentum box ($\vec{\boldsymbol{P}}$).

	$\vec{\boldsymbol{P}}$	
	$\Omega=\frac{\partial \vec{\boldsymbol{P}}}{\partial \vec{\boldsymbol{x}}}$	

Diagram 26: The relationship between the impedance box and the momentum box

Impedance on the grid

This diagram shows a 4x4 piece of the grid with the impedance box placed correctly.

Power	Energy		Moment of Inertia
	Force	Momentum	Moment
	Spring Constant	Impedance	Mass
	Pressure		Linear Density

Diagram 27: Impedance placed on the grid

Action and energy

Action (A) is the time integral of energy (E):

$$A = \int E\,dt \ .$$

According to the rules by which the grid is constructed the action box (A) is located to the right of the energy box.

	E	$A=\int E\,dt$

Diagram 28: The relationship between the action box and the energy box

Action on the grid

This diagram shows a 4x4 piece of the grid with the action box placed correctly.

Power	Energy	Action	Moment of Inertia
	Force	Momentum	Moment
	Spring Constant	Impedance	Mass
	Pressure		Linear Density

Diagram 29: Action placed on the grid

Unnamed elements on the grid

For the sake of discussion the four as yet unnamed boxes on this 4x4 piece of the grid have been labeled with arbitrary names X_1 , X_2 , X_3 and X_4 .

Power	Energy	Action	Moment of Inertia
X_1	Force	Momentum	Moment
X_2	Spring Constant	Impedance	Mass
X_3	Pressure	X_4	Linear Density

Diagram 30: Unnamed elements labeled on the grid

According to the rules by which the grid is constructed X_1 is both the space derivative of power ($X_1 = \frac{\partial P}{\partial x}$). and the time derivative of force ($X_1 = \frac{\partial F}{\partial t}$). But, X_1 is also related to six other neighbors. Similarly X_2 , X_3 and X_4 are each related to their eight neighbors.

Labeling one of these unnamed elements with it's relationship to a particular neighbor emphasizes one aspect of the element's nature, and diminishes the importance of the other seven.

Mechanical Units

Power $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{0}$	Energy $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{1}$	Action $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{2}$	Moment of Inertia $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{3}$
$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{-1}$	Force $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{0}$	Momentum $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{1}$	Moment $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{2}$
$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{-2}$	Spring Constant $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{-1}$	Impedance $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{0}$	Mass $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{1}$
$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{-3}$	Pressure $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{-2}$	$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{-1}$	Linear Density $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{0}$
$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{-4}$	$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{-3}$	$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{-2}$	Areal Density $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{-1}$
$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{-5}$	$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{-4}$	$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{-3}$	Volumetric Density $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{-2}$

Diagram 31: The Periodic Table of Mechanical Elements labeled with canonical units

The force between two masses

The force (F) between two masses (m_1 , m_2) is defined as the gravitational constant (G) times the ratio of the product of the two masses divided by the square of the distance (r) between their centroids:

$$F = G\frac{m_1 m_2}{r^2} \quad .$$

If the mass m_1 is known by some means then the mass of m_2 can, in principle, be determined by measuring the force of attraction it exerts on m_1 . Note that this determination depends critically upon the distance between the centroids of the two masses.

Consider the situation where both the mass of m_1 and the force F are known. It is not possible to determine the mass of m_2 without making some assumption about the distance r between the centroids of the two masses. From this it can be deduced that the force between two masses is only defined in terms of the distance between them.

The distributed nature of mass

Consider the situation of two point masses. As these masses are brought infinitesimally close together the force of attraction will become infinite. At the same time the potential energy of this two-particle system will become infinite. From this it can be deduced that mass must always be distributed in space.

The definition of the force between two masses can be rewritten to reflect this fact:

$$F = G\frac{m_1}{r}\frac{m_2}{r} \quad .$$

This reformulation reflects the fundamentally distributed nature of mass in terms of the product of two linear densities.

Unit analysis of mechanical elements

To reflect physicality the distributed nature of mass should be applied rigorously to the unit analysis of mechanical elements. A mass unit should not appear without a linear density. In the SI system of units the kilogram (kg) should only appear as $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)$.

The joule, commonly written as:

$$\mathrm{kg}\frac{\mathrm{m}^2}{\mathrm{s}^2} \quad ,$$

should be written as:

$$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{m}^1 \quad .$$

The newton, commonly written as:

$$\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2} \quad ,$$

should be written as:

$$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{m}^0 \quad .$$

The watt, commonly written as:

$$\mathrm{kg}\frac{\mathrm{m}^2}{\mathrm{s}^3} \quad ,$$

should be written as:

$$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^3 \mathrm{m}^0 \quad .$$

These formulations reflect the distributed nature of mass.

Canonical units for mechanical elements

The units for power, energy and force have been written above in terms of three components. There is a density component: $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)$,

a velocity component: $\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{a}$, and a spatial component: m^{b} .

Expressing the units of mechanical elements in this form will be referred to as the canonical form for reasons that will become clear.

Placing the canonical units for power, energy and force in the appropriate boxes on the periodic table of mechanical elements reveals useful patterns for filling in the canonical units of the remaining elements on the grid.

Power $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{0}$	Energy $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{1}$	Action	Moment of Inertia
	Force $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{0}$	Momentum	Moment
	Spring Constant	Impedance	Mass
	Pressure		Linear Density

Diagram 32: Power, energy and force labeled with canonical units on the grid

The energy/force column

According to the rules by which the grid is constructed all of the elements in the energy/force column have spatial relationships. This fact is reflected by changes in the exponent of the spatial component of the canonical units. The exponent of the velocity component must remain unchanged in this column.

Examination of the canonical units for energy and force shows this relationship clearly. Energy has a spatial exponent of 1, while force has a spatial exponent of 0. The velocity exponent remains fixed at 2, as required by the rules.

Extending this pattern downward indicates that spring constant has a spatial exponent of -1, and pressure has a spatial exponent of -2. The velocity exponent remains fixed at 2 for all elements in this column.

Power	Energy $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{m}^1$	Action	Moment of Inertia
	Force $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{m}^0$	Momentum	Moment
	Spring Constant $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{m}^{-1}$	Impedance	Mass
	Pressure $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{m}^{-2}$		Linear Density

Diagram 33: The energy/force column labeled with canonical units on the grid

The power/force diagonal

According to the rules by which the grid is constructed all of the elements in the power/force diagonal have velocity relationships. This fact is reflected by changes in the exponent of the velocity component of the canonical units. The exponent of the spatial component must remain unchanged along this diagonal.

Examination of the canonical units for power and force shows this relationship clearly. Power has a velocity exponent of 3, while force has a velocity exponent of 2. The spatial exponent remains fixed at 0, as required by the rules.

Extending this pattern downward along the diagonal indicates that impedance has a velocity exponent of 1, and linear density has a velocity exponent of 0. The spatial exponent remains fixed at 0 for all elements along this diagonal.

Power $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{0}$	Energy	Action	Moment of Inertia
	Force $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{0}$	Momentum	Moment
	Spring Constant	Impedance $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{0}$	Mass
	Pressure		Linear Density $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{0}$

Diagram 34: The power/force diagonal labeled with canonical units on the grid

The power/energy row

According to the rules by which the grid is constructed all of the elements in the power/energy row have temporal (time) relationships. This fact is reflected by simultaneous changes in the exponents of both the velocity and spatial components of the canonical units.

Examination of the canonical units for power and energy shows this relationship clearly. Power has a velocity exponent of 3 and a spatial exponent of 0, while energy has a velocity exponent of 2 and a spatial exponent of 1. The velocity and spatial exponents change in opposite directions, as required by the rules.

Extending this pattern to the right indicates that action has a velocity exponent of 1 and a spatial exponent of 2, while moment of inertia has a velocity exponent of 0 and a spatial exponent of 3.

Power $\left(\frac{\text{kg}}{\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{3}\text{m}^{0}$	Energy $\left(\frac{\text{kg}}{\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{2}\text{m}^{1}$	Action $\left(\frac{\text{kg}}{\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{1}\text{m}^{2}$	Moment of Inertia $\left(\frac{\text{kg}}{\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{3}$
	Force	Momentum	Moment
	Spring Constant	Impedance	Mass
	Pressure		Linear Density

Diagram 35: The power/energy row labeled with canonical units on the grid

The remaining mechanical elements

All of the elements in a given column share the same exponent for the velocity component of their canonical units. Proceeding downward within that column the exponents of the spatial components decrease.

All of the elements in a given velocity diagonal share the same exponent in the spatial component of their canonical units. Proceeding downward along that diagonal the exponents of the velocity components decrease.

Proceeding from left to right along a given row the exponents of the velocity components decrease, while the exponents of the spatial components increase.

Power $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{0}$	Energy $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{1}$	Action $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{2}$	Moment of Inertia $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{3}$
$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{-1}$	Force $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{0}$	Momentum $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{1}$	Moment $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{2}$
$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{-2}$	Spring Constant $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{-1}$	Impedance $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{0}$	Mass $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{1}$
$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{-3}$	Pressure $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{-2}$	$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{-1}$	Linear Density $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{0}$

Diagram 36: The remaining mechanical elements labeled with canonical units on the grid

Alternative canonical systems

The canonical system presented is based on a spatial distribution of mass, indicated by the presence of the leading spatial density component, $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)$. The spatial density perspective yields a periodic table where the exponents of the velocity and spatial components are both 0 in the linear density box.

An alternative, but equivalent, system can be constructed based on a temporal distribution of mass, indicated by a leading temporal density component, $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)$. The temporal density perspective yields a periodic table where the exponents of the velocity and spatial components are both 0 in the impedance box.

Power $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{0}$	Energy $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{1}$	Action $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{2}$	Moment of Inertia $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-1}\mathrm{m}^{3}$
$\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{-1}$	Force $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{0}$	Momentum $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{1}$	Moment $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-1}\mathrm{m}^{2}$
$\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{-2}$	Spring Constant $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{-1}$	Impedance $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{0}$	Mass $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-1}\mathrm{m}^{1}$
$\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{-3}$	Pressure $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{-2}$	$\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{-1}$	Linear Density $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-1}\mathrm{m}^{0}$

Diagram 37: The mechanical elements labeled with temporal canonical units on the grid

Yet another alternative system could be constructed by replacing the spatial component, $\left(\mathrm{m}^{\mathrm{b}}\right)$ with a temporal component, $\left(\mathrm{s}^{\mathrm{b}}\right)$. In this system each row has a constant exponent for the velocity component while the exponent of the temporal component increases when proceeding from left to right, and each column has a decreasing exponent for the velocity component while the exponent of the temporal component increases when proceeding downward.

Power $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{s}^0$	Energy $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{s}^1$	Action $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{s}^2$	Moment of Inertia $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{s}^3$
$\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^1 \mathrm{s}^{-1}$	Force $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^1 \mathrm{s}^0$	Momentum $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^1 \mathrm{s}^1$	Moment $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^1 \mathrm{s}^2$
$\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^0 \mathrm{s}^{-2}$	Spring Constant $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^0 \mathrm{s}^{-1}$	Impedance $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^0 \mathrm{s}^0$	Mass $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^0 \mathrm{s}^1$
$\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-1} \mathrm{s}^{-3}$	Pressure $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-1} \mathrm{s}^{-2}$	$\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-1} \mathrm{s}^{-1}$	Linear Density $\left(\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-1} \mathrm{s}^0$

Diagram 38: The mechanical elements labeled with yet another system of canonical units on the grid

There are several other possible canonical systems. Each of these systems emphasizes certain types of relationships while making other types of relationships more obscure. The choice of using a particular canonical system of units depends on the kinds of problems to be addressed. This choice is similar to expressing geometric relationships using a rectangular, cylindrical or spherical coordinate system.

Interpreting physical expressions

There are four major directions on the periodic table of mechanical elements: vertically up, horizontally right, diagonally up/left and diagonally up/right.

Elements that are arranged vertically (those that are in the same column) have spatial relationships. Adjacent elements within a column are separated by lines that represent the spatial-dimension relationship between the two elements.

Elements that are arranged horizontally (those that are in the same row) have temporal (time) relationships. Adjacent elements within a row are separated by lines that represent the temporal relationship between the two elements.

Elements that are arranged diagonally up/left (those that are in the same up/left diagonal) have velocity relationships (the ratio of a spatial relationship over a temporal relationship). Adjacent elements within an up/left diagonal are separated by lines that represent the velocity-dimension relationship between the two elements.

Elements that are arranged diagonally up/right (those that are in the same up/right diagonal) have space-time relationships (the product of a spatial relationship and a temporal relationship). Adjacent elements within an up/right diagonal are separated by lines that represent the space-time-dimension relationship between those two elements.

The periodic table of mechanical elements provides a powerful means for interpreting physical expressions. This is best illustrated by looking at some examples.

Hooke's law

Hooke's law states that the magnitude of the force ($|\vec{F}|$) on a spring is the product of the spring constant (k) times the magnitude of displacement ($|\vec{x}|$) from equilibrium of the spring:

$$|\vec{F}| = k|\vec{x}| \quad .$$

This expression can be interpreted on the periodic table of mechanical elements. Start in the spring constant box. Go up one box. This ends up in the force box:

$$|\vec{F}| = \int k\, d|\vec{x}| \quad .$$

Crossing one spatial-dimension line describes a distance relationship.

Power	Energy	Action	Moment of Inertia
	Force $|\vec{F}|=k|\vec{x}|$	Momentum	Moment
	Spring Constant $k=\frac{|\vec{F}|}{|\vec{x}|}$	Impedance	Mass
	Pressure		Linear Density

Diagram 39: Interpreting Hooke's law using the periodic table of mechanical elements

Pressure and force

The magnitude of the force ($|\vec{F}|$) on an area can be calculated as the double integral of pressure (P) with respect to area:

$$|\vec{F}| = \iint P \; dx \; dx \; .$$

If the pressure (P) is constant over the entire area (A) this reduces to:

$$|\vec{F}| = P A \; .$$

Both expressions can be interpreted on the periodic table of mechanical elements. Start in the pressure box. Go up two boxes. This ends up in the force box. Crossing two consecutive spatial-dimension lines describes an area relationship.

Power	Energy	Action	Moment of Inertia
	Force $\lvert\vec{F}\rvert = \iint P \; dx \; dx$ $\lvert\vec{F}\rvert = P A$	Momentum	Moment
	Spring Constant	Impedance	Mass
	Pressure $P = \frac{\partial \lvert\vec{F}\rvert}{\partial A}$ $P = \frac{\lvert\vec{F}\rvert}{A}$		Linear Density

Diagram 40: Interpreting $|\vec{F}| = \iint P \; dx \; dx$ *and* $|\vec{F}| = P A$ *using the periodic table of mechanical elements*

A mass-spring system

The frequency (f) of a mass-spring system is the square root of the spring constant (k) divided by the mass (m):

$$f = \sqrt{\frac{k}{m}} \; .$$

This can be rearranged to give:

$$k = m f^2 \; .$$

This expression can be interpreted on the periodic table of mechanical elements by recognizing that multiplying by frequency is the same as dividing by time (period). Start in the mass box. Go two boxes to the left. This ends up in the spring constant box:

$$k = \frac{\partial^2 m}{\partial t^2} \; .$$

Crossing two temporal-dimension lines describes a frequency squared relationship.

Power	Energy	Action	Moment of Inertia
	Force	Momentum	Moment
	Spring Constant $k = m f^2$	Impedance	Mass $m = \frac{k}{f^2}$
	Pressure		Linear Density

Diagram 41: Interpreting the frequency of a mass-spring system using the periodic table of mechanical elements

A pulse on a rope

The magnitude of the velocity ($|\vec{\boldsymbol{v}}|$) of a pulse on a stretched rope is the square root of the magnitude of the tension ($|\vec{\boldsymbol{T}}|$) on the rope divided by the linear density (μ) of the rope:

$$|\vec{\boldsymbol{v}}| = \sqrt{\frac{|\vec{\boldsymbol{T}}|}{\mu}} \ .$$

This can be rearranged to give:

$$|\vec{\boldsymbol{T}}| = \mu |\vec{\boldsymbol{v}}|^2 \ .$$

This expression can be interpreted on the periodic table of mechanical elements, by recognizing that tension is another name for force. Start in the linear density box. Go two boxes up along the velocity diagonal. This ends up in the force (tension) box:

$$|\vec{\boldsymbol{F}}| = \iint \mu \, \overrightarrow{d\boldsymbol{v}} \cdot \overrightarrow{d\boldsymbol{v}} \ .$$

Crossing two velocity-dimension lines describes a velocity squared relationship.

Power	Energy	Action	Moment of Inertia
	Force $|\vec{\boldsymbol{T}}| = \mu |\vec{\boldsymbol{v}}|^2$	Momentum	Moment
	Spring Constant	Impedance	Mass
	Pressure		Linear Density $\mu = \frac{|\vec{\boldsymbol{T}}|}{|\vec{\boldsymbol{v}}|^2}$

Diagram 42: Interpreting the velocity of a pulse on a rope using the periodic table of mechanical elements

Power, force and impedance

Consider the situation of a body moving against an impedance, such as a block sliding on a frictional surface. For the body to proceed at a constant velocity ($|\vec{v}|$) a force ($|\vec{F}|$) is required to overcome the impedance(Ω):

$$|\vec{F}| = \Omega|\vec{v}| \quad .$$

This expression can be interpreted on the periodic table of mechanical elements. Start in the impedance box. Go up the velocity diagonal one box. This ends up in the force box.

This expression can be rewritten as:

$$\Omega = \frac{|\vec{F}|}{|\vec{v}|}$$

This new expression can be interpreted on the periodic table of mechanical elements. Start in the force box. Go down the velocity diagonal one box. This ends up in the impedance box.

Power	Energy	Action	Moment of Inertia
	Force $|\vec{F}|=\Omega|\vec{v}|$	Momentum	Moment
	Spring Constant	Impedance $\Omega=\frac{|\vec{F}|}{|\vec{v}|}$	Mass
	Pressure		Linear Density

Diagram 43: Interpreting $|\vec{F}| = \Omega|\vec{v}|$ *and* $\Omega = \frac{|\vec{F}|}{|\vec{v}|}$ *using the periodic table of mechanical elements*

The force does work on the body. The power ($\mathcal{P}$) provided by this force is:

$$\mathcal{P} = |\vec{F}||\vec{v}| \quad .$$

This expression can be interpreted on the periodic table of mechanical elements. Start in the force box. Go up the velocity diagonal one box. This ends up in the the power box.

This expression can be rewritten as:

$$|\vec{F}| = \frac{\mathcal{P}}{|\vec{v}|} \quad .$$

This new expression can be interpreted on the periodic table of mechanical elements. Start in the power box. Go down the velocity diagonal one box. This ends up in the force box.

Power $\mathcal{P}=\|\vec{F}\|\|\vec{v}\|$	Energy	Action	Moment of Inertia
	Force $\|\vec{F}\|=\frac{\mathcal{P}}{\|\vec{v}\|}$	Momentum	Moment
	Spring Constant	Impedance	Mass
	Pressure		Linear Density

Diagram 44: Interpreting $\mathcal{P} = |\vec{F}||\vec{v}|$ *and* $|\vec{F}| = \frac{\mathcal{P}}{|\vec{v}|}$ *using the periodic table of mechanical elements*

These two equations can be combined to yield:

$$\mathcal{P} = \Omega|\vec{\mathbf{v}}|^2 \ .$$

This expression can be interpreted on the periodic table of mechanical elements. Start in the impedance box. Go up the velocity diagonal two boxes. This ends up in the power box. Crossing two velocity-dimension lines describes a velocity squared relationship.

This expression can be rewritten as:

$$\Omega = \frac{\mathcal{P}}{|\vec{\mathbf{v}}|^2} \ .$$

This new expression can be interpreted on the periodic table of mechanical elements. Start in the power box. Go down the velocity diagonal two boxes. This ends up in the impedance box.

Power $\mathcal{P}=\Omega\lvert\vec{\mathbf{v}}\rvert^2$	Energy	Action	Moment of Inertia
	Force	Momentum	Moment
	Spring Constant	Impedance $\Omega=\frac{\mathcal{P}}{\lvert\vec{\mathbf{v}}\rvert^2}$	Mass
	Pressure		Linear Density

Diagram 45: Interpreting $\mathcal{P} = \Omega|\vec{\mathbf{v}}|^2$ *and* $\Omega = \frac{\mathcal{P}}{|\vec{\mathbf{v}}|^2}$ *using the periodic table of mechanical elements*

Maxwell's kinetic theory of gas

A basic tenet of Maxwell's kinetic theory of gas states that the pressure(P) of the gas times the volume(V) of the gas is equal to the mass (m) of the gas times the square of the velocity (v) of the gas particles:

$$PV = mv^2 .$$

The mass and velocity are defined to be consistent with the universal gas law:

$$PV = nRT .$$

There is no need to go into such details for the purpose of this discussion.

It is instructive to interpret the left and right side of the equation $PV = mv^2$ separately, and then return to the entire equation for further interpretation.

Interpreting the left side of $PV = mv^2$

The left side of this equation, PV, is an energy expression. It describes the potential energy contained in a volume of gas under pressure. Pressure can be considered as a volumetric energy density.

The product of pressure times volume can be interpreted on the periodic table of mechanical elements. Start in the pressure box. Go up by a volume (three boxes). This ends up in the energy box. Crossing three consecutive spatial-dimension lines describes a volume relationship.

Power	Energy $E=PV$	Action	Moment of Inertia
	Force	Momentum	Moment
	Spring Constant	Impedance	Mass
	Pressure $P=\frac{E}{V}$		Linear Density

Diagram 46: Interpreting the left side of $PV = mv^2$ *from Maxwell's kinetic theory of gas using the periodic table of mechanical elements*

Interpreting the right side of $PV = mv^2$

The right side of this equation, mv^2 , is also an energy expression. It describes the kinetic energy contained in a volume of gas under pressure.

This can be interpreted on the periodic table of mechanical elements. Start in the mass box. Go up the velocity diagonal by two boxes. This ends up in the energy box. Crossing two velocity-dimension lines describes a velocity squared relationship.

Power	Energy $E = mv^2$	Action	Moment of Inertia
	Force	Momentum	Moment
	Spring Constant	Impedance	Mass $m = \frac{E}{v^2}$
	Pressure		Linear Density

Diagram 47: Interpreting the right side of $PV = mv^2$ *from Maxwell's kinetic theory of gas using the periodic table of mechanical elements*

Interpreting the equation $PV = mv^2$

This equation describes the energy of a gas in two ways. The left side describes the potential energy view, from pressure to energy by volume:

$$E = \iiint P\,dx\,dx\,dx \quad .$$

The right side describes the kinetic energy view, from mass to energy by velocity squared:

$$E = \iint m\,dv\,dv \quad .$$

Both sides are describing the same energy.

This equation states that the potential energy contained in a gas under pressure is the same as the kinetic energy of the particles in the gas.

Power	Energy $E = PV = mv^2$	Action	Moment of Inertia
	Force	Momentum	Moment
	Spring Constant	Impedance	Mass $m = \frac{E}{v^2}$
	Pressure $P = \frac{E}{V}$		Linear Density

Diagram 48: Interpreting $PV = mv^2$ *from Maxwell's kinetic theory of gas using the periodic table of mechanical elements*

The Euler-Lagrange equation

The Euler-Lagrange equation states that the space derivative of the Lagrangian function ($\mathcal{L}$) is equal to the time derivative of the velocity derivative of the Lagrangian function:

$$\frac{\partial}{\partial x}(\mathcal{L}) = \frac{d}{dt}\left[\frac{\partial}{\partial v}(\mathcal{L})\right] .$$

This can be interpreted on the periodic table of mechanical elements.

Interpreting the Lagrangian function

The Lagrangian function is defined as the difference between the kinetic energy in the system (T) and the potential energy in the system (U):

$$\mathcal{L} = T - U .$$

For the purpose of this discussion the Lagrangian function is simply an energy expression.

Interpreting the left side of the Euler-Lagrange equation

The left side of the Euler-Lagrange equation, $\frac{\partial}{\partial x}(\mathcal{L})$, can be interpreted on the periodic table of mechanical elements. Start in the energy box. Go down (in the inverse spatial direction on the grid) one box (take the partial derivative with respect to space). This ends up in the force box.

Power	Energy $E=\mathcal{L}$	Action	Moment of Inertia
	Force $F=\frac{\partial}{\partial x}(\mathcal{L})$	Momentum	Moment
	Spring Constant	Impedance	Mass
	Pressure		Linear Density

Diagram 49: Interpreting the left side of the Euler-Lagrange equation using the periodic table of mechanical elements

The left side of the Euler-Lagrange equation, $\frac{\partial}{\partial x}(\mathcal{L})$, is a force expression.

Interpreting the right side of the Euler-Lagrange equation

The right side of the Euler-Lagrange equation, $\frac{d}{dt}\left[\frac{\partial}{\partial v}(\mathcal{L})\right]$, can be interpreted on the periodic table of mechanical elements. Start in the energy box. Go down the velocity diagonal one box (take the partial derivative with respect to velocity) to arrive in the momentum box. Go left one box (take the derivative with respect to time) to arrive in the force box.

Power	Energy $E=\mathcal{L}$	Action	Moment of Inertia
	Force $F=\frac{d}{dt}\left[\frac{\partial}{\partial v}(\mathcal{L})\right]$	Momentum $P=\frac{\partial}{\partial v}(\mathcal{L})$	Moment
	Spring Constant	Impedance	Mass
	Pressure		Linear Density

Diagram 50: Interpreting the right side of the Euler-Lagrange equation using the periodic table of mechanical elements

The right side of the Euler-Lagrange equation, $\frac{d}{dt}\left[\frac{\partial}{\partial v}(\mathcal{L})\right]$, is a force expression.

Interpreting the Euler-Lagrange equation

The Euler-Lagrange equation describes two different ways to get force from energy. The left side describes a static view of force in terms of the work-energy theorem. The right side describes a dynamic view of force in terms of the time derivative of momentum. Both sides are describing the same force in relation to the same energy. Both sides of the Euler-Lagrange equation are describing the same physical situation, just from different perspectives. The Euler-Lagrange equation states that the forces arising from the two perspectives are equivalent.

Power	Energy $E=\mathcal{L}$	Action	Moment of Inertia
	Force $F=\frac{\partial}{\partial x}(\mathcal{L})$ $F=\frac{d}{dt}\left[\frac{\partial}{\partial v}(\mathcal{L})\right]$	Momentum $P=\frac{\partial}{\partial v}(\mathcal{L})$	Moment
	Spring Constant	Impedance	Mass
	Pressure		Linear Density

Diagram 51: Interpreting the Euler-Lagrange equation using the periodic table of mechanical elements

Interpreting physical relationships

The periodic table of mechanical elements provides a graphical representation of the relationships between physical elements. The periodic table of mechanical elements is a topological map of the landscape of physical relationships.

The rules by which the periodic table of mechanical elements is constructed specify the mathematical relationships between a particular box and each of the eight adjacent boxes on the grid. Each of the eight boxes adjacent to a given box has a different geometric relationship on the grid to the given box.

The geometric relationships between boxes on the grid can be described in terms of navigation instructions. There are many sequences of navigation instructions that have the net result of describing a particular geometric relationship between two boxes on the grid. For example, going left one box followed by going up one box has the same net result as going up one box followed by going left one box. In other words, there is a many-to-one relationship between sequences of navigation instructions that have the net result of going from one box to another and the geometric relationship between those two boxes on the grid.

A pair of boxes, A and B, have a particular geometric relationship on the grid. This geometric relationship on the grid is described by many sequences of navigation instructions that have the net result of going from box A to box B. If another pair of boxes, C and D, have the same geometric relationship as A and B then all of the sequences of navigation instructions that have the net result of going from box A to box B will also go from box C to box D.

The concept of navigation on the periodic table of mechanical elements provides a powerful tool for describing and interpreting physical relationships. The rules by which the grid is constructed allow sequences of navigation instructions, and the corresponding geometric relationships on the grid, to be interpreted in terms of mathematical and physically meaningful relationships. Any two boxes that have a particular geometric relationship on the grid also have a particular mathematical and physical relationship. This is best illustrated by looking at some examples.

Spatial relationships

Spatial relationships are described by any sequence of navigation instructions that have the net result of moving up (in the spatial direction on the grid) one box within a particular column on the grid. Inverse spatial relationships are described by any sequence of navigation instructions that have the net result of moving down (in the inverse spatial direction on the grid) one box within a particular column on the grid. A pair of elements that have a spatial relationship when considered in one order have an inverse spatial relationship when considered in the reverse order.

Hooke's law

Hooke's law describes the spatial relationship between force ($\vec{F}$), spring constant (k) and displacement($\vec{x}$):

$$\vec{F} = k\vec{x} \quad .$$

This formula can be interpreted in terms of navigation instructions. Start in the spring constant box. Go up (in the spatial direction on the grid) one box to arrive in the force box:

$$\vec{F} = \int k\,d\vec{x} \quad .$$

The net result of this sequence of navigation instructions is to move up (in the spatial direction on the grid) one box. This is characteristic of a spatial relationship.

Power	Energy	Action	Moment of Inertia
	Force $\vec{\boldsymbol{F}}=k\,\vec{\boldsymbol{x}}$	Momentum	Moment
	Spring Constant k	Impedance	Mass
	Pressure		Linear Density

Diagram 52: The spatial relationship $\vec{\boldsymbol{F}} = k\,\vec{\boldsymbol{x}}$

Examining the periodic table of mechanical elements shows that the force box is one box above the spring constant box.

Mass and linear density

Linear density (μ) can be defined as the partial derivative of mass (m) with respect to distance (x):

$$\mu = \frac{\partial m}{\partial x} \; .$$

This can be interpreted in terms of navigation instructions. Start in the mass box. Go down (in the inverse spatial direction on the grid) one box to arrive in the linear density box. The net result of this sequence of navigation instructions is to move down (in the inverse spatial direction on the grid) one box. This is characteristic of an inverse spatial relationship.

Power	Energy	Action	Moment of Inertia
	Force	Momentum	Moment
	Spring Constant	Impedance	Mass m
	Pressure		Linear Density $\mu=\frac{\partial m}{\partial x}$

Diagram 53: The inverse spatial relationship $\mu = \frac{\partial m}{\partial x}$

Examining the periodic table of mechanical elements shows that the linear density box is one box below the mass box.

Temporal relationships

Temporal relationships are described by any sequence of navigation instructions that have the net result of moving right (in the temporal direction on the grid) one box within a particular row on the grid. A temporal relationship is a time or period relationship. Inverse temporal relationships are described by any sequence of navigation instructions that have the net result of moving left (in the inverse temporal direction on the grid) one box within a particular row on the grid. An inverse temporal relationship is a frequency relationship. A pair of elements that have a temporal relationship when considered in one order have an inverse temporal relationship when considered in the reverse order.

Power and energy

Power ($\mathcal{P}$) is defined as the time rate of change of energy (E):

$$\mathcal{P} = \frac{\partial E}{\partial t} \ .$$

This formula can be interpreted in terms of navigation instructions. Start in the energy box. Go left (in the inverse temporal direction on the grid) one box to arrive in the power box. The net result of this sequence of navigation instructions is to move left (in the inverse temporal direction on the grid) one box. This is characteristic of an inverse temporal relationship.

Power $\mathcal{P}=\frac{\partial E}{\partial t}$	Energy E	Action	Moment of Inertia
	Force	Momentum	Moment
	Spring Constant	Impedance	Mass
	Pressure		Linear Density

Diagram 54: The inverse temporal relationship $\mathcal{P} = \frac{\partial E}{\partial t}$

Examining the periodic table of mechanical elements shows that the power box is one box to the left of the energy box.

Force, impulse and frequency

The force ($\vec{F}$) due to a stream of impulses of momentum ($\vec{P}$) arriving at a rate (f) is given by the formula:

$$\vec{F} = \vec{P} f \ .$$

This can be interpreted in terms of navigation instructions. Start in the momentum box. Go left (in the frequency direction on the grid) one box to arrive in the force box. The net result of this sequence of navigation instructions is to move left (in the frequency direction on the grid) one box. This is characteristic of a frequency relationship.

Power	Energy	Action	Moment of Inertia
	Force $\vec{F}=\vec{P}f$	Momentum $\vec{P}$	Moment
	Spring Constant	Impedance	Mass
	Pressure		Linear Density

Diagram 55: The frequency relationship $\vec{F} = \vec{P} f$

Examining the periodic table of mechanical elements shows that the force box is one box to the left of the momentum box.

Velocity relationships

Velocity relationships are described by any sequence of navigation instructions that have the net result of moving up (in the spatial direction on the grid) one box and left (in the inverse temporal direction on the grid) one box. This can also be described as moving diagonally up and left (in the velocity direction on the grid) one box. Inverse velocity relationships are described by any sequence of navigation instructions that have the net result of moving down (in the inverse spatial direction on the grid) one box, and right (in the temporal direction on the grid) one box. This can also be described as moving diagonally down and right (in the inverse velocity direction on the grid) one box. A pair of elements that have a velocity relationship when considered in one order have an inverse velocity relationship when considered in the reverse order.

Momentum and mass

Momentum ($\vec{P}$) is defined as the product of mass (m) times velocity. ($\vec{v}$):

$$\vec{P} = m\vec{v}$$.

This can be interpreted in terms of navigations instructions. Start in the mass box. Go diagonally up and left (in the velocity direction on the grid) one box to arrive in the momentum box. The net result of this sequence of navigation instructions is to go diagonally up and left (in the velocity direction on the grid) one box. This is characteristic of a velocity relationship.

Power	Energy	Action	Moment of Inertia
	Force	Momentum $\vec{P}=m\vec{v}$	Moment
	Spring Constant	Impedance	Mass m
	Pressure		Linear Density

Diagram 56: The velocity relationship $\vec{P} = m\vec{v}$

Examining the periodic table of mechanical elements shows that the momentum box is diagonally up and left (in the velocity direction on the grid) one box.

Impedance and force

The impedance (Ω) encountered when a force ($\vec{F}$) is applied to maintain a velocity ($\vec{v}$) is:

$$\Omega = \frac{\vec{F}}{\vec{v}} \; .$$

This can be interpreted in terms of navigation instructions. Start in the force box. Go diagonally down and right (in the inverse velocity direction on the grid) one box to arrive in the impedance box. The net result of this sequence of navigation instructions is to move diagonally down and right (in the inverse velocity direction on the grid) one box. This is characteristic of an inverse velocity relationship.

Power	Energy	Action	Moment of Inertia
	Force $\vec{F}$	Momentum	Moment
	Spring Constant	Impedance $\Omega=\frac{\vec{F}}{\vec{v}}$	Mass
	Pressure		Linear Density

Diagram 57: The inverse velocity relationship $\Omega = \frac{\vec{F}}{\vec{v}}$

Examining the periodic table of mechanical elements shows that the impedance box is diagonally down and right (in the inverse velocity direction on the grid) one box from the force box.

Acceleration relationships

Acceleration relationships are described by any sequence of navigation instructions that have the net result of moving up (in the spatial direction on the grid) one box and left (in the inverse temporal direction on the grid) two boxes. Inverse acceleration relationships are described by any sequence of navigation instructions that have the net result of moving down (in the inverse spatial direction on the grid) one box, and right (in the temporal direction on the grid) two boxes. A pair of elements that have an acceleration relationship when considered in one order have an inverse acceleration relationship when considered in the reverse order.

Force and mass

Force ($\vec{F}$) is defined as the time derivate of momentum ($\vec{P}$):

$$\vec{F} = \frac{\partial \vec{P}}{\partial t} \quad .$$

Momentum is defined as the product of mass (m) and velocity ($\vec{v}$):

$$\vec{P} = m\vec{v} \quad .$$

These two definitions can be combined to yield the equation:

$$\vec{F} = \frac{\partial}{\partial t}(m\vec{v}) \quad .$$

This formula can be interpreted in terms of navigation instructions. Start in the mass box. Go diagonally up and left (in the velocity direction on the grid) one box (multiply by velocity) to arrive in the momentum box. Go left (in the inverse temporal direction on the grid) one box (take the partial derivative with respect to time) to arrive in the force box. The net result of this sequence of navigation instructions is to move up (in the spatial direction on the grid) one box, and left (in the inverse temporal direction on the grid) two boxes. This is characteristic of an acceleration relationship.

Power	Energy	Action	Moment of Inertia
	Force $\vec{\boldsymbol{F}}=\frac{\partial}{\partial t}(m\vec{\mathbf{v}})$	Momentum $\vec{\boldsymbol{P}}=m\vec{\mathbf{v}}$	Moment
	Spring Constant	Impedance	Mass m
	Pressure		Linear Density

Diagram 58: The acceleration relationship $\vec{\boldsymbol{F}} = \frac{\partial}{\partial t}(m\vec{\mathbf{v}})$

Examining the periodic table of mechanical elements shows that the force box is up (in the spatial direction on the grid) one box, and left (in the inverse temporal direction on the grid) two boxes from the mass box.

By applying the product rule for derivatives the equation $\vec{F} = \frac{\partial}{\partial t}(m\vec{v})$ can be rewritten as:

$$\vec{F} = \frac{\partial m}{\partial t}\vec{v} + m\frac{\partial \vec{v}}{\partial t} \quad .$$

The first term, $\frac{\partial m}{\partial t}\vec{v}$, can be interpreted in terms of navigation instructions. Start in the mass box. Go left (in the inverse temporal direction on the grid) one box (take the partial derivative with respect to time) to arrive in the impedance box. Go diagonally up and left (in the velocity direction on the grid) one box (multiply by velocity) to arrive in the force box. The net result of this sequence of navigation instructions is to move up (in the spatial direction on the grid) one box, and left (in the inverse temporal direction) two boxes. This is characteristic of an acceleration relationship.

Power	Energy	Action	Moment of Inertia
	Force $\vec{F}=\frac{\partial m}{\partial t}\vec{v}$	Momentum	Moment
	Spring Constant	Impedance $\Omega=\frac{\partial m}{\partial t}$	Mass m
	Pressure		Linear Density

Diagram 59: The acceleration relationship $\frac{\partial m}{\partial t}\vec{v}$

The second term, $m\frac{\partial \vec{\mathbf{v}}}{\partial t}$, can be interpreted in terms of navigation instructions. Start in the mass box. Go diagonally up and left (in the velocity direction on the grid) one box (multiply by velocity) to arrive in the momentum box. Go left (in the inverse temporal direction on the grid) one box (take the partial derivative with respect to time) to arrive in the force box. The net result of this sequence of navigation instructions is to move up (in the spatial direction on the grid) one box, and left (in the inverse temporal direction on the grid) two boxes. This is characteristic of an acceleration relationship.

Power	Energy	Action	Moment of Inertia
	Force $\vec{\boldsymbol{F}}=m\frac{\partial \vec{\mathbf{v}}}{\partial t}$	Momentum $\vec{\boldsymbol{P}}=m\vec{\mathbf{v}}$	Moment
	Spring Constant	Impedance	Mass m
	Pressure		Linear Density

Diagram 60: The acceleration relationship $m\frac{\partial \vec{\mathbf{v}}}{\partial t}$

Uniform circular motion

Consider a constant (non-time-varying) mass moving in a circular path at a constant (non-time-varying) speed. A constant speed (linear velocity) means the magnitude of the velocity vector ($|\vec{\mathbf{v}}|$) does not change with time:

$$\frac{\partial|\vec{\mathbf{v}}|}{\partial t} = 0 \quad .$$

However, the direction of the velocity vector is changing with time. This is not a constant (non-time-varying) velocity situation. There is a non-zero acceleration associated with the time-varying direction of the velocity vector.

When the position of the mass with respect to the center of the circular path is represented using a radius vector ($\vec{\boldsymbol{r}}$) the acceleration is:

$$\vec{\boldsymbol{a}} = \frac{|\vec{\mathbf{v}}|^2}{\vec{\boldsymbol{r}}} \quad .$$

A force must be present to cause the momentum, and the kinetic energy, to change direction. The force that causes the direction of the velocity vector to change (to maintain the circular path of motion) is:

$$\vec{\boldsymbol{F}} = m\frac{|\vec{\mathbf{v}}|^2}{\vec{\boldsymbol{r}}} \quad .$$

This formula shows that the force vector is in the inverse radius direction (radially inward). The magnitude of the force vector comes from the formula:

$$m\vec{\mathbf{v}}\cdot\vec{\mathbf{v}} = m|\vec{\mathbf{v}}|^2 \quad .$$

The formula $\vec{\boldsymbol{F}} = m\frac{|\vec{\mathbf{v}}|^2}{\vec{\boldsymbol{r}}}$ can be interpreted in terms of navigation instructions. Start in the mass box. Go up and left along the velocity diagonal (in the velocity direction on the grid) two boxes (multiply by the square of velocity), passing through the momentum box, to arrive in the energy box. Go down (in the inverse spatial direction on the grid) one box (divide by radius) to arrive in the force box. The net result of this sequence of navigation instructions is to move up (in the spatial direction on the grid) one box, and left (in the inverse temporal direction on the grid) two boxes. This is characteristic of an acceleration relationship.

Power	Energy $E = m\|\vec{\boldsymbol{v}}\|^2$	Action	Moment of Inertia
	Force $\vec{\boldsymbol{F}} = m\frac{\|\vec{\boldsymbol{v}}\|^2}{\vec{\boldsymbol{r}}}$	Momentum $\vec{\boldsymbol{P}} = m\vec{\boldsymbol{v}}$	Moment
	Spring Constant	Impedance	Mass m
	Pressure		Linear Density

Diagram 61: The acceleration relationship $\vec{\boldsymbol{F}} = m\frac{|\vec{\boldsymbol{v}}|^2}{\vec{\boldsymbol{r}}}$

Examining the periodic table of mechanical elements shows that force is up (in the spatial direction on the grid) one box and left (in the inverse temporal direction on the grid) two boxes from the mass box.

Circulation relationships

Circulation relationships are described by any sequence of navigation instructions that have the net result of moving up (in the spatial direction on the grid) two boxes, and left (in the inverse temporal direction on the grid) one box. Inverse circulation relationships are described by any sequence of navigation instructions that have the net result of moving down (in the inverse spatial direction on the grid) two boxes, and right (in the temporal direction on the grid) one box. A pair of elements that have a circulation relationship when considered in one order have an inverse circulation relationship when considered in the reverse order.

The differential operator $\nabla\times$ is sometimes used to describe circulation relationships. For this reason circulation is also known as curl.

A brief look at some pairs of mechanical elements that have circulation relationships helps to illustrate the concept of a circulation relationship on the periodic table of mechanical elements.

Energy and impedance

Examination of the periodic table of mechanical elements shows that energy (E) and impedance (Ω) have a circulation relationship:

$$E = \int \Omega \vec{\boldsymbol{v}} \cdot \overrightarrow{d\boldsymbol{x}} \quad .$$

This can be interpreted as a sequence of navigation instructions. Start in the impedance box. Go diagonally up and left (in the velocity direction on the grid) one box to arrive in the force box. Go up (in the spatial direction on the grid) one box to arrive in the energy box.

Power	Energy $E = \int \Omega \vec{\boldsymbol{v}} \cdot \overrightarrow{d\boldsymbol{x}}$	Action	Moment of Inertia
	Force $\vec{\boldsymbol{F}} = \Omega \vec{\boldsymbol{v}}$	Momentum	Moment
	Spring Constant	Impedance Ω	Mass
	Pressure		Linear Density

Diagram 62: The circulation relationship $E = \int \Omega \vec{\boldsymbol{v}} \cdot \overrightarrow{d\boldsymbol{x}}$

Action and mass

Examination of the periodic table of mechanical elements shows that action (A) and mass (m) have a circulation relationship:

$$A = \int m|\vec{\mathbf{v}}|^2 dt \ .$$

This can be interpreted as a sequence of navigation instructions. Start in the mass box. Go diagonally up and left (in the velocity direction on the grid) two boxes, passing through the momentum box, to arrive in the energy box. Go right (in the temporal direction on the grid) one box to arrive in the action box.

Power	Energy $E = m|\vec{\mathbf{v}}|^2$	Action $A=\int m|\vec{\mathbf{v}}|^2 dt$	Moment of Inertia
	Force	Momentum $\vec{\boldsymbol{P}}=\int m\vec{\mathbf{v}}\,dt$	Moment
	Spring Constant	Impedance	Mass m
	Pressure		Linear Density

Diagram 63: The circulation relationship $A = \int m|\vec{\mathbf{v}}|^2 dt$

Power and spring constant

Examination of the periodic table of mechanical elements shows that power ($\mathcal{P}$) and spring constant (k) have a circulation relationship:

$$\mathcal{P} = \frac{\partial}{\partial t}\left(\iint k \; \overrightarrow{d\boldsymbol{x}}\cdot\overrightarrow{d\boldsymbol{x}}\right) \; .$$

This can be interpreted as a sequence of navigation instructions. Start in the spring constant box. Go up (in the spatial direction on the grid) two boxes, passing through the force box, to arrive in the energy box. Go left (in the inverse temporal direction on the grid) one box to arrive in the power box.

Power $\mathcal{P} = \frac{\partial}{\partial t}\left(\iint k \; \overrightarrow{d\boldsymbol{x}}\cdot\overrightarrow{d\boldsymbol{x}}\right)$	Energy $E = \iint k \; \overrightarrow{d\boldsymbol{x}}\cdot\overrightarrow{d\boldsymbol{x}}$	Action	Moment of Inertia
	Force $\vec{\boldsymbol{F}} = \int k \; \overrightarrow{d\boldsymbol{x}}$	Momentum	Moment
	Spring Constant k	Impedance	Mass
	Pressure		Linear Density

Diagram 64: The circulation relationship $\mathcal{P} = \frac{\partial}{\partial t}\left(\iint k \; \overrightarrow{d\boldsymbol{x}}\cdot\overrightarrow{d\boldsymbol{x}}\right)$

Momentum and linear density

Examination of the periodic table of mechanical elements shows that momentum ($\vec{\boldsymbol{P}}$) and linear density (μ) have a circulation relationship:

$$\vec{\boldsymbol{P}} = \vec{\mathbf{v}} \int \mu \, dx \quad .$$

This can be interpreted as a sequence of navigation instructions. Start in the linear density box. Go up (in the spatial direction on the grid) one box to arrive in the mass box. Go diagonally up and left (in the velocity direction on the grid) one box to arrive in the momentum box.

Power	Energy	Action	Moment of Inertia
	Force	Momentum $\vec{\boldsymbol{P}} = \vec{\mathbf{v}} \int \mu \, dx$	Moment
	Spring Constant	Impedance	Mass $m = \int \mu \, dx$
	Pressure		Linear Density μ

Diagram 65: The circulation relationship $\vec{\boldsymbol{P}} = \vec{\mathbf{v}} \int \mu \, dx$

Analyzing the grid

The rules by which the grid on which the periodic table of mechanical elements is constructed describes an infinite set of simultaneous partial differential equations.

$\int \star\, d(x/t)$	$\int \star\, dx$	$\int \star\, d(xt)$
$\frac{\partial \star}{\partial t}$	Any Element ★	$\int \star\, dt$
$\frac{\partial \star}{\partial (xt)}$	$\frac{\partial \star}{\partial x}$	$\frac{\partial \star}{\partial (x/t)}$

Diagram 66: The rules by which the grid is constructed

The grid must extend infinitely in all directions, because each element is related to eight neighbors.

The elements on the grid form a basis set for the linear space of physical relationships described by the grid. This linear space of physical relationships has an infinite number of dimensions.

Each element on the grid must have an infinite number of integrals and derivatives, because each element on the grid is related by both space and time derivatives to other elements on the grid. This is a logical consequence of the rules by which the grid is constructed.

This is consistent with the fundamental assumption in quantum electrodynamics (QED) that all physical quantities have infinite integrals and derivatives in both time and space. This fundamental, and unproven, assumption in QED is a logical consequence of the rules by which the grid is constructed.

The distribution of energy

One of the most fundamental assumptions in all of physics is that energy is distributed in space. This fundamental, and unproven, assumption of physics is a logical consequence of the rules by which the grid is constructed.

According to the rules by which the grid is constructed the element energy (E) is directly related to eight neighboring elements on the grid.

$\int E\,d(x/t)$	$\int E\,dx$	$\int E\,d(xt)$
Power $\frac{\partial E}{\partial t}$	Energy E	Action $\int E\,dt$
$\frac{\partial E}{\partial (xt)}$	Force $\frac{\partial E}{\partial x}$	Momentum $\frac{\partial E}{\partial (x/t)}$

Diagram 67: Energy and it's eight immediate neighbors

The spatial distribution of energy

One of the rules by which the grid is constructed describes how energy is distributed in space:

$$\frac{\partial E}{\partial x} \ .$$

The spatial distribution of energy is represented by the box called force. Force can be considered as the spatial density of energy.

The temporal distribution of energy

One of the rules by which the grid is constructed describes how energy is distributed in time:

$$\frac{\partial E}{\partial t} \; .$$

The temporal distribution of energy is represented by the box called power. Power can be considered as the temporal density of energy.

The velocity distribution of energy

One of the rules by which the grid is constructed describes how energy is distributed in velocity:

$$\frac{\partial E}{\partial (x/t)} \; .$$

The velocity distribution of energy is represented by the box called called momentum. Momentum can be considered as the velocity density of energy.

The space-time distribution of energy

One of the rules by which the grid is constructed describes how energy is distributed in space-time:

$$\frac{\partial E}{\partial (xt)} \; .$$

The space-time distribution of energy is represented by the unnamed box below the box called power, and to the left of the box called force. This unnamed box can be considered as the space-time density of energy.

Reference frames

All of the elements on the periodic table of mechanical elements must be described in terms of a particular frame of reference. This frame of reference must be characterized by having a centroid that is stationary. The periodic table of mechanical elements describes fully accounted (closed) systems.

Energy can neither enter nor leave the system. Energy can only be transferred from one part of the system to another.

Linear momentum can be exchanged between parts of the system. The linear momentum of the centroid of the system must at all times be zero.

Angular momentum can be exchanged between parts of the system. The angular momentum of the centroid of the system must at all times be zero.

All forces must occur in balanced pairs.

Many seeming paradoxes will arise if these edicts are not adhered to rigorously. This is a consequence of mixing physical quantities described with respect to different reference frames without properly accounting for the relationship between those reference frames.

Crossing dimension lines

Interesting and useful relationships come from taking the ratio of two elements that have particular geometric relationships on the grid. These relationships arise solely from the geometric relationship on the grid between the two elements. These relationships are a logical consequence of the rules by which the grid is constructed.

The boxes on the grid are separated by dimension lines. There are spatial-dimension lines, temporal-dimension lines, velocity-dimension lines, and space-time-dimension lines. The ratio of two elements is related to the dimension lines that separate that pair of elements on the grid.

Each dimension line on the grid represents a particular aspect of the physical situation described by the elements on the grid. Following a sequence of navigation instructions that go from one element on the grid to another involves crossing dimension lines. Each step in the sequence of navigation instructions describes how particular dimension lines are crossed. The concept of crossing dimension lines provides a powerful tool for analyzing physical relationships. This is best illustrated by looking at some examples.

Consider an arbitrary 3x3 piece of the grid. The boxes have been labeled with arbitrary names to facilitate this discussion.

$\xi_{-1,1}$	$\xi_{0,1}$	$\xi_{1,1}$
$\xi_{-1,0}$	$\xi_{0,0}$	$\xi_{1,0}$
$\xi_{-1,-1}$	$\xi_{0,-1}$	$\xi_{1,-1}$

Diagram 68: An arbitrary 3x3 piece of the grid

Place this piece of the grid anywhere on the periodic table of mechanical elements and replace these labels with real elements. The relationships discussed here apply to the real elements wherever this piece of the grid is placed on the periodic table of mechanical elements.

Crossing one spatial-dimension line

Consider two elements ($\xi_{0,1}$ and $\xi_{0,0}$) that have a spatial relationship (x_1):

$$\xi_{0,1} = \left(\xi_{0,0}\right)\left(x_1\right) \quad .$$

These two elements lie on opposite sides of the spatial-dimension line between the two elements.

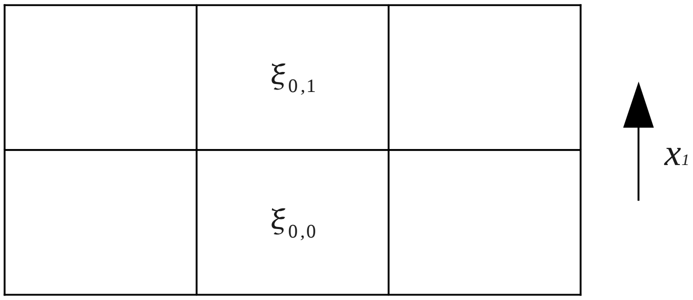


Diagram 69: An arbitrary pair of elements that have a spatial relationship

The ratio of these two elements yields the spatial relationship represented by the spatial-dimension line between the two elements:

$$\frac{\xi_{0,1}}{\xi_{0,0}} = x_1 \quad .$$

The ratio of any other pair of elements that are only separated by this spatial-dimension line have exactly the same spatial relationship.

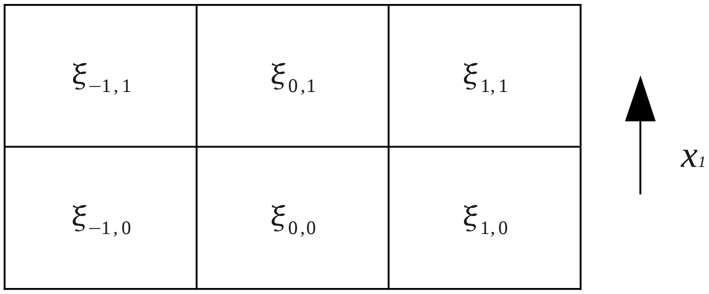


Diagram 70: Three pairs of elements that share a particular spatial relationship

For example:

$$\frac{\xi_{-1,1}}{\xi_{-1,0}} = x_1 \text{ , and } \frac{\xi_{1,1}}{\xi_{1,0}} = x_1 \quad .$$

As a concrete example, the work-energy theorem states that energy (E) is the space integral of force in the direction of motion ($\vec{F}_{\|}$) times the path length of that motion ($\|\vec{x}\|$):

$$E = \int \left|\vec{F}_{\|}\right| d\|\vec{x}\| \quad .$$

Examining the differential case:

$$dE = \left|\vec{F}_{\|}\right| d\|\vec{x}\| \quad ,$$

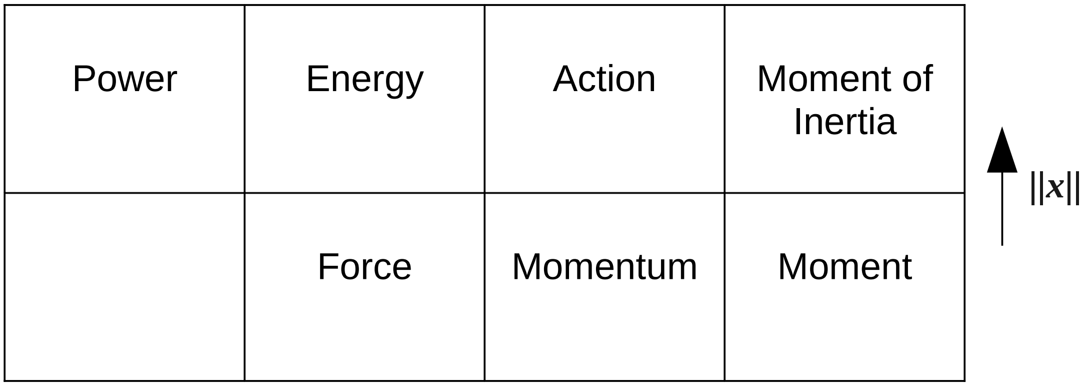
and applying some simple algebraic manipulation yields:

$$d\|\vec{x}\| = \frac{dE}{\left|\vec{F}_{\|}\right|} \quad .$$

This formula states that the differential path length is the ratio of the differential energy over force in the direction of motion. Removing the differentials yields the path length of the motion as the ratio of energy over force in the direction of motion:

$$\|\vec{x}\| = \frac{E}{\left|\vec{F}_{\|}\right|} \quad .$$

The spatial-dimension line that separates the energy box and the force box represents the path length of the motion caused by the force. The ratio of action over momentum, the ratio of moment of inertia over moment, in fact the ratio of any two boxes that are only separated by this spatial-dimension line will yield the same path length.



Power	Energy	Action	Moment of Inertia
	Force	Momentum	Moment

Diagram 71: Some pairs of elements that share the same spatial relationship as energy and force

Crossing two spatial-dimension lines

Consider three elements ($\xi_{0,1}$ $\xi_{0,0}$, $\xi_{0,-1}$) that are only separated by a pair of consecutive spatial-dimension lines (x_1 , x_{-1}).

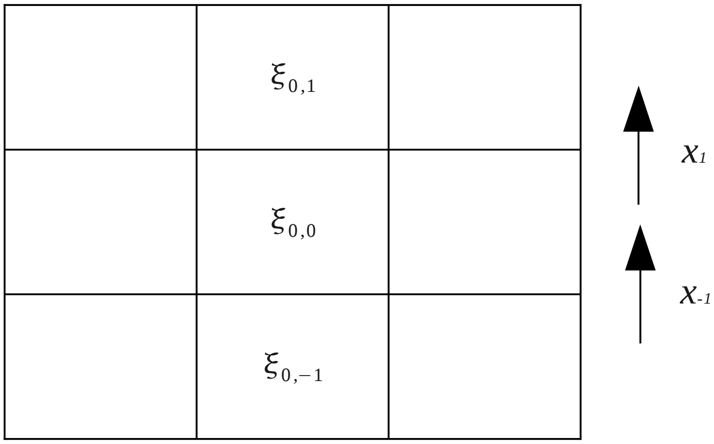


Diagram 72: Three arbitrary elements that are only separated by a pair of spatial-dimension lines

The first two elements have the spatial relationship x_1 :

$$\xi_{0,1} = \left(\xi_{0,0}\right)\left(x_1\right) \ .$$

The ratio of the first two elements yields the spatial relationship between them:

$$\frac{\xi_{0,1}}{\xi_{0,0}} = x_1 \ .$$

The last two elements have the spatial relationship x_{-1} :

$$\xi_{0,0} = \left(\xi_{0,-1}\right)\left(x_{-1}\right) \ .$$

The ratio of the last two elements yields the spatial relationship between them:

$$\frac{\xi_{0,0}}{\xi_{0,-1}} = x_{-1} \ .$$

Algebraic manipulation shows that the ratio of the first and last elements yields the areal relationship between them:

$$\frac{\xi_{0,1}}{\xi_{0,-1}} = \left(x_1\right)\left(x_{-1}\right) \ .$$

The ratio of any pair of elements that are only separated by the same pair of spatial-dimension lines will yield the same areal relationship.

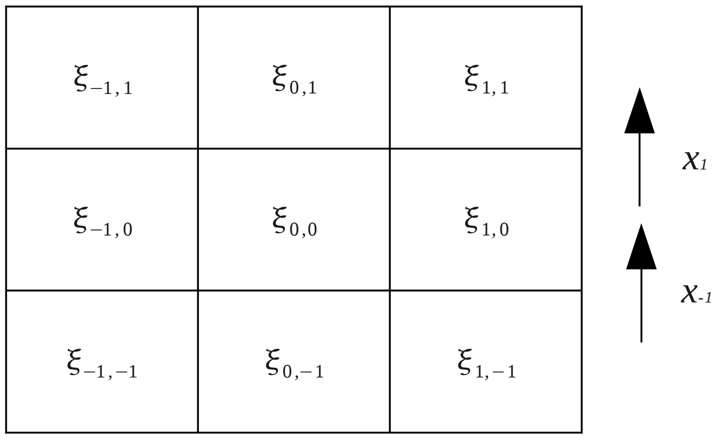


Diagram 73: Three sets of elements that are only separated by a pair of spatial-dimension lines

For example:

$$\frac{\xi_{-1,1}}{\xi_{-1,-1}} = (x_1)(x_{-1}) \text{ , and } \frac{\xi_{1,1}}{\xi_{1,-1}} = (x_1)(x_{-1}) \text{ .}$$

Algebraic manipulation also shows that the product of the first and last elements yields the square of the second element times the ratio of the first and second spatial relationships:

$$\xi_{0,1}\xi_{0,-1} = (\xi_{0,0})^2\left(\frac{x_1}{x_{-1}}\right) \text{ .}$$

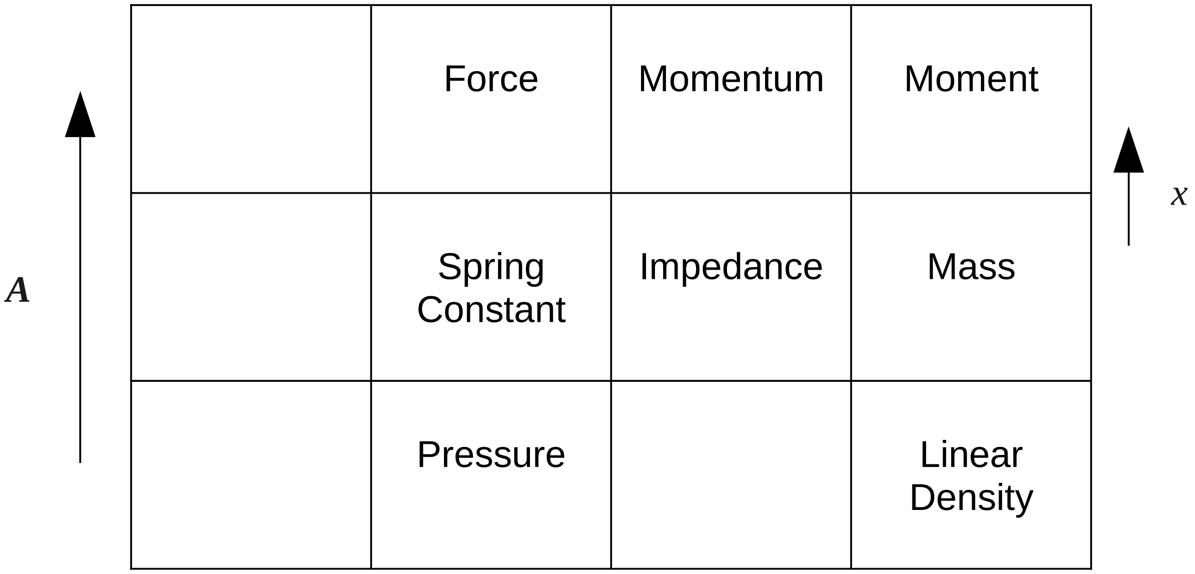
As a concrete example, Hooke's law states that force ($|\vec{F}|$) is equal to the product of spring constant (k) times the displacement (x) of the spring from it's equilibrium position:

$$|\vec{F}| = k\,x \quad .$$

The ratio of force over spring constant yields the spatial relationship between force and spring constant:

$$\frac{|\vec{F}|}{k} = x \quad .$$

The spatial-dimension line that separates the force box and the spring constant box represents the displacement of the spring from equilibrium.



	Force	Momentum	Moment
	Spring Constant	Impedance	Mass
	Pressure		Linear Density

Diagram 74: Force, spring constant and pressure are only separated by a pair of spatial-dimension lines

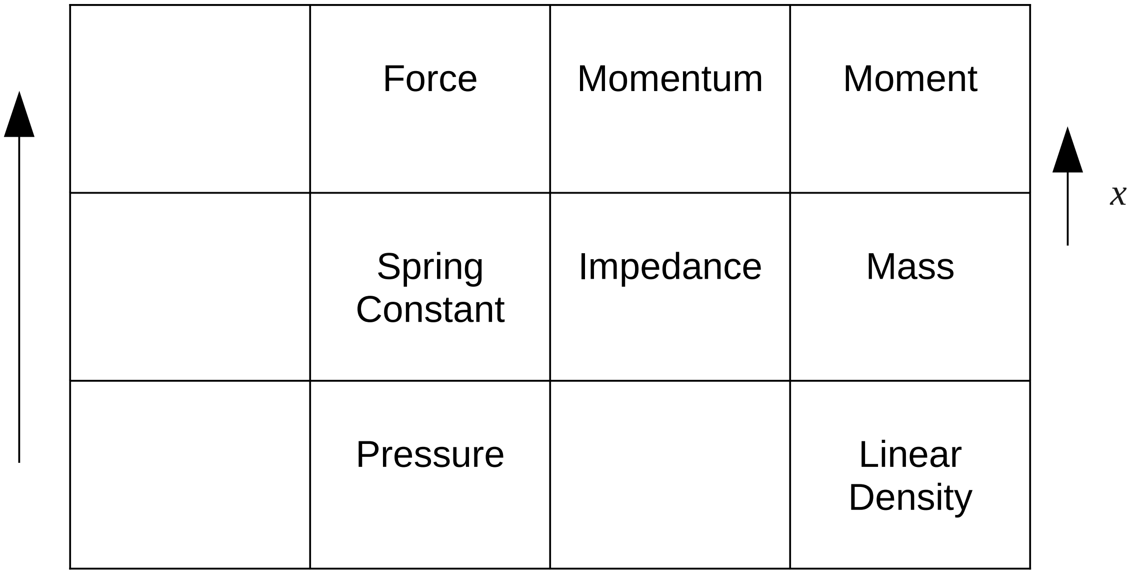
Pressure (P) can be defined as the ratio of force ($\vec{F}$) over area ($\vec{A}$):

$$P = \frac{\vec{F}}{\vec{A}} \quad .$$

The ratio of force over pressure yields the areal relationship between force and pressure:

$$\frac{P}{\vec{F}} = \vec{A} \quad .$$

The two spatial-dimension lines that separate the force box and the pressure box represent the areal relationship between force and pressure. The ratio of any other pair of elements that are only separated by these two spatial-dimension lines will yield the same areal relationship. For example, the ratio of moment over linear density yields this same areal relationship.



	Force	Momentum	Moment
	Spring Constant	Impedance	Mass
	Pressure		Linear Density

Diagram 75: Some other elements that are only separated by the same pair of spatial-dimension lines as force and pressure

The spatial relationship between spring constant and pressure (the ratio of spring constant over pressure) can be found by dividing the areal relationship between force and pressure by the spatial relationship between force and spring constant:

$$\frac{k}{P} = \frac{|\vec{\mathbf{A}}|}{x} .$$

The product of force and pressure yields the square of spring constant times the ratio of the spatial relationship between force and spring constant over the spatial relationship between spring constant and pressure:

$$|\vec{\mathbf{F}}|P = k^2\left(\frac{x}{\left(\frac{|\vec{\mathbf{A}}|}{x}\right)}\right) = \frac{k^2 x^2}{|\vec{\mathbf{A}}|} .$$

Spring constant can now be written as:

$$k = \sqrt{\frac{|\vec{\mathbf{F}}|P|\vec{\mathbf{A}}|}{x^2}} = \frac{P|\vec{\mathbf{A}}|}{x} = \frac{|\vec{\mathbf{F}}|}{x} .$$

This is just a restatement of Hooke's law in terms of pressure and area.

Crossing three spatial-dimension lines

Crossing three consecutive spatial-dimension lines represents a volume relationship. The ratio of any pair of elements that are only separated by the same three consecutive spatial-dimension lines yields the volume relationship that is represented by those three spatial-dimension lines.

As a concrete example, the potential energy (E) contained in a volume of gas is the product of the pressure (P) of the gas times the volume (V) of the gas:

$$E = PV \ .$$

The ratio of energy over pressure yields the volume relationship represented by the three spatial-dimension lines that separate energy and pressure:

$$\frac{E}{P} = V \ .$$

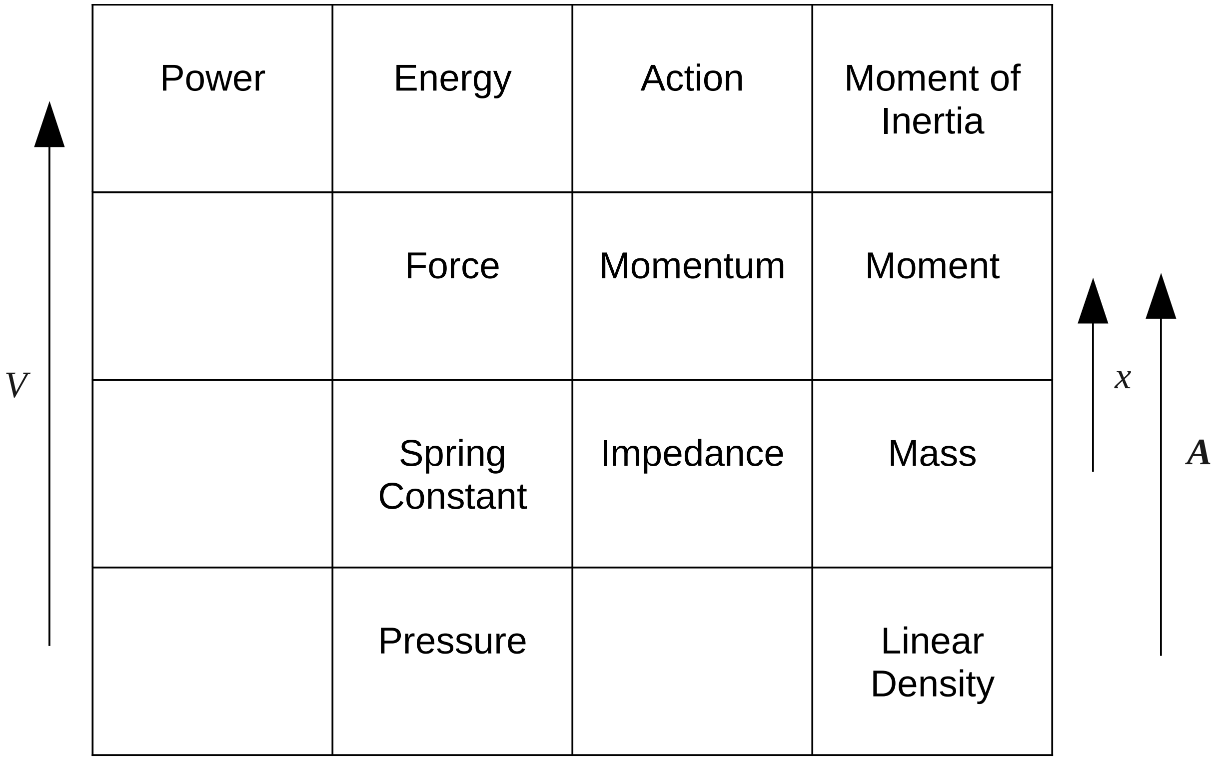
The ratio of moment of inertia over linear density, in fact the ratio of any two elements that are only separated by these three spatial-dimension lines will also yield this volume relationship.

From the previous concrete example of crossing two spatial-dimension lines the spatial-dimension line that separates force from spring constant represented the displacement:

$$x \ ,$$

and the spatial-dimension line that separates spring constant and pressure represented the ratio of area over displacement:

$$\frac{|\vec{\mathbf{A}}|}{x} \ .$$



Power	Energy	Action	Moment of Inertia
	Force	Momentum	Moment
	Spring Constant	Impedance	Mass
	Pressure		Linear Density

Diagram 76: The volume relationship between energy and pressure, and some other elements that are only separated by the same three spatial-dimension lines

The spatial-dimension line that separates energy from force can be calculated as:

$$\frac{V}{|\vec{\boldsymbol{A}}|} \;.$$

The three consecutive spatial-dimension lines that separate energy and pressure represent the volume relationship between energy and pressure. The product of the three dimension-lines $\frac{|\vec{\boldsymbol{A}}|}{x}$, x , and $\frac{V}{|\vec{\boldsymbol{A}}|}$ yields the volume relationship:

$$V = \left(\frac{|\vec{\boldsymbol{A}}|}{x}\right)(x)\left(\frac{V}{|\vec{\boldsymbol{A}}|}\right) \;.$$

Crossing one temporal-dimension line

Consider two elements ($\xi_{0,0}$ and $\xi_{-1,0}$) that have a temporal relationship (τ_{-1}):

$$\xi_{0,0} = (\xi_{-1,0})(\tau_{-1}) \; .$$

These two elements lie on opposite sides of the temporal-dimension line that separates them.

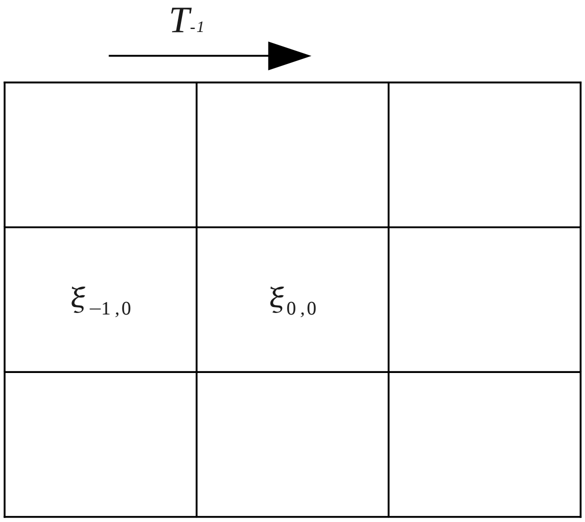
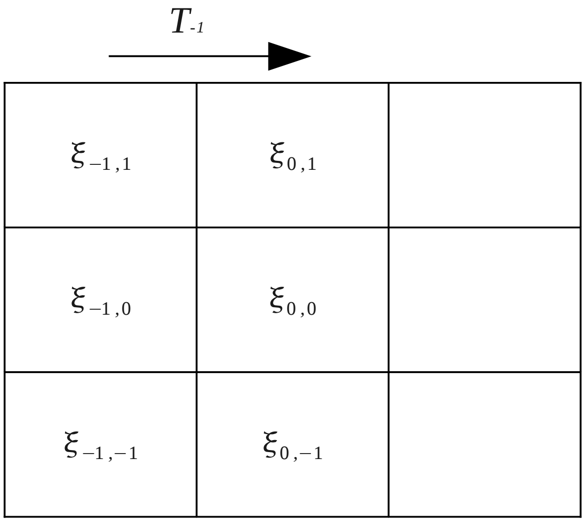


Diagram 77: An arbitrary pair of elements that have a temporal relationship

The ratio of these two elements yields the temporal relationship that is represented by the temporal-dimension line between the two elements:

$$\frac{\xi_{0,0}}{\xi_{-1,0}} = \tau_{-1} \; .$$

The ratio of any pair of elements that are only separated by this temporal-dimension line yields the same temporal relationship.



$\xi_{-1,1}$	$\xi_{0,1}$	
$\xi_{-1,0}$	$\xi_{0,0}$	
$\xi_{-1,-1}$	$\xi_{0,-1}$	

Diagram 78: Three pairs of elements that share a particular temporal relationship

For example:

$$\frac{\xi_{0,1}}{\xi_{-1,1}} = \tau_{-1} \quad \text{and} \quad \frac{\xi_{0,-1}}{\xi_{-1,-1}} = \tau_{-1} \ .$$

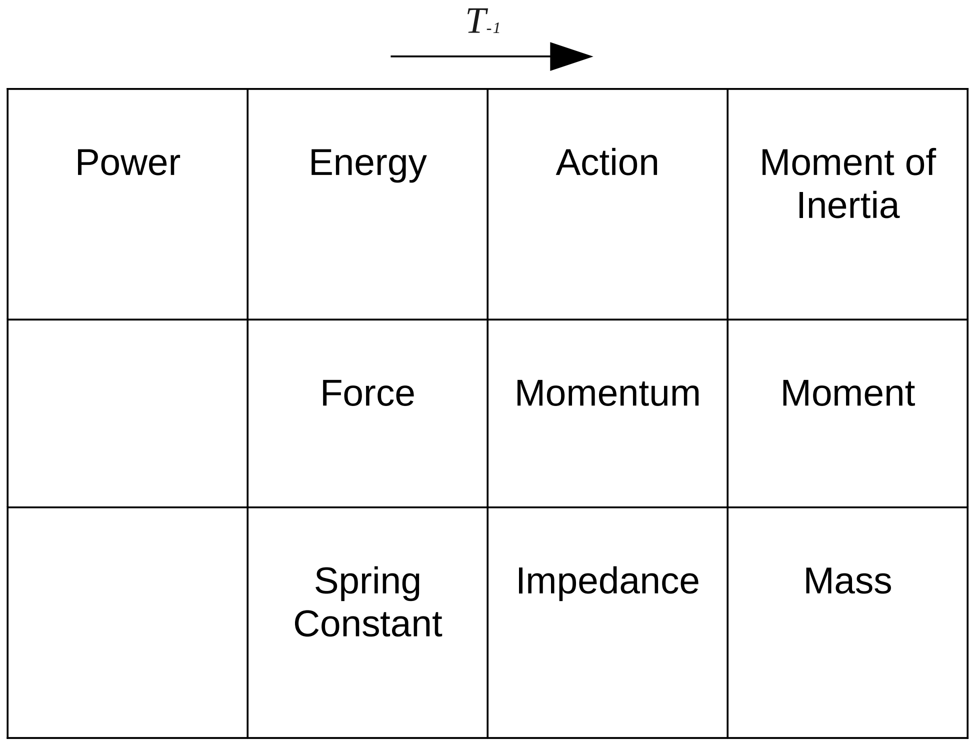
As a concrete example, force ($\vec{F}$) can be calculated as the product of impulses of momentum ($\vec{P}$) times the frequency (f) at which those impulses are delivered:

$$\vec{F} = \vec{P} f \ .$$

The ratio of force over momentum yields the frequency at which the impulses of momentum are delivered:

$$\frac{\vec{F}}{\vec{P}} = f \ .$$

The temporal-dimension line that separates the force box and the momentum box represents the frequency of the impulses that cause the force.



Power	Energy	Action	Moment of Inertia
	Force	Momentum	Moment
	Spring Constant	Impedance	Mass

Diagram 79: Some pairs of elements that share the same temporal relationship as force and momentum

The ratio of energy over action, the ratio of spring constant over impedance, in fact the ratio of any two boxes that are only separated by this temporal-dimension line will yield this same frequency.

Crossing two temporal-dimension lines

Consider three elements ($\xi_{-1,0}$ $\xi_{0,0}$, $\xi_{1,0}$) that are only separated by two consecutive temporal-dimension lines (τ_{-1} , τ_1).

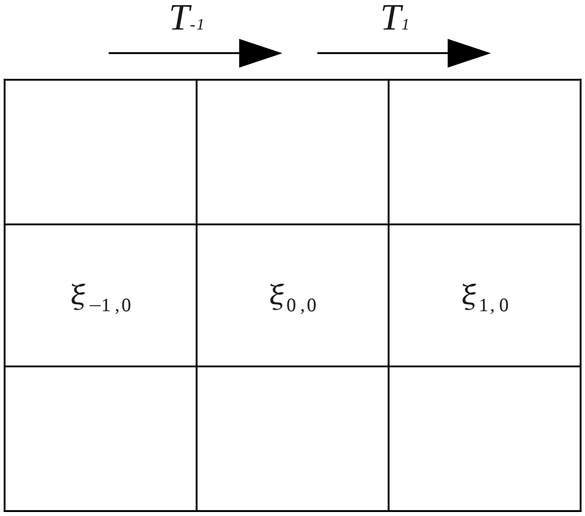


Diagram 80: Three arbitrary elements that are only separated by a pair of temporal-dimension lines

The first two elements have a temporal relationship:

$$\xi_{0,0} = (\xi_{-1,0})(\tau_{-1}) \quad .$$

The ratio of the first two elements yields the inverse temporal relationship between the two elements:

$$\frac{\xi_{-1,0}}{\xi_{0,0}} = \frac{1}{\tau_{-1}} \quad .$$

The last two elements have a temporal relationship:

$$\xi_{1,0} = (\xi_{0,0})(\tau_1) \quad .$$

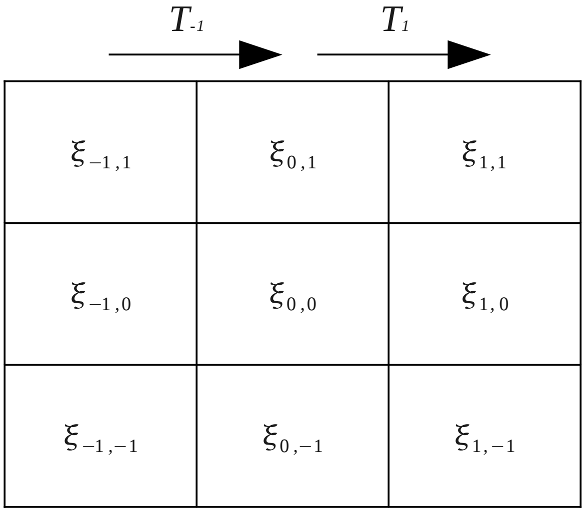
The ratio of the last two elements yields the inverse temporal relationship between the two elements:

$$\frac{\xi_{0,0}}{\xi_{1,0}} = \frac{1}{\tau_1} \quad .$$

Algebraic manipulation shows the ratio of the first element over the last element is the product of the frequency relationship between the first two elements times the frequency relationship between the last two elements:

$$\frac{\xi_{-1,0}}{\xi_{1,0}} = \frac{1}{\tau_{-1}}\frac{1}{\tau_{1}} \ .$$

The ratio of any pair of elements that are only separated by this pair of temporal-dimension lines will yield the same product of frequencies relationship.



$\xi_{-1,1}$	$\xi_{0,1}$	$\xi_{1,1}$
$\xi_{-1,0}$	$\xi_{0,0}$	$\xi_{1,0}$
$\xi_{-1,-1}$	$\xi_{0,-1}$	$\xi_{1,-1}$

Diagram 81: Three sets of elements that are only separated by a pair of temporal-dimension lines

Algebraic manipulation also shows the product of the first and last elements as the square of the second element times the ratio of the frequency relationship between the first two elements over the frequency relationship between the last two elements:

$$\xi_{-1,0}\xi_{1,0} = \frac{\left(\frac{1}{\tau_{-1}}\right)}{\left(\frac{1}{\tau_{1}}\right)} \ .$$

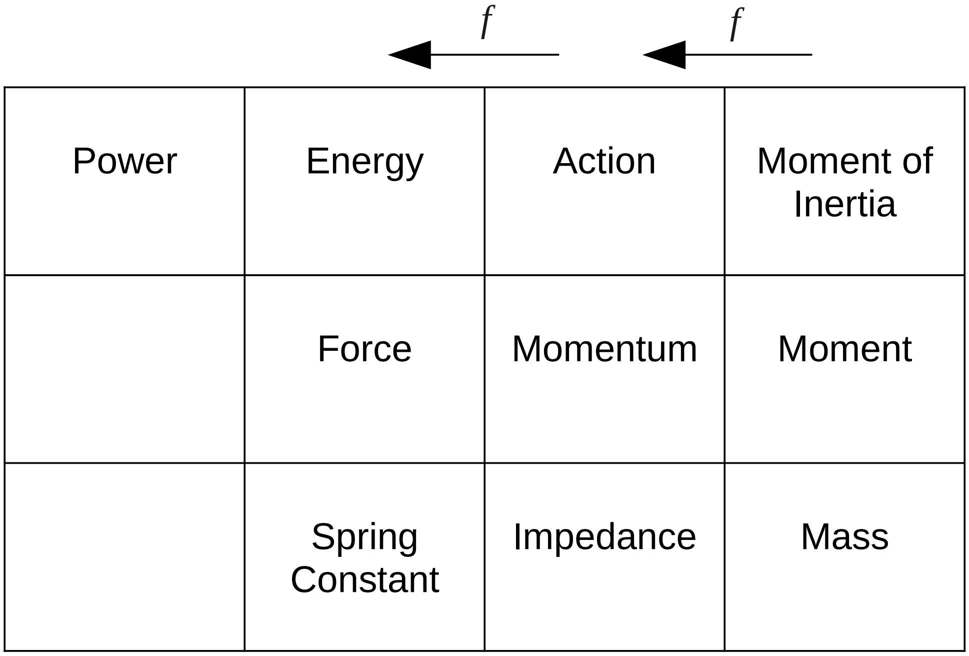
As a concrete example, the self-resonant frequency (f) of a mass-spring system is the square root of the ratio of spring constant (k) over mass (m):

$$f = \sqrt{\frac{k}{m}} \; .$$

This can be rewritten as:

$$f^2 = \frac{k}{m} \; .$$

Each of the two temporal-dimension lines that separate the spring constant box and the mass box represents the self-resonant frequency of the mass-spring system.



Power	Energy	Action	Moment of Inertia
	Force	Momentum	Moment
	Spring Constant	Impedance	Mass

Diagram 82: The frequency squared relationship between mass and spring constant, and some other elements that share the same frequency squared relationship

The ratio of energy over moment of inertia, the ratio of force over moment, in fact the ratio of any two elements that are only separated by this pair of temporal-dimension lines will yield the same frequency squared relationship.

The product of spring constant and mass yields the square of the impedance (Ω) of the system times the ratio of the frequencies represented by the two temporal-dimension lines that separate the spring constant box and the mass box:

$$k\,m \;=\; \Omega^2 \frac{f}{f} \;=\; \Omega^2 \quad .$$

The impedance of a mass-spring system is the square root of the product of the spring constant times the mass:

$$\Omega \;=\; \sqrt{k\,m} \quad .$$

Crossing velocity dimension lines

Crossing a velocity-dimension line from one element to another is the same as simultaneously crossing the spatial-dimension line and the temporal-dimension line which intersect at the velocity-dimension line between the two elements. If the velocity-dimension line is crossed in a forward direction then the spatial-dimension line is crossed in a forward direction and the temporal-dimension line is crossed in a backwards (or inverse) direction. If the velocity-dimension line is crossed in a backwards direction then the spatial-dimension line is crossed in a backwards direction while the temporal-dimension line is crossed in a forward direction.

Crossing one velocity-dimension line

Consider two elements ($\xi_{-1,1}$ and $\xi_{0,0}$) that have a velocity relationship (v_1):

$$\xi_{-1,1} = (\xi_{0,0})(v_1) \quad .$$

These two elements line on opposite sides of the velocity-dimension line that separates them.

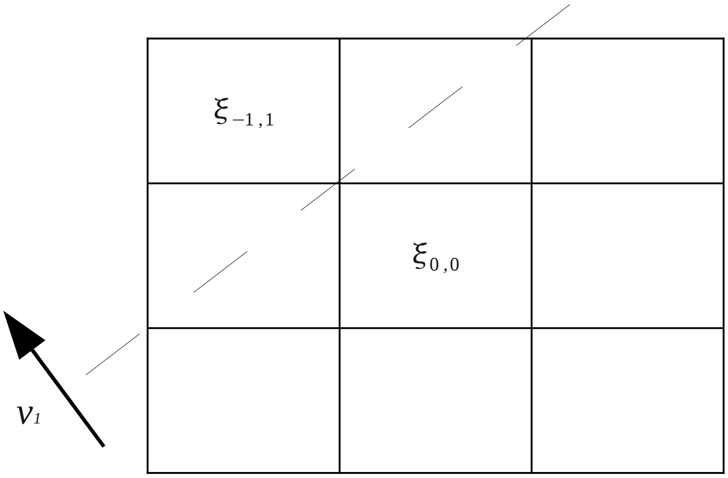


Diagram 83: A pair of elements that are only separated by a velocity-dimension line

The ratio of the two elements yields the velocity relationship represented by the velocity-dimension line that separates the two elements:

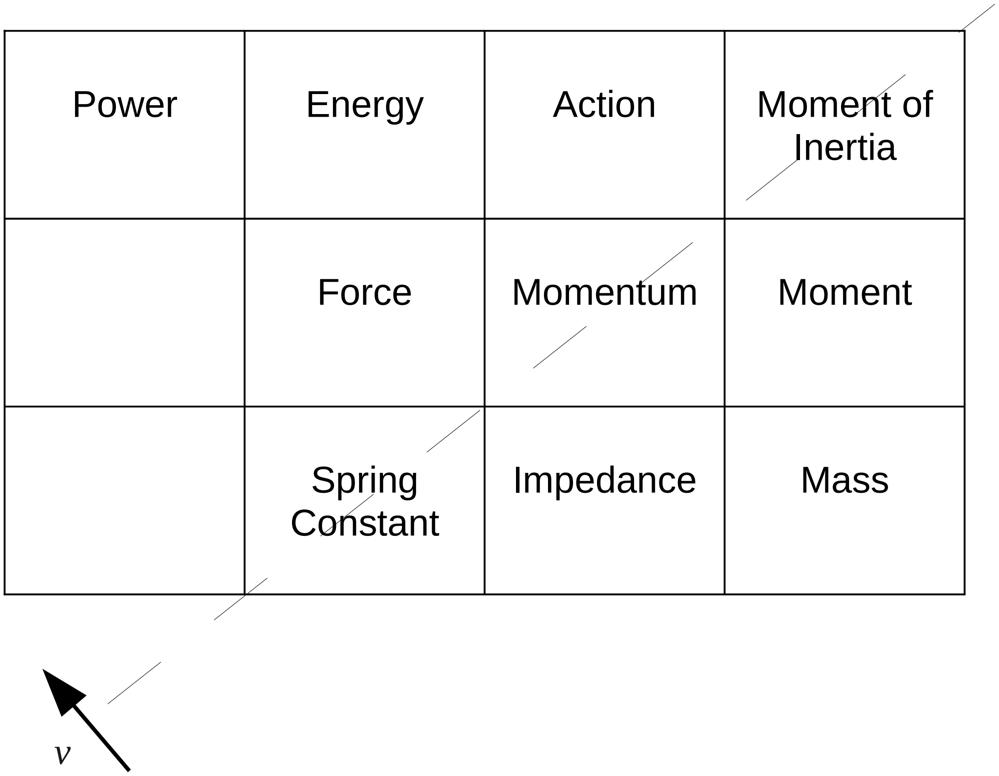
$$\frac{\xi_{-1,1}}{\xi_{0,0}} = v_1 \quad .$$

The ratio of any two elements that are only separated by this velocity-dimension line yields the same velocity relationship.

As a concrete example, the ratio of force ($\vec{F}$) over impedance (Ω) yields the velocity of motion ($\vec{v}$):

$$\frac{\vec{F}}{\Omega} = \vec{v} \; .$$

The velocity-dimension line that separates the force box and the impedance box represents the velocity of motion. The ratio of action over moment yields the same velocity relationship. The ratio of any pair of elements that are only separated by this velocity-dimension line will yield the same velocity relationship.



Power	Energy	Action	Moment of Inertia
	Force	Momentum	Moment
	Spring Constant	Impedance	Mass

Diagram 84: The velocity relationship between force and impedance, and some other elements that share the same velocity relationship

Crossing two velocity-dimension lines

Consider three elements ($\xi_{-1,1}$, $\xi_{0,0}$, $\xi_{1,-1}$) that are only separated by two consecutive velocity-dimension lines (v_1 , v_{-1}).

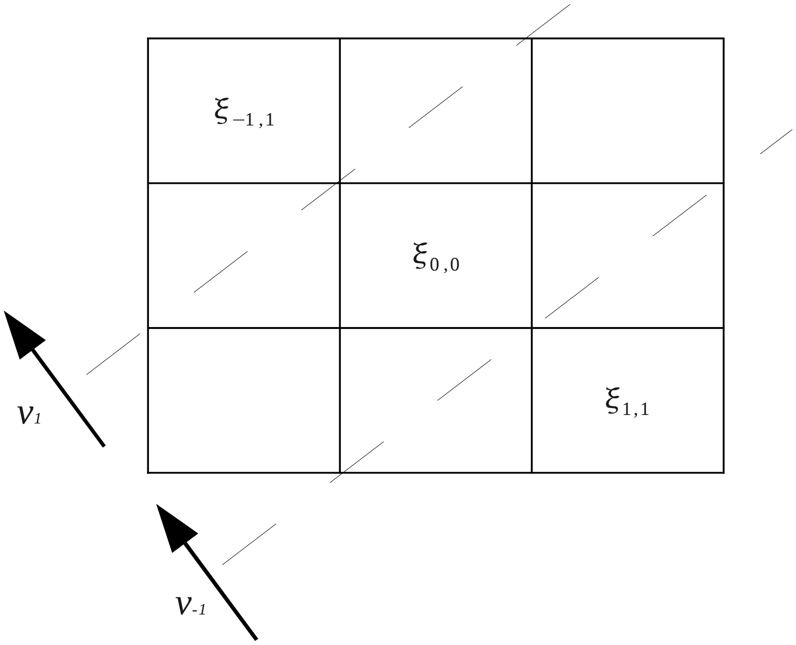


Diagram 85: Three elements that are only separated by a pair of velocity-dimension lines

The first two elements have a velocity relationship:

$$\xi_{-1,1} = (\xi_{0,0})(v_1) \quad .$$

The ratio of the first two elements yields the velocity relationship between the two elements:

$$\frac{\xi_{-1,1}}{\xi_{0,0}} = v_1 \quad .$$

The last two elements have a velocity relationship:

$$\xi_{0,0} = (\xi_{1,1})(v_{-1}) \quad .$$

The ratio of the last two elements yields the velocity relationship between the two elements:

$$\frac{\xi_{0,0}}{\xi_{1,-1}} = v_{-1} \quad .$$

Algebraic manipulation shows the ratio of the first element over the and last element as the product of the velocity relationship between the first two elements times the velocity relationship between the last two elements:

$$\frac{\xi_{-1,1}}{\xi_{1,-1}} = (v_1)(v_{-1}) \quad .$$

Algebraic manipulation also shows the product of the first and last elements as the square of the second element times the ratio of the velocity relationship between the first two elements over the velocity relationship between the last two elements:

$$(\xi_{-1,1})(\xi_{1,-1}) = (\xi_{0,0})^2\left(\frac{v_1}{v_{-1}}\right) \quad .$$

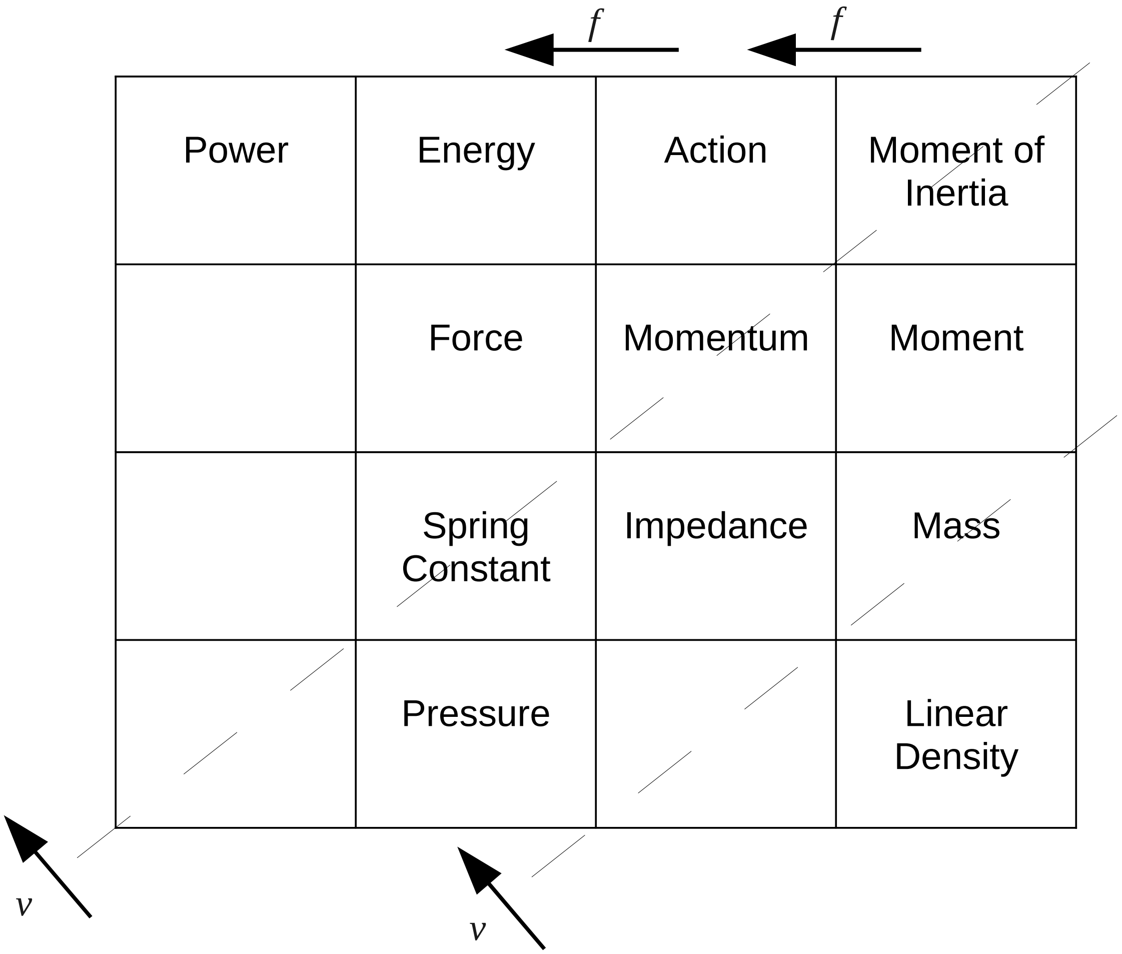
As a concrete example, the velocity (v) of a pulse traveling along a stretched rope is the square root of the ratio of the force of tension (T) in the rope over the linear density (μ) of the rope:

$$v = \sqrt{\frac{T}{\mu}} \quad .$$

This can be rewritten as:

$$v^2 = \frac{T}{\mu} \quad .$$

The ratio of force over linear density is the square of velocity.



Power	Energy	Action	Moment of Inertia
	Force	Momentum	Moment
	Spring Constant	Impedance	Mass
	Pressure		Linear Density

Diagram 86: The velocity and frequency relationships between force, impedance and linear density in a rope

The product of force times linear density yields the impedance (Ω) squared times the ratio of the resonant frequency (f) over the resonant frequency:

$$T\mu = \Omega^2 \frac{f}{f} = \Omega^2 .$$

Crossing space-time dimension lines

Crossing a space-time-dimension line between two elements is the same as simultaneously crossing the spatial-dimension line and the temporal-dimension line which intersect at the space-time-dimension line between the two elements. If the space-time-dimension line is crossed in a forward direction then the spatial-dimension line and the temporal-dimension line are both crossed in a forward direction. If the space-time-dimension line is crossed in a backwards (or inverse) direction then both the spatial-dimension line and the temporal-dimension line are crossed in a backwards (or inverse) direction.

Crossing one space-time-dimension line

Consider two elements ($\xi_{0,0}$ and $\xi_{1,1}$) that have a space-time relationship. These two elements are separated by a spatial-dimension line (x_1) and a temporal-dimension line (τ_1).

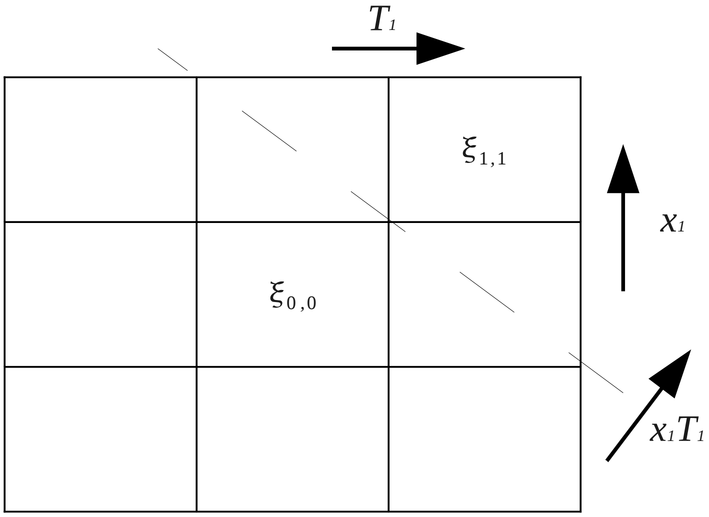


Diagram 87: A pair of elements that are only separated by a space-time-dimension line

The ratio of the second element over the first element yields the space-time relationship represented by the space-time-dimension line that separates the two elements:

$$\frac{\xi_{1,1}}{\xi_{0,0}} = \left(x_1 \tau_1\right) \quad .$$

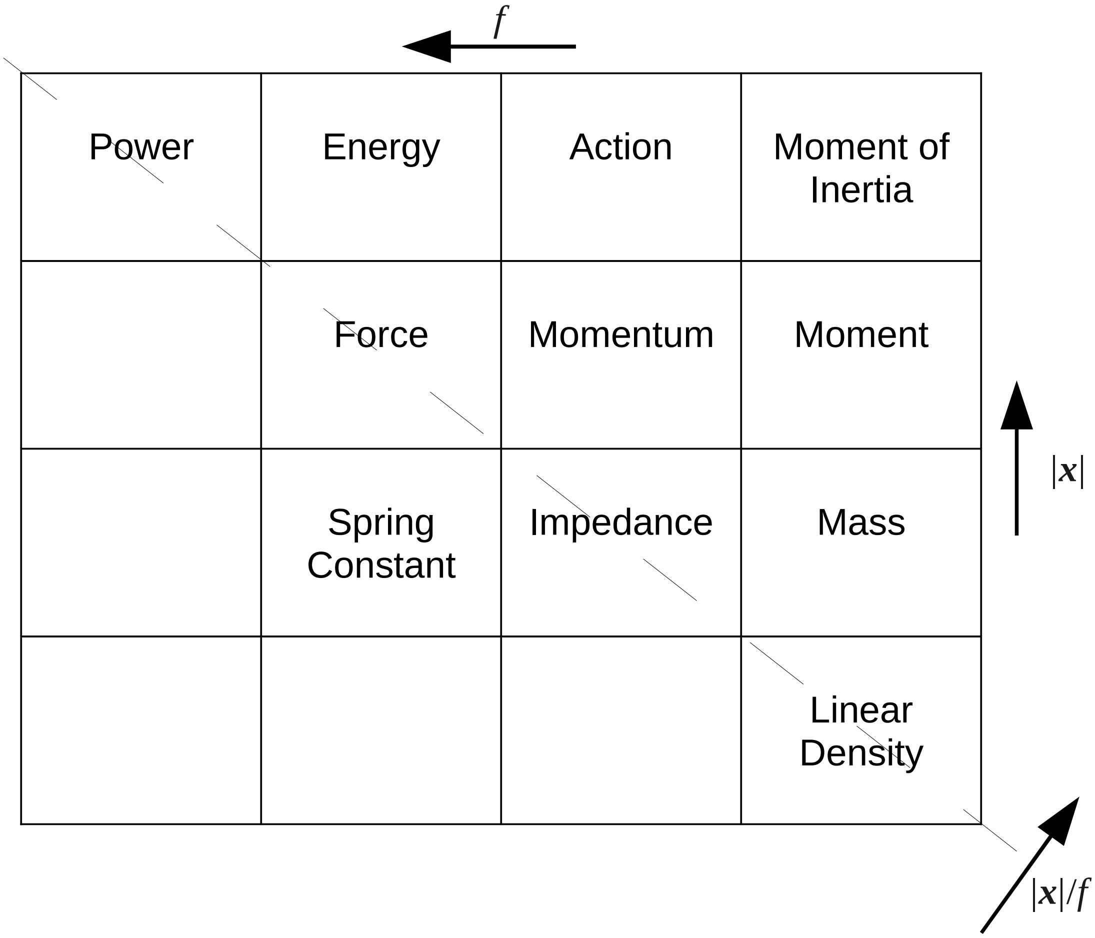
As a concrete example, momentum ($\vec{P}$) and spring constant (k) have a space-time relationship ($\vec{x}\tau$):

$$\vec{P} = k\vec{x}\tau \quad .$$

The ratio of momentum over spring constant yields the space-time relationship between the momentum box and the spring constant box:

$$\frac{\vec{P}}{k} = \vec{x}\tau \quad .$$

The space-time-dimension line between spring constant and momentum represents the product of displacement and period.



Power	Energy	Action	Moment of Inertia
	Force	Momentum	Moment
	Spring Constant	Impedance	Mass
			Linear Density

Diagram 88: The spatial, temporal and space-time relationships between spring constant and momentum

Crossing two space-time-dimension lines

Consider three elements ($\xi_{-1,-1}$ $\xi_{0,0}$, $\xi_{1,1}$) that are only separated by a pair of consecutive space-time-dimension lines ($x_{-1}\tau_{-1}$, $x_1\tau_1$).

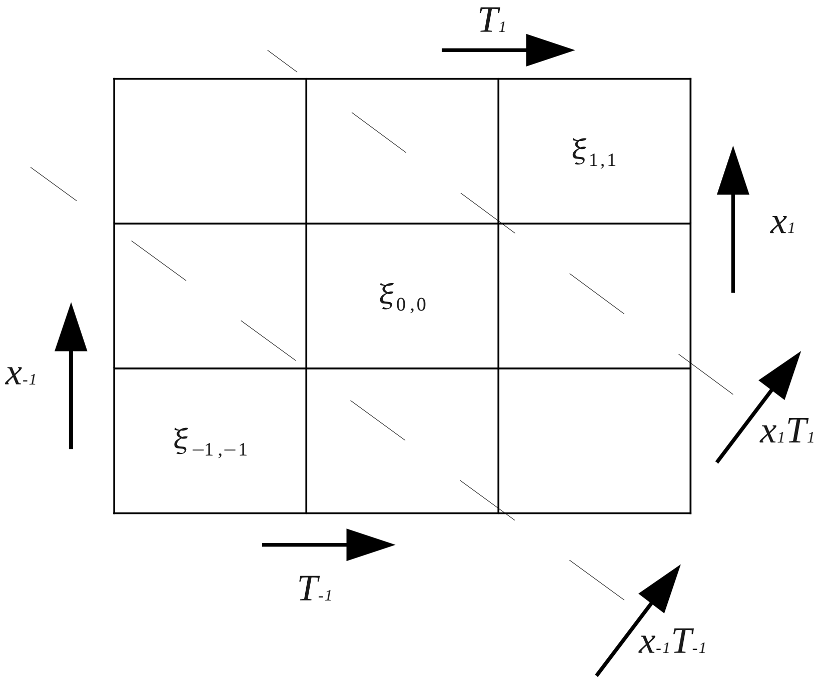


Diagram 89: Three elements that are only separated by a pair of space-time-dimension lines

The first element and the second element have a space-time relationship:

$$\xi_{0,0} = (\xi_{-1,-1})(x_{-1}\tau_{-1}) .$$

The ratio of the second element over the first element yields the space-time relationship between the first element and the second element:

$$\frac{\xi_{0,0}}{\xi_{-1,-1}} = (x_{-1}\tau_{-1}) .$$

The second element and the third element have a space-time relationship:

$$\xi_{1,1} = (\xi_{0,0})(x_1\tau_1) .$$

The ratio of the third element over the second element yields the space-time relationship between the second element and the third element:

$$\frac{\xi_{1,1}}{\xi_{0,0}} = \left(x_1 \tau_1\right) \quad .$$

Algebraic manipulation shows that the ratio of the third element over the first element yields the product of the space-time relationship between the first element and the second element times the space-time relationship between the second element and the third element:

$$\frac{\xi_{1,1}}{\xi_{-1,-1}} = \left(x_{-1} \tau_{-1}\right)\left(x_1 \tau_1\right) \quad .$$

Algebraic manipulation also shows that the product of the first element times the third element yields the square of the second element times the ratio of the space-time relationship between the first element and the second element over the space-time relationship between the second element and the third element:

$$\xi_{-1,-1} \xi_{1,1} = \frac{\left(x_{-1} \tau_{-1}\right)}{\left(x_1 \tau_1\right)} \quad .$$

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Electromagnetic Elements

$\frac{d\Phi_E}{dt}$	Φ_E		
$\frac{dV}{dt}$	$V=\frac{d\Phi_B}{dt}$	Φ_B	
$\frac{\overrightarrow{d\boldsymbol{E}}}{dt}$	$\vec{\boldsymbol{E}}$	$\frac{1}{c^2}\frac{d\Phi_E}{dt}$	
	$\frac{\overrightarrow{d\boldsymbol{B}}}{dt}$	$\vec{\boldsymbol{B}}$	

Diagram 90: The Periodic Table of Electromagnetic Elements

Introduction

Maxwell's equations define the relationships between the electromagnetic elements. Maxwell's equations are a set of simultaneous equations. They must all be satisfied at the same time.

The integral form of Maxwell's equations will be used for this presentation:

$$\oint \vec{\boldsymbol{E}}\cdot\overrightarrow{d\boldsymbol{A}} = \Phi_E$$

$$\oint \vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{A}} = \Phi_B$$

$$\oint \vec{\boldsymbol{E}}\cdot\overrightarrow{d\boldsymbol{s}} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}} = \frac{1}{c^2}\frac{d\Phi_E}{dt}$$

Constructing the grid

Consider a two-dimensional grid of boxes where each box is related to it's immediate neighbors by a specific set of rules. The same set of rules applies to each and every box on the grid. Each box on the grid represents a particular electromagnetic element ($\frac{d\Phi_E}{dt}$, Φ_E , voltage, …).

Using a suitable set of rules all of the electromagnetic elements can be arranged on the grid in accordance with Maxwell's equations in an unambiguous fashion. This arrangement constitutes the periodic table of electromagnetic elements. The periodic quality of the electromagnetic elements will become clear in the course of the presentation.

Rules of the grid

This diagram describes how any element, represented by ★, is related to it's immediate neighbors.

$\int \star\, d(x/t)$	$\int \star\, dx$	$\int \star\, d(xt)$
$\frac{\partial \star}{\partial t}$	Any Element ★	$\int \star\, dt$
$\frac{\partial \star}{\partial (xt)}$	$\frac{\partial \star}{\partial x}$	$\frac{\partial \star}{\partial (x/t)}$

Diagram 91: How an element is related to it's immediate neighbors

This is the complete set of rules by which the grid is constructed.

Φ_E on the grid

This diagram shows a 4x4 piece of the grid containing a box labeled Φ_E .

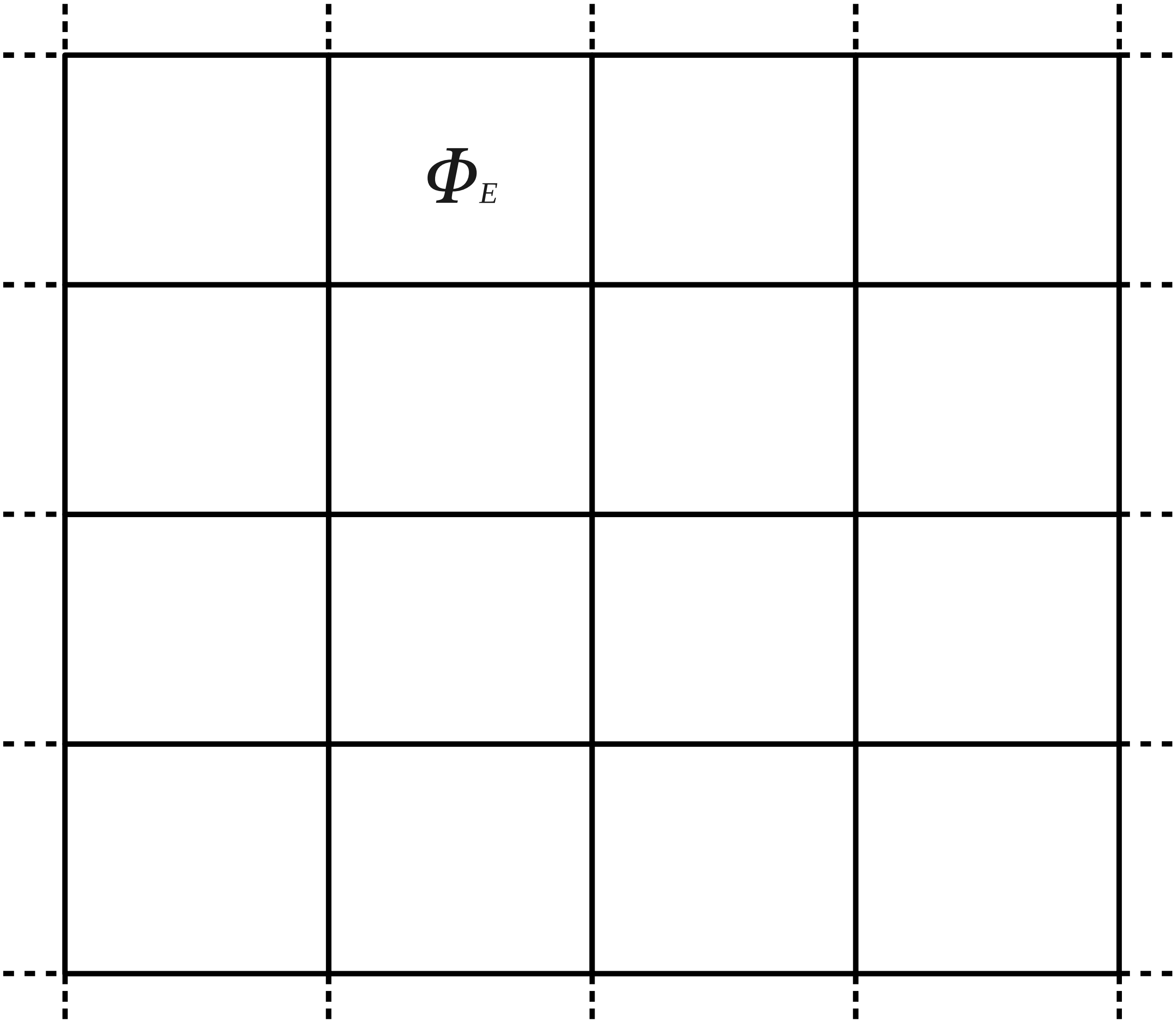

Diagram 92: Φ_E *placed on the grid*

The grid must extend infinitely in all directions. This is a logical consequence of the fact that every box is related to eight immediate neighbors.

E and *Φ*$_E$

The first Maxwell equation states that Φ_E is the product of $\vec{E}$ times an area:

$$\oint \vec{E}\cdot\overrightarrow{dA} = \Phi_E \quad .$$

This is an area integral which must be calculated as a double integral with respect to area:

$$\iint |\vec{E}|\, dx\, dx = \Phi_E \quad ,$$

with appropriate limits of integration.

According to the rules by which the grid is constructed the Φ_E box is located two boxes above the $\vec{E}$ box.

	$\oint \vec{E}\cdot\overrightarrow{dA} = \Phi_E$	
	$\vec{E}$	

Diagram 93: The relationship between the $\vec{E}$ *box and the* Φ_E *box*

E on the grid

This diagram shows a 4x4 piece of the grid with the $\vec{E}$ box placed correctly.

	Φ_E		
	$\vec{E}$		

Diagram 94: $\vec{E}$ *placed on the grid*

Voltage and *E*

The third Maxwell equation describes electric potential, or voltage (V):

$$V = \oint \vec{E} \cdot \overrightarrow{ds} = -\frac{d\Phi_B}{dt} \quad .$$

Examining the left part of this three way equality shows voltage is the product of $\vec{E}$ times the length of the electric path.

According to the rules by which the grid is constructed the V box is located one box above the $\vec{E}$ box.

	$V = \oint \vec{E} \cdot \overrightarrow{ds}$	
	$\vec{E}$	

Diagram 95: The relationship between the V box and the $\vec{E}$ box

Voltage on the grid

This diagram shows a 4x4 piece of the grid with the V box placed correctly.

	Φ_E		
	V		
	$\vec{\mathbf{E}}$		

Diagram 96: V *placed on the grid*

Φ_B and voltage

The third Maxwell equation describes electric potential, or voltage (V):

$$V = \oint \vec{E} \cdot \vec{ds} = -\frac{d\Phi_B}{dt} .$$

Examining the left and right parts of this three way equality shows voltage is the time derivative of Φ_B .

According to the rules by which the grid is constructed the V box is located one box to the left of the Φ_B box.

$V = -\frac{d\Phi_B}{dt}$	Φ_B	

Diagram 97: The relationship between the Φ_B box and the V box

This means the Φ_B box must be placed on the grid one box to the right of the V box.

Φ_B on the grid

This diagram shows a 4x4 piece of the grid with the Φ_B box placed correctly.

	Φ_E		
	V	Φ_B	
	$\vec{\boldsymbol{E}}$		

Diagram 98: Φ_B *placed on the grid*

B **and Φ_B**

The third Maxwell equation states Φ_B is the product of $\vec{B}$ times an area:

$$\oint \vec{B}\cdot\overrightarrow{dA} = \Phi_B \quad .$$

This is an area integral which must be calculated as a double integral with respect to area:

$$\iint |\vec{B}|\, dx\, dx = \Phi_B \quad ,$$

with appropriate limits of integration.

According to the rules by which the grid is constructed the Φ_B box is located two boxes above the $\vec{B}$ box.

	$\oint \vec{B}\cdot\overrightarrow{dA}=\Phi_B$	
	$\vec{B}$	

Diagram 99: The relationship between the $\vec{B}$ *box and the* Φ_B *box*

This means the $\vec{B}$ box must be placed on the grid two boxes below the Φ_B box.

B on the grid

This diagram shows a 4x4 piece of the grid with the $\vec{\boldsymbol{B}}$ box placed correctly.

	Φ_E		
	V	Φ_B	
	$\vec{\boldsymbol{E}}$		
		$\vec{\boldsymbol{B}}$	

Diagram 100: $\vec{\boldsymbol{B}}$ *placed on the grid*

d/dt-Φ_E and Φ_E

According to the rules by which the grid is constructed $\frac{d\Phi_E}{dt}$ is located one box to the left of the Φ_E box.

$\frac{d\Phi_E}{dt}$	Φ_E	

Diagram 101: The relationship between the $\frac{d\Phi_E}{dt}$ *box and the* Φ_E *box*

This means the $\frac{d\Phi_E}{dt}$ box must be placed on the grid one box to the left of the Φ_E box.

d/dt-Φ_E on the grid

This diagram shows a 4x4 piece of the grid with the $\frac{d\Phi_E}{dt}$ box placed correctly.

$\frac{d\Phi_E}{dt}$	Φ_E		
	V	Φ_B	
	$\vec{E}$		
		$\vec{B}$	

Diagram 102: $\frac{d\Phi_E}{dt}$ *placed on the grid*

B*.ds, Φ_E and *B

The fourth Maxwell equation states the relationship between $\oint \vec{B} \cdot \vec{ds}$ and $\frac{d\Phi_E}{dt}$:

$$\oint \vec{B} \cdot \vec{ds} = \frac{1}{c^2} \frac{d\Phi_E}{dt} \quad .$$

The left side of this equation is the product of $\vec{B}$ times a distance. According to the rules by which the grid is constructed the $\oint \vec{B} \cdot \vec{ds}$ box is located one box above the $\vec{B}$ box. This describes crossing one spatial-dimension line.

The right side of this equation is $\frac{d\Phi_E}{dt}$ over a velocity squared. According to the rules by which the grid is constructed the $\oint \vec{B} \cdot \vec{ds}$ box is located two boxes down and to the right of the $\frac{d\Phi_E}{dt}$ box. This describes crossing two velocity-dimension lines.

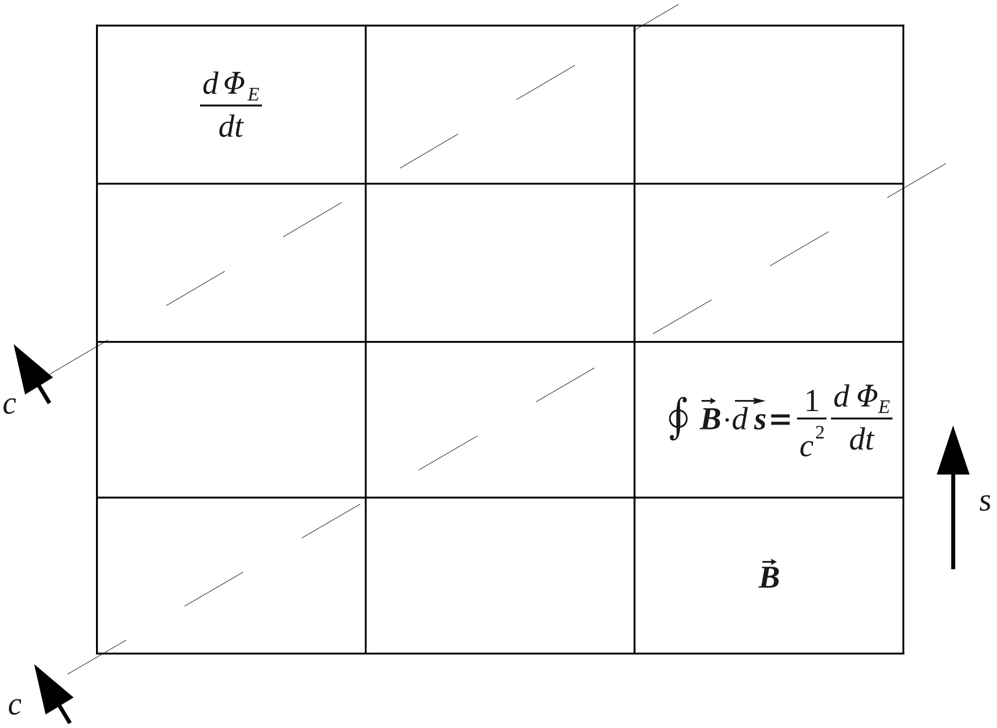

Diagram 103: The relationship between the $\oint \vec{B} \cdot \vec{ds}$ box, the $\frac{d\Phi_E}{dt}$ box and the $\vec{B}$ box

***B*.ds on the grid**

This diagram shows a 4x4 piece of the grid with the $\oint \vec{B}\cdot\overrightarrow{ds}$ box placed correctly.

$\frac{d\Phi_E}{dt}$	Φ_E		
	V	Φ_B	
	$\vec{E}$	$\oint \vec{B}\cdot\overrightarrow{ds}$	
		$\vec{B}$	

Diagram 104: $\oint \vec{B}\cdot\overrightarrow{ds}$ *placed on the grid*

dv/dt and voltage

According to the rules by which the grid is constructed the $\frac{dV}{dt}$ box is located one box to the left of the V box.

$\frac{dV}{dt}$	V	

Diagram 105: The relationship between the $\frac{dV}{dt}$ *box and the* V *box*

dv/dt on the grid

This diagram shows a 4x4 piece of the grid with the $\frac{dV}{dt}$ box placed correctly.

$\frac{d\Phi_E}{dt}$	Φ_E		
$\frac{dV}{dt}$	V	Φ_B	
	$\vec{E}$	$\oint \vec{B}\cdot\overrightarrow{ds}$	
		$\vec{B}$	

Diagram 106: $\frac{dV}{dt}$ *placed on the grid*

d*E*/dt and *E*

According to the rules by which the grid is constructed the $\frac{d\vec{E}}{dt}$ box is located one box to the left of the $\vec{E}$ box.

$\frac{d\vec{E}}{dt}$	$\vec{E}$	

Diagram 107: The relationship between the $\frac{d\vec{E}}{dt}$ *box and the* $\vec{E}$ *box*

d*E*/dt on the grid

This diagram shows a 4x4 piece of the grid with the $\frac{\overrightarrow{d\mathbf{E}}}{dt}$ box placed correctly.

$\frac{d\Phi_E}{dt}$	Φ_E		
$\frac{dV}{dt}$	V	Φ_B	
$\frac{\overrightarrow{d\mathbf{E}}}{dt}$	$\vec{\mathbf{E}}$	$\oint \vec{\mathbf{B}}\cdot\overrightarrow{d\mathbf{s}}$	
		$\vec{\mathbf{B}}$	

Diagram 108: $\frac{\overrightarrow{d\mathbf{E}}}{dt}$ *placed on the grid*

d*B*/dt and *B*

According to the rules by which the grid is constructed the $\frac{\overrightarrow{d\boldsymbol{B}}}{dt}$ box is located one box to the left of the $\vec{\boldsymbol{B}}$ box.

$\frac{\overrightarrow{d\boldsymbol{B}}}{dt}$	$\vec{\boldsymbol{B}}$	

Diagram 109: The relationship between the $\frac{\overrightarrow{d\boldsymbol{B}}}{dt}$ *box and the* $\vec{\boldsymbol{B}}$ *box*

d*B*/dt on the grid

This diagram shows a 4x4 piece of the grid with the $\frac{\overrightarrow{d\mathbf{B}}}{dt}$ box placed correctly.

$\frac{d\Phi_E}{dt}$	Φ_E		
$\frac{dV}{dt}$	V	Φ_B	
$\frac{\overrightarrow{d\mathbf{E}}}{dt}$	$\vec{\mathbf{E}}$	$\oint \vec{\mathbf{B}} \cdot \overrightarrow{d\mathbf{s}}$	
	$\frac{\overrightarrow{d\mathbf{B}}}{dt}$	$\vec{\mathbf{B}}$	

Diagram 110: $\frac{\overrightarrow{d\mathbf{B}}}{dt}$ *placed on the grid*

Maxwell's velocity prediction

Maxwell predicted:

$$\left|\vec{\boldsymbol{E}}\right| = c\left|\vec{\boldsymbol{B}}\right| \quad .$$

Examining the periodic table of electromagnetic elements shows the $\vec{\boldsymbol{E}}$ box and the $\vec{\boldsymbol{B}}$ box have a velocity relationship.

$\frac{d\Phi_E}{dt}$	Φ_E		
$\frac{dV}{dt}$	V	Φ_B	
$\frac{d\vec{\boldsymbol{E}}}{dt}$	$\lvert\vec{\boldsymbol{E}}\rvert = c\lvert\vec{\boldsymbol{B}}\rvert$	$\oint \vec{\boldsymbol{B}} \cdot \vec{d\boldsymbol{s}}$	
	$\frac{d\vec{\boldsymbol{B}}}{dt}$	$\lvert\vec{\boldsymbol{B}}\rvert = \frac{1}{c}\lvert\vec{\boldsymbol{E}}\rvert$	

Diagram 111: The velocity relationship between the $\vec{\boldsymbol{E}}$ *box and the* $\vec{\boldsymbol{B}}$ *box on the periodic table of electromagnetic elements*

Maxwell's prediction indicates the velocity-dimension line separating the $\vec{\boldsymbol{E}}$ box from the $\vec{\boldsymbol{B}}$ box represents the velocity of propagation of electromagnetic waves, and must have the value c .

Electromagnetic Units

$\frac{d\Phi_E}{dt}$ (Power) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{3}\text{m}^{0}$	Φ_E (Energy) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{2}\text{m}^{1}$	(Action) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{1}\text{m}^{2}$	(Moment of Inertia) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{3}$
$\frac{dV}{dt}$ $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{3}\text{m}^{-1}$	V (Force) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{2}\text{m}^{0}$	Φ_B (Momentum) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{1}\text{m}^{1}$	(Moment) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{2}$
$\frac{\overrightarrow{d\boldsymbol{E}}}{dt}$ $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{3}\text{m}^{-2}$	$\vec{\boldsymbol{E}}$ (Spring Constant) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{2}\text{m}^{-1}$	$\oint \vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}$ (Impedance) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{1}\text{m}^{0}$	(Mass) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{1}$
$\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{3}\text{m}^{-3}$	$\frac{\overrightarrow{d\boldsymbol{B}}}{dt}$ (Pressure) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{2}\text{m}^{-2}$	$\vec{\boldsymbol{B}}$ $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{1}\text{m}^{-1}$	(Linear Density) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{0}$
$\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{3}\text{m}^{-4}$	$\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{2}\text{m}^{-3}$	$\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{1}\text{m}^{-2}$	(Areal Density) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{-1}$
$\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{3}\text{m}^{-5}$	$\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{2}\text{m}^{-4}$	$\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{1}\text{m}^{-3}$	(Volumetric Density) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{-2}$

Diagram 112: The Periodic Table of Electromagnetic Elements labeled with canonical units

The force between two charges

The force (F) between two charges (q_1 , q_2) is defined as the product of a proportionality constant ($\frac{1}{4\pi\epsilon_0}$) times the ratio of the product of the two charges divided by the square of the distance (r) between their centroids:

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \quad .$$

If the charge q_1 is known by some means then the charge of q_2 can, in principle, be determined by measuring the force of attraction it exerts on q_1 . Note that this determination depends critically upon the distance between the centroids of the two charges.

Consider the situation where both the charge of q_1 and the force F are known. It is not possible to determine the charge of q_2 without making some assumption about the distance r between the centroids of the two charges. From this it can be deduced that the force between two charges is only defined in terms of the distance between them.

The distributed nature of charge

Consider the situation of two point charges. As these charges are brought infinitesimally close together the force of attraction, or repulsion, will become infinite. At the same time the potential energy of this two-particle system will become infinite. From this it can be deduced that charge must always be distributed in space.

The definition of the force between two charges can be rewritten to reflect this fact:

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r}\frac{q_2}{r} \quad .$$

This reformulation reflects the fundamentally distributed nature of charge in terms of the product of two linear densities.

Unit analysis of electromagnetic elements

To reflect physicality the distributed nature of charge should be applied rigorously to the unit analysis of electromagnetic elements. A charge unit should not appear without a linear density. In the SI system of units the coulomb (C) should only appear as $\left(\frac{\mathrm{C}}{\mathrm{m}}\right)$.

The volt (V) is commonly written as a joule (J) per coulomb:

$$\mathrm{V} = \frac{\mathrm{J}}{\mathrm{C}} \ .$$

The joule can be written as a newton-meter:

$$\mathrm{J} = \mathrm{N}\cdot\mathrm{m} \ .$$

This allows the volt to be written as a newton-meter per coulomb:

$$\mathrm{V} = \frac{\mathrm{N}\cdot\mathrm{m}}{\mathrm{C}} \ .$$

However, in order to reflect the distributed nature of charge, the volt should be written as a newton per coulomb per meter:

$$\mathrm{V} = \frac{\mathrm{N}}{(\mathrm{C}/\mathrm{m})} \ .$$

This formulation reflects the distributed nature of charge.

The volt is the electromagnetic unit of potential. The newton is the mechanical unit of potential. This formulation of the volt expresses electromagnetic potential as the ratio of mechanical potential over linear charge density.

From the section on mechanical units the newton should be written as:

$$\mathrm{N} = \left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{m}^0 \ .$$

The volt should be written as:

$$\mathrm{V} = \frac{\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{m}^0}{\left(\frac{\mathrm{C}}{\mathrm{m}}\right)} \text{ , or } \quad \mathrm{V} = \left(\frac{\mathrm{kg}/\mathrm{m}}{\mathrm{C}/\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{m}^0 \ .$$

Canonical units for electromagnetic elements

The units for the volt have been written above in terms of three components. There is a density component: $\left(\frac{\mathrm{kg/m}}{\mathrm{C/m}}\right)$, a velocity component: $\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{a}$, and a spatial component: m^{b} .

Expressing the units of electromagnetic elements in this form will be referred to as the canonical form for reasons that will become clear.

Placing the units for voltage in the appropriate box on the periodic table of electromagnetic elements provides a starting point for assigning units to the other elements.

$\frac{d\Phi_E}{dt}$ (Power)	Φ_E (Energy)	(Action)	(Moment of Inertia)
$\frac{dV}{dt}$	V (Force) $\left(\frac{\mathrm{kg/m}}{\mathrm{C/m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{0}$	Φ_B (Momentum)	(Moment)
$\frac{d\vec{\boldsymbol{E}}}{dt}$	$\vec{\boldsymbol{E}}$ (Spring Constant)	$\oint \vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{s}}$ (Impedance)	(Mass)
	$\frac{d\vec{\boldsymbol{B}}}{dt}$ (Pressure)	$\vec{\boldsymbol{B}}$	(Linear Density)

Diagram 113: Voltage labeled with canonical units on the grid

The voltage column

Just as seen when assigning units to the elements within a column on the periodic table of mechanical elements, the only changes will be in the exponent of the spatial component which decreases when proceeding from top to bottom.

The Φ_E box is one box above the voltage box; the exponent of the spatial component will be one greater than the voltage box. The $\vec{E}$ box is one box below the voltage box; the exponent of the spatial component will be one less than the voltage box. The $\frac{d\vec{B}}{dt}$ box is two boxes below the voltage box; the exponent of the spatial component will be two less than the voltage box. The exponent of the velocity component will remain fixed at 2 for all of the elements in this column.

$\frac{d\Phi_E}{dt}$ (Power)	Φ_E (Energy) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^2 \text{m}^1$	(Action)	(Moment of Inertia)
$\frac{dV}{dt}$	V (Force) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^2 \text{m}^0$	Φ_B (Momentum)	(Moment)
$\frac{d\vec{E}}{dt}$	$\vec{E}$ (Spring Constant) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^2 \text{m}^{-1}$	$\oint \vec{B}\cdot\vec{ds}$ (Impedance)	(Mass)
	$\frac{d\vec{B}}{dt}$ (Pressure) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^2 \text{m}^{-2}$	$\vec{B}$	(Linear Density)

Diagram 114: The voltage column labeled with canonical units on the grid

The voltage velocity-diagonal

Just as seen when assigning units to the elements along a velocity-diagonal on the periodic table of mechanical elements the only changes will be in the exponent of the velocity component which decreases when proceeding from upper left to lower right.

The $\frac{d\Phi_E}{dt}$ box is one box up and to the left of the voltage box; the exponent of the velocity component will be one greater than the voltage box. The $\oint \vec{B}\cdot\overrightarrow{ds}$ box is one box down and to the right of the voltage box; the exponent of the velocity component will be one less than the voltage box. The exponent of the spatial component will remain fixed at 0 for all of the elements along this velocity-diagonal.

$\frac{d\Phi_E}{dt}$ (Power) $\left(\frac{\mathrm{kg/m}}{\mathrm{C/m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^3\mathrm{m}^0$	Φ_E (Energy)	(Action)	(Moment of Inertia)
$\frac{dV}{dt}$	V (Force) $\left(\frac{\mathrm{kg/m}}{\mathrm{C/m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2\mathrm{m}^0$	Φ_B (Momentum)	(Moment)
$\frac{d\vec{E}}{dt}$	$\vec{E}$ (Spring Constant)	$\oint \vec{B}\cdot\overrightarrow{ds}$ (Impedance) $\left(\frac{\mathrm{kg/m}}{\mathrm{C/m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^1\mathrm{m}^0$	(Mass)
	$\frac{d\vec{B}}{dt}$ (Pressure)	$\vec{B}$	(Linear Density) $\left(\frac{\mathrm{kg/m}}{\mathrm{C/m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^0\mathrm{m}^0$

Diagram 115: The voltage velocity-diagonal labeled with canonical units on the grid

The voltage row

Just as seen when assigning units to the elements within a row on the periodic table of mechanical elements the exponent of the velocity component decreases by one while the exponent of the spatial component increases by one when proceeding from left to right.

The $\frac{dV}{dt}$ box is one box to the left of the voltage box; the exponent of the velocity component will be one greater than the voltage box while the exponent of the spatial component will be one less than the voltage box. The Φ_B box is one box to the right of the voltage box; the exponent of the velocity component will be one less than the voltage box while the exponent of the spatial component will be one greater than the voltage box.

$\frac{d\Phi_E}{dt}$ (Power)	Φ_E (Energy)	(Action)	(Moment of Inertia)
$\frac{dV}{dt}$ $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^3 \text{m}^{-1}$	V (Force) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^2 \text{m}^0$	Φ_B (Momentum) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^1 \text{m}^1$	(Moment) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^0 \text{m}^2$
$\frac{\overrightarrow{d\boldsymbol{E}}}{dt}$	$\vec{\boldsymbol{E}}$ (Spring Constant)	$\oint \vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}$ (Impedance)	(Mass)
	$\frac{\overrightarrow{d\boldsymbol{B}}}{dt}$ (Pressure)	$\vec{\boldsymbol{B}}$	(Linear Density)

Diagram 116: The voltage row labeled with canonical units on the grid

The remaining electromagnetic elements

Just as seen when assigning units to the remaining elements on the periodic table of mechanical elements the column pattern, the velocity-diagonal pattern and/or the row pattern can be used to assign units to the remaining elements on the periodic table of electromagnetic elements.

$\frac{d\Phi_E}{dt}$ (Power) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^3\text{m}^0$	Φ_E (Energy) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^2\text{m}^1$	(Action) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^1\text{m}^2$	(Moment of Inertia) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^0\text{m}^3$
$\frac{dV}{dt}$ $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^3\text{m}^{-1}$	V (Force) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^2\text{m}^0$	Φ_B (Momentum) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^1\text{m}^1$	(Moment) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^0\text{m}^2$
$\frac{\overrightarrow{d\boldsymbol{E}}}{dt}$ $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^3\text{m}^{-2}$	$\vec{\boldsymbol{E}}$ (Spring Constant) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^2\text{m}^{-1}$	$\oint \vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}$ (Impedance) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^1\text{m}^0$	(Mass) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^0\text{m}^1$
$\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^3\text{m}^{-3}$	$\frac{\overrightarrow{d\boldsymbol{B}}}{dt}$ (Pressure) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^2\text{m}^{-2}$	$\vec{\boldsymbol{B}}$ $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^1\text{m}^{-1}$	(Linear Density) $\left(\frac{\text{kg/m}}{\text{C/m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^0\text{m}^0$

Diagram 117: The remaining electromagnetic elements labeled with canonical units on the grid

Mechanical correspondence

Simple inspection shows that every element on the periodic table of electromagnetic elements corresponds to an element on the periodic table of mechanical elements divided by linear charge density. For example voltage corresponds to force divided by linear charge density, Φ_E corresponds to energy divided by linear charge density, and Φ_B corresponds to momentum divided by linear charge density.

Power $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^3\mathrm{m}^0$	Energy $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2\mathrm{m}^1$	Action $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^1\mathrm{m}^2$	Moment of Inertia $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^0\mathrm{m}^3$
$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^3\mathrm{m}^{-1}$	Force $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2\mathrm{m}^0$	Momentum $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^1\mathrm{m}^1$	Moment $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^0\mathrm{m}^2$
$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^3\mathrm{m}^{-2}$	Spring Constant $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2\mathrm{m}^{-1}$	Impedance $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^1\mathrm{m}^0$	Mass $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^0\mathrm{m}^1$
$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^3\mathrm{m}^{-3}$	Pressure $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2\mathrm{m}^{-2}$	$\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^1\mathrm{m}^{-1}$	Linear Density $\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^0\mathrm{m}^0$

Diagram 118: The periodic table of mechanical elements labeled with canonical units on the grid

The correspondence between electromagnetic and mechanical systems is not a coincidence. It is a logical consequence of the mechanical foundation on which electromagnetics is constructed.

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Thermodynamic elements

(Power)	(Energy)	(Action)	(Moment of Inertia)
	(Force)	(Momentum)	(Moment)
	(Spring Constant)	(Impedance)	(Mass)
	(Pressure)		(Linear Density)

Diagram 119: The periodic table of thermodynamic elements

Introduction

The relationship between electromagnetic elements and mechanical elements poses an interesting question. Are thermodynamic elements related to mechanical elements in the same way that electromagnetic elements are related to mechanical elements?

Assuming that thermodynamic elements are related to mechanical elements in the same way they will fit the following pattern.

Constructing the grid

Consider a two-dimensional grid of boxes where each box is related to it's immediate neighbors by a specific set of rules. The same set of rules applies to each and every box on the grid. Each box on the grid represents a particular thermodynamic element.

Using a suitable set of rules all of the thermodynamic elements can be arranged on the grid in accordance with the fundamental definitions and theorems of thermodynamics in an unambiguous fashion. This arrangement constitutes the periodic table of thermodynamic elements. The periodic quality of the thermodynamic elements will become clear in the course of the presentation.

Rules of the grid

This diagram describes how any element, represented by ★, is related to it's immediate neighbors.

$\int \star \, d(x/t)$	$\int \star \, dx$	$\int \star \, d(xt)$
$\frac{\partial \star}{\partial t}$	Any Element ★	$\int \star \, dt$
$\frac{\partial \star}{\partial (xt)}$	$\frac{\partial \star}{\partial x}$	$\frac{\partial \star}{\partial (x/t)}$

Diagram 120: How an element is related to it's immediate neighbors

This is the complete set of rules by which the grid is constructed.

Thermodynamic units

(Power) $\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{3}\text{m}^{0}$	(Energy) $\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{2}\text{m}^{1}$	(Action) $\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{1}\text{m}^{2}$	(Moment of Inertia) $\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{3}$
$\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{3}\text{m}^{-1}$	(Force) $\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{2}\text{m}^{0}$	(Momentum) $\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{1}\text{m}^{1}$	(Moment) $\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{2}$
$\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{3}\text{m}^{-2}$	(Spring Constant) $\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{2}\text{m}^{-1}$	(Impedance) $\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{1}\text{m}^{0}$	(Mass) $\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{1}$
$\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{3}\text{m}^{-3}$	(Pressure) $\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{2}\text{m}^{-2}$	$\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{1}\text{m}^{-1}$	(Linear Density) $\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{0}$
$\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{0}$	$\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{0}$	$\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{0}$	(Areal Density) $\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{0}$
$\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{0}$	$\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{0}$	$\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{0}$	(Volumetric Density) $\left(\frac{\text{kg}/\text{m}}{\text{K}/\text{m}}\right)\left(\frac{\text{m}}{\text{s}}\right)^{0}\text{m}^{0}$

Diagram 121: The periodic table of thermodynamic elements labeled with canonical units

The force between two temperatures

In mechanics the force (F) between two particles of mass is defined as the product of a proportionality constant (G) times the ratio of the product of the masses (m_1 , m_2) of the two particles divided by the square of the distance (r) between their centroids:

$$F = G\frac{m_1 m_2}{r^2} = G\frac{m_1}{r}\frac{m_2}{r} \quad .$$

In electromagnetics the mechanical force (F) between two particles of charge is defined as the product of a proportionality constant ($\frac{1}{4\pi\epsilon_0}$) times the ratio of the product of the charges (q_1 , q_2) of the two particles divided by the square of the distance (r) between their centroids:

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r}\frac{q_2}{r} \quad .$$

Suppose there is a mechanical force that causes two particles of temperature to be repelled. Suppose this force (F) is defined as the product of a proportionality constant (ζ) times the ratio of the product of the temperatures (T_1 , T_2) of the two particles divided by the square of the distance (r) between their centroids:

$$F = \zeta\frac{T_1 T_2}{r^2} \quad .$$

The distributed nature of temperature

Consider the situation of two point temperatures. As these temperatures are brought infinitesimally close together the force of repulsion will become infinite. At the same time the potential energy of this two-particle system will become infinite. From this it can be deduced that temperature must always be distributed in space.

The definition of the force between two temperatures can be rewritten to reflect this fact:

$$F = \zeta\frac{T_1}{r}\frac{T_2}{r} \quad .$$

This reformulation reflects the fundamentally distributed nature of temperature in terms of the product of two linear densities.

Unit analysis of thermodynamic elements

To reflect physicality the distributed nature of temperature should be applied rigorously to the unit analysis of thermodynamic elements. A temperature unit should not appear without a linear density. In the SI system of units the kelvin (K) should only appear as $\left(\frac{\mathrm{K}}{\mathrm{m}}\right)$.

The universal gas law constant

The constant (R) from the universal gas law:

$$PV = nRT$$

commonly has the units joules per mole kelvin:

$$R = \frac{\mathrm{J}}{\mathrm{mol}\cdot\mathrm{K}} \ .$$

A joule is also known as a newton-meter:

$$\mathrm{J} = \mathrm{N}\cdot\mathrm{m} \ .$$

The units for the universal gas law constant can be written as:

$$R = \frac{\mathrm{N}\cdot\mathrm{m}}{\mathrm{mol}\cdot\mathrm{K}} \ .$$

The units for the universal gas law constant should be written as:

$$R = \frac{\mathrm{N}}{\left(\frac{\mathrm{K}}{\mathrm{m}}\right)\mathrm{mol}} \ .$$

This formulation reflects the distributed nature of temperature.

From the section on mechanical units the newton should be written as:

$$\mathrm{N} = \left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{m}^0 \ .$$

The units for the universal gas law constant should be written as:

$$R = \frac{\left(\frac{\mathrm{kg/m}}{\mathrm{K/m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{m}^0}{\mathrm{mol}} \ .$$

Boltzmann's constant

The units for Boltzmann's constant (κ_B) are commonly written as joules per kelvin:

$$\kappa_B = \frac{\mathrm{J}}{\mathrm{K}} \quad .$$

From the section on mechanical units the joule should be written as:

$$\mathrm{J} = \left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{m}^1 \quad .$$

Boltzmann's constant should be written as:

$$\kappa_B = \frac{\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{m}^1}{\mathrm{K}} \quad .$$

To reflect the distributed nature of temperature Boltzmann's constant should be written as:

$$\kappa_B = \frac{\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{m}^0}{\left(\frac{\mathrm{K}}{\mathrm{m}}\right)} \text{ , or } \quad \kappa_B = \left(\frac{\mathrm{kg/m}}{\mathrm{K/m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \mathrm{m}^0 \quad .$$

Temperature is an intrinsic property of mass. The mass and temperature must be distributed identically over the same space.

Canonical units for thermodynamic elements

The units for the universal gas law constant and Boltzmann's constant have been written above in terms of three components. There is a density component: $\left(\frac{\mathrm{kg/m}}{\mathrm{K/m}}\right)$, a velocity component: $\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{a}$, and a spatial component: m^{b} .

Expressing the units for thermodynamic elements in this form will be referred to as the canonical form for reasons that will become clear.

Mechanical correspondence

Simple inspection shows that every element on the periodic table of thermodynamic elements corresponds to an element on the periodic table of mechanical elements divided by linear temperature density.

Final thoughts

In the course of developing the periodic tables of mechanical and electromagnetic elements the thermodynamic proposition presented itself.

The fundamental proposition behind the periodic table of thermodynamic elements presented above clearly needs to be backed up with theoretical work and experimental evidence.

Other questions also arise. Are there more fundamental quantities and associated periodic tables in physics? Do quantum theories fit on the tables presented?

Analyzing physical problems

The periodic tables of physics are a powerful tool for analyzing physical problems. The analysis process consists of labeling the boxes (which represent physical elements) and the dimension-lines (which represent the physical relationships between elements) on the grid in accordance with the fundamental definitions and theorems that apply to the problem at hand. The analysis is complete when consistency is achieved no matter what path (sequence of navigation instructions) is used to get from one box (element) on the grid to another.

This is best illustrated by looking at some examples.

The classic inductor

It is very instructive to analyze the classic inductor using the periodic table of electromagnetic elements.

The geometry of the cylindrical/helical inductor

Start by examining the geometry of the cylindrical/helical inductor.

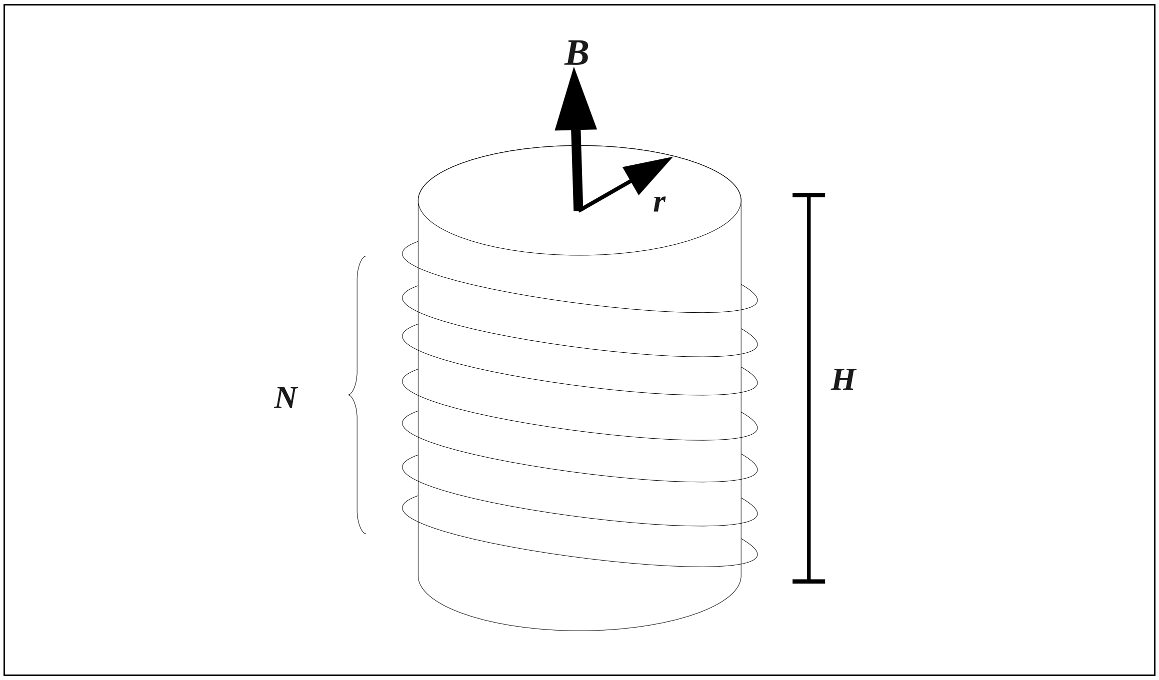


Diagram 122: The geometry of the cylindrical/helical inductor

The cylinder has a radius r , and a height H .

The cross section area is:

$$A = \pi r^2 .$$

The surface area is:

$$S = 2\pi r H .$$

In the classic derivation the wire is assumed to have an infinitesimal pitch. This allows the use of the stack of loops approximation. There are N turns of wire which yields a wire length of:

$$w = 2\pi r N .$$

The *B* field

In the classic derivation the $\vec{B}$ field is assumed to be uniform within the solenoid, having a value of:

$$\vec{B} = \mu_0 \frac{N}{H} \vec{i}_0 \quad .$$

Place this in the $\vec{B}$ box on the periodic table of electromagnetic elements.

$\frac{d\Phi_E}{dt}$ (Power)	Φ_E (Energy)	(Action)	(Moment of Inertia)
$\frac{dV}{dt}$	V (Force)	Φ_B (Momentum)	(Moment)
$\frac{\overrightarrow{dE}}{dt}$	$\vec{E}$ (Spring Constant)	$\oint \vec{B}\cdot\overrightarrow{ds}$ (Impedance)	(Mass)
	$\frac{\overrightarrow{dB}}{dt}$ (Pressure)	$\vec{B}$ $\vec{B}=\mu_0 \frac{N}{H} \vec{i}_0$	(Linear Density)

Diagram 123: Magnetic field, $\vec{B}$ *, placed on the grid*

Total magnetic flux

The magnetic flux inside a single loop is the product of the $\vec{\boldsymbol{B}}$ field times the area of the loop:

$$\left|\vec{\boldsymbol{B}}\right| A \;=\; \left|\mu_0 \frac{N}{H} \vec{\boldsymbol{i}}_{\mathbf{0}}\right| \left(\pi r^2\right) \quad .$$

The total magnetic flux within the solenoid is the sum of the magnetic flux in each of the N loops:

$$\Phi_B \;=\; N\left(\left|\vec{\boldsymbol{B}}\right| A\right) \;=\; \mu_0 \frac{N}{H}\left(N \pi r^2\right)\left|\vec{\boldsymbol{i}}_{\mathbf{0}}\right| \quad .$$

Multiply by the factor $\frac{4\pi}{4\pi}$:

$$\Phi_B \;=\; \mu_0 \frac{4\pi}{4\pi} \frac{N}{H}\left(N \pi r^2\right)\left|\vec{\boldsymbol{i}}_{\mathbf{0}}\right| \quad ,$$

then regroup:

$$\Phi_B \;=\; \mu_0 \frac{\left(2 \pi r N\right)^2}{4 \pi H}\left|\vec{\boldsymbol{i}}_{\mathbf{0}}\right| \quad .$$

Place this in the Φ_B box on the periodic table of electromagnetic elements.

$\frac{d\Phi_E}{dt}$ (Power)	Φ_E (Energy)	(Action)	(Moment of Inertia)
$\frac{dV}{dt}$	V (Force)	Φ_B (Momentum) $\Phi_B=\mu_0\frac{(2\pi r N)^2}{4\pi H}\lvert\vec{\boldsymbol{i}}_0\rvert$	(Moment)
$\frac{d\vec{\boldsymbol{E}}}{dt}$	$\vec{\boldsymbol{E}}$ (Spring Constant)	$\oint \vec{\boldsymbol{B}}\cdot\vec{d\boldsymbol{s}}$ (Impedance)	(Mass)
	$\frac{d\vec{\boldsymbol{B}}}{dt}$ (Pressure)	$\vec{\boldsymbol{B}}$ $\vec{\boldsymbol{B}}=\mu_0\frac{N}{H}\vec{\boldsymbol{i}}_0$	(Linear Density)

Diagram 124: Total magnetic flux, Φ_B , placed on the grid

This shows an important and physically meaningful geometric relationship. The total magnetic flux within the solenoid is a function of the ratio of the square of the wire length divided by the height of the solenoid. Recall that the wire length is $2\pi r N$.

Voltage

From Maxwell's third equation voltage is the time derivative of total magnetic flux:

$$V = \frac{d\Phi_B}{dt} = \mu_0 \frac{(2\pi r N)^2}{4\pi H}\left|\frac{d\vec{\boldsymbol{i}}_{\mathbf{0}}}{dt}\right| .$$

The only quantity changing with time is current. The geometry remains fixed.

Place this in the V box on the periodic table of electromagnetic elements.

$\frac{d\Phi_E}{dt}$ (Power)	Φ_E (Energy)	(Action)	(Moment of Inertia)
$\frac{dV}{dt}$	V (Force) $V=\mu_0\frac{(2\pi r N)^2}{4\pi H}\left\lvert\frac{d\vec{\boldsymbol{i}}_{\mathbf{0}}}{dt}\right\rvert$	Φ_B (Momentum) $\Phi_B=\mu_0\frac{(2\pi r N)^2}{4\pi H}\lvert\vec{\boldsymbol{i}}_{\mathbf{0}}\rvert$	(Moment)
$\frac{d\vec{\boldsymbol{E}}}{dt}$	$\vec{\boldsymbol{E}}$ (Spring Constant)	$\oint \vec{\boldsymbol{B}}\cdot\vec{d\boldsymbol{s}}$ (Impedance)	(Mass)
	$\frac{d\vec{\boldsymbol{B}}}{dt}$ (Pressure)	$\vec{\boldsymbol{B}}$ $\vec{\boldsymbol{B}}=\mu_0\frac{N}{H}\vec{\boldsymbol{i}}_{\mathbf{0}}$	(Linear Density)

Diagram 125: Voltage, V *, placed on the grid*

The current is changing direction as it flows along the wire. There will be a non-zero time derivative of current flow even when the magnitude of the current does not vary with time.

The E field

The potential (voltage) along a path can be calculated as:

$$V = \int \vec{\boldsymbol{E}}\cdot\overrightarrow{d\boldsymbol{s}} \quad .$$

In the classic derivation $\vec{\boldsymbol{E}}$ is in the direction of the wire at all points along the wire. The voltage from one end of the wire to the other is given by the equation:

$$V = \left|\vec{\boldsymbol{E}}\right|(2\pi r N) \quad .$$

Simple algebraic manipulation and substitution yields:

$$\vec{\boldsymbol{E}} = \frac{V}{2\pi r N} = \mu_0 \frac{(2\pi r N)}{4\pi H}\frac{d\vec{\boldsymbol{i}}_{\boldsymbol{0}}}{dt} \quad .$$

Place this in the $\vec{\boldsymbol{E}}$ box on the periodic table of electromagnetic elements.

$\frac{d\Phi_E}{dt}$ (Power)	Φ_E (Energy)	(Action)	(Moment of Inertia)
$\frac{dV}{dt}$	V (Force) $V=\mu_0\frac{(2\pi r N)^2}{4\pi H}\left\lvert\frac{d\vec{\boldsymbol{i}}_{\boldsymbol{0}}}{dt}\right\rvert$	Φ_B (Momentum) $\Phi_B=\mu_0\frac{(2\pi r N)^2}{4\pi H}\lvert\vec{\boldsymbol{i}}_{\boldsymbol{0}}\rvert$	(Moment)
$\frac{d\vec{\boldsymbol{E}}}{dt}$	$\vec{\boldsymbol{E}}$ (Spring Constant) $\vec{\boldsymbol{E}}=\mu_0\frac{(2\pi r N)}{4\pi H}\frac{d\vec{\boldsymbol{i}}_{\boldsymbol{0}}}{dt}$	$\oint \vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}$ (Impedance)	(Mass)
	$\frac{d\vec{\boldsymbol{B}}}{dt}$ (Pressure)	$\vec{\boldsymbol{B}}$ $\vec{\boldsymbol{B}}=\mu_0\frac{N}{H}\vec{\boldsymbol{i}}_{\boldsymbol{0}}$	(Linear Density)

Diagram 126: Electric field, $\vec{\boldsymbol{E}}$ *, placed on the grid*

Total electric flux

In the classic derivation the $\vec{\boldsymbol{E}}$ field is distributed over the surface area of the solenoid:

$$A = 2\pi r H \quad .$$

The total electric flux is the product of the $\vec{\boldsymbol{E}}$ field times the surface area:

$$\Phi_E = |\vec{\boldsymbol{E}}| A \quad .$$

Substitution and algebraic manipulation yields:

$$\Phi_E = \left|\mu_0 \frac{(2\pi r N)}{4\pi H}\frac{d\vec{\boldsymbol{i}}_{\mathbf{0}}}{dt}\right|(2\pi r H)\left(\frac{N}{N}\right) = \mu_0 \frac{(2\pi r N)^2}{4\pi N}\left|\frac{d\vec{\boldsymbol{i}}_{\mathbf{0}}}{dt}\right| \quad .$$

Place this in the Φ_E box on the periodic table of electromagnetic elements.

$\frac{d\Phi_E}{dt}$ (Power)	Φ_E (Energy) $\Phi_E = \mu_0 \frac{(2\pi r N)^2}{4\pi N}\left\|\frac{d\vec{\boldsymbol{i}}_{\mathbf{0}}}{dt}\right\|$	(Action)	(Moment of Inertia)
$\frac{dV}{dt}$	V (Force) $V = \mu_0 \frac{(2\pi r N)^2}{4\pi H}\left\|\frac{d\vec{\boldsymbol{i}}_{\mathbf{0}}}{dt}\right\|$	Φ_B (Momentum) $\Phi_B = \mu_0 \frac{(2\pi r N)^2}{4\pi H}\left\|\vec{\boldsymbol{i}}_{\mathbf{0}}\right\|$	(Moment)
$\frac{d\vec{\boldsymbol{E}}}{dt}$	$\vec{\boldsymbol{E}}$ (Spring Constant) $\vec{\boldsymbol{E}} = \mu_0 \frac{(2\pi r N)}{4\pi H}\frac{d\vec{\boldsymbol{i}}_{\mathbf{0}}}{dt}$	$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{s}}$ (Impedance)	(Mass)
	$\frac{d\vec{\boldsymbol{B}}}{dt}$ (Pressure)	$\vec{\boldsymbol{B}}$ $\vec{\boldsymbol{B}} = \mu_0 \frac{N}{H}\vec{\boldsymbol{i}}_{\mathbf{0}}$	(Linear Density)

Diagram 127: Total electric flux, Φ_E *, placed on the grid*

The spatial-dimension lines between total magnetic flux and magnetic field

The ratio of Φ_B over $\vec{\boldsymbol{B}}$ yields the areal relationship between the Φ_B box and the $\vec{\boldsymbol{B}}$ box:

$$\frac{\Phi_B}{\left|\vec{\boldsymbol{B}}\right|} = \frac{\mu_0 \frac{(2\pi r N)^2}{4\pi H} \left|\vec{\boldsymbol{i}}_{\mathbf{0}}\right|}{\mu_0 \frac{N}{H} \left|\vec{\boldsymbol{i}}_{\mathbf{0}}\right|} = N \pi r^2 \quad .$$

The areal relationship between the Φ_B box and the $\vec{\boldsymbol{B}}$ box represents the total area of magnetic flux. This is simply the total area enclosed by the N loops of the inductor.

The ratio of V over $\vec{\boldsymbol{E}}$ yields the spatial relationship between the V box and the $\vec{\boldsymbol{E}}$ box:

$$\frac{V}{\left|\vec{\boldsymbol{E}}\right|} = \frac{\mu_0 \frac{(2\pi r N)^2}{4\pi H} \left|\frac{d\vec{\boldsymbol{i}}_{\mathbf{0}}}{dt}\right|}{\mu_0 \frac{(2\pi r N)}{4\pi H} \left|\frac{d\vec{\boldsymbol{i}}_{\mathbf{0}}}{dt}\right|} = 2\pi r N \quad .$$

The spatial relationship between the V box and the $\vec{\boldsymbol{E}}$ box represents the length of the path along which the $\vec{\boldsymbol{E}}$ field stretches. The current also flows along the length of this path.

The product of the two spatial-dimension lines between the Φ_B box and the $\vec{\boldsymbol{B}}$ box must yield the areal relationship between these two boxes. The ratio of the areal relationship between the Φ_B box and the $\vec{\boldsymbol{B}}$ box over the spatial relationship between the V box and the $\vec{\boldsymbol{E}}$ box yields the spatial relationship represented by the spatial-dimension line that separates the $\vec{\boldsymbol{E}}$ box from the $\vec{\boldsymbol{B}}$ box:

$$\frac{\left(\frac{\Phi_B}{\left|\vec{\boldsymbol{B}}\right|}\right)}{\left(\frac{V}{\left|\vec{\boldsymbol{E}}\right|}\right)} = \frac{(N\pi r^2)}{(2\pi r N)} = \frac{r}{2} \quad .$$

Which is the ratio of the area over the circumference for any circle.

The spatial dimension lines between total electric flux and electric field

The ratio of Φ_E over $\vec{\boldsymbol{E}}$ yields the areal relationship between the Φ_E box and $\vec{\boldsymbol{E}}$ box:

$$\frac{\Phi_E}{\left|\vec{\boldsymbol{E}}\right|} = \frac{\mu_0 \frac{\left(2\pi r N\right)^2}{4\pi N}\left|\frac{d\vec{\boldsymbol{i}}_{\boldsymbol{0}}}{dt}\right|}{\mu_0 \frac{\left(2\pi r N\right)}{4\pi H}\left|\frac{d\vec{\boldsymbol{i}}_{\boldsymbol{0}}}{dt}\right|} = 2\pi r H \ .$$

The areal relationship between the Φ_E box and the $\vec{\boldsymbol{E}}$ box represents the area over which the $\vec{\boldsymbol{E}}$ field is distributed.

The ratio of V over $\vec{\boldsymbol{E}}$ yields the spatial relationship between the V box and the $\vec{\boldsymbol{E}}$ box:

$$\frac{V}{\left|\vec{\boldsymbol{E}}\right|} = \frac{\mu_0 \frac{\left(2\pi r N\right)^2}{4\pi H}\left|\frac{d\vec{\boldsymbol{i}}_{\boldsymbol{0}}}{dt}\right|}{\mu_0 \frac{\left(2\pi r N\right)}{4\pi H}\left|\frac{d\vec{\boldsymbol{i}}_{\boldsymbol{0}}}{dt}\right|} = 2\pi r N \ .$$

The spatial relationship between the V box and the $\vec{\boldsymbol{E}}$ box represents the length of the path over which the $\vec{\boldsymbol{E}}$ field stretches.

The product of the two spatial-dimension lines between the Φ_E box and the $\vec{\boldsymbol{E}}$ box must yield the areal relationship between these two boxes. The ratio of the areal relationship between the Φ_E box and the $\vec{\boldsymbol{E}}$ box over the spatial relationship between the V box and the $\vec{\boldsymbol{E}}$ box yields the spatial relationship represented by the spatial-dimension line that separates the Φ_E box from the V box:

$$\frac{\Phi_E}{V} = \frac{\mu_0 \frac{\left(2\pi r N\right)^2}{4\pi N}\left|\frac{d\vec{\boldsymbol{i}}_{\boldsymbol{0}}}{dt}\right|}{\mu_0 \frac{\left(2\pi r N\right)^2}{4\pi H}\left|\frac{d\vec{\boldsymbol{i}}_{\boldsymbol{0}}}{dt}\right|} = \frac{2\pi r H}{2\pi r N} = \frac{H}{N} \ .$$

Which is the ratio of the surface area over the wire length of the solenoid. Physically this represents the area over which the electric field is distributed divided by the distance over which the electric field stretches.

Clarifying the magnetic field

The $\vec{B}$ field within the solenoid is classically written as:

$$\vec{B} = \mu_0 \frac{N}{H} \vec{i}_0 \ .$$

Multiplying the numerator and denominator by the factor $2\pi r$ can be used to clarify the initial formula for the $\vec{B}$ field within the solenoid:

$$\vec{B} = \mu_0 \frac{N}{H} \vec{i}_0 = \mu_0 \frac{2\pi r N}{2\pi r H} \vec{i}_0 \ .$$

This now reflects the physical geometry of the situation.

$\frac{d\Phi_E}{dt}$ (Power)	Φ_E (Energy) $\Phi_E = \mu_0 \frac{(2\pi r N)^2}{4\pi N} \left\lvert \frac{d\vec{i}_0}{dt} \right\rvert$	(Action)	(Moment of Inertia)
$\frac{dV}{dt}$	V (Force) $V = \mu_0 \frac{(2\pi r N)^2}{4\pi H} \left\lvert \frac{d\vec{i}_0}{dt} \right\rvert$	Φ_B (Momentum) $\Phi_B = \mu_0 \frac{(2\pi r N)^2}{4\pi H} \lvert \vec{i}_0 \rvert$	(Moment)
$\frac{d\vec{E}}{dt}$	$\vec{E}$ (Spring Constant) $\vec{E} = \mu_0 \frac{(2\pi r N)}{4\pi H} \frac{d\vec{i}_0}{dt}$	$\oint \vec{B} \cdot \vec{ds}$ (Impedance)	(Mass)
	$\frac{d\vec{B}}{dt}$ (Pressure)	$\vec{B}$ $\vec{B} = \mu_0 \frac{2\pi r N}{2\pi r H} \vec{i}_0$	(Linear Density)

Diagram 128: Magnetic field, $\vec{B}$ *, clarified on the grid*

Filling in the table

Examination of the periodic table of electromagnetic elements shows that each element labeled so far is the product of the permeability constant (μ_0) times a geometric constant times a particular time derivative of charge. Remember that current is defined as the time derivative of charge.

All of the elements in a given column contain the same particular time derivative of charge. The time derivatives decrease when proceeding from left to right across a given row. The geometric constants are the same for each element in a given row because the only quantities that change with time are the charge, and it's time derivatives.

$\frac{d\Phi_E}{dt}$ (Power) $\mu_0\frac{(2\pi r N)^2}{4\pi N}\left\lvert\frac{d^2\vec{\boldsymbol{i}}_{\mathbf{0}}}{dt^2}\right\rvert$	Φ_E (Energy) $\mu_0\frac{(2\pi r N)^2}{4\pi N}\left\lvert\frac{d\vec{\boldsymbol{i}}_{\mathbf{0}}}{dt}\right\rvert$	(Action) $\mu_0\frac{(2\pi r N)^2}{4\pi N}\lvert\vec{\boldsymbol{i}}_{\mathbf{0}}\rvert$	(Moment of Inertia) $\mu_0\frac{(2\pi r N)^2}{4\pi N}\lvert\vec{\boldsymbol{q}}_{\mathbf{0}}\rvert$
$\frac{dV}{dt}$ $\mu_0\frac{(2\pi r N)^2}{4\pi H}\left\lvert\frac{d^2\vec{\boldsymbol{i}}_{\mathbf{0}}}{dt^2}\right\rvert$	V (Force) $\mu_0\frac{(2\pi r N)^2}{4\pi H}\left\lvert\frac{d\vec{\boldsymbol{i}}_{\mathbf{0}}}{dt}\right\rvert$	Φ_B (Momentum) $\mu_0\frac{(2\pi r N)^2}{4\pi H}\lvert\vec{\boldsymbol{i}}_{\mathbf{0}}\rvert$	(Moment) $\mu_0\frac{(2\pi r N)^2}{4\pi H}\lvert\vec{\boldsymbol{q}}_{\mathbf{0}}\rvert$
$\frac{\overrightarrow{d\boldsymbol{E}}}{dt}$ $\mu_0\frac{(2\pi r N)}{4\pi H}\frac{d^2\vec{\boldsymbol{i}}_{\mathbf{0}}}{dt^2}$	$\vec{\boldsymbol{E}}$ (Spring Constant) $\mu_0\frac{(2\pi r N)}{4\pi H}\frac{d\vec{\boldsymbol{i}}_{\mathbf{0}}}{dt}$	$\oint\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}$ (Impedance) $\mu_0\frac{(2\pi r N)}{4\pi H}\vec{\boldsymbol{i}}_{\mathbf{0}}$	(Mass) $\mu_0\frac{(2\pi r N)}{4\pi H}\vec{\boldsymbol{q}}_{\mathbf{0}}$
$\frac{\overrightarrow{d^2\boldsymbol{B}}}{dt^2}$ $\mu_0\frac{2\pi r N}{2\pi r H}\frac{d^2\vec{\boldsymbol{i}}_{\mathbf{0}}}{dt^2}$	$\frac{\overrightarrow{d\boldsymbol{B}}}{dt}$ (Pressure) $\mu_0\frac{2\pi r N}{2\pi r H}\frac{d\vec{\boldsymbol{i}}_{\mathbf{0}}}{dt}$	$\vec{\boldsymbol{B}}$ $\mu_0\frac{2\pi r N}{2\pi r H}\vec{\boldsymbol{i}}_{\mathbf{0}}$	(Linear Density) $\mu_0\frac{2\pi r N}{2\pi r H}\vec{\boldsymbol{q}}_{\mathbf{0}}$

Diagram 129: Completed analysis of the classic inductor

A template for electromagnetic analysis

A careful analysis of the geometric relationships embodied in the construction of the periodic table of electromagnetic elements leads to a useful template for the analysis of electromagnetic problems.

	$\vec{\boldsymbol{E}}(vol)$		
$\frac{d\Phi_E}{dt}$	Φ_E	$\vec{\boldsymbol{B}}(vol)$	
$\frac{dV}{dt}$	V	Φ_B	
$\frac{d\vec{\boldsymbol{E}}}{dt}$	$\vec{\boldsymbol{E}}$	$\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}$	
	$\frac{\Phi_E}{(vol)}$	$\vec{\boldsymbol{B}}$	
	$\frac{V}{(vol)}$	$\frac{\Phi_B}{(vol)}$	
	$\frac{\vec{\boldsymbol{E}}}{(vol)}$	$\frac{\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}}{(vol)}$	
		$\frac{\vec{\boldsymbol{B}}}{(vol)}$	

ΦE

Start by examining the first Maxwell equation:

$$\oint \vec{\boldsymbol{E}} \cdot \overrightarrow{d\boldsymbol{A}} = \Phi_E = \frac{Q}{\epsilon_0} .$$

Place the right-hand expression in the Φ_E box on the periodic table of electromagnetic elements.

	$\vec{\boldsymbol{E}}(vol)$		
$\frac{d\Phi_E}{dt}$	Φ_E $\frac{Q}{\epsilon_0}$	$\vec{\boldsymbol{B}}(vol)$	
$\frac{dV}{dt}$	V	Φ_B	
$\frac{d\vec{\boldsymbol{E}}}{dt}$	$\vec{\boldsymbol{E}}$	$\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}$	
	$\frac{\Phi_E}{(vol)}$	$\vec{\boldsymbol{B}}$	
	$\frac{V}{(vol)}$	$\frac{\Phi_B}{(vol)}$	
	$\frac{\vec{\boldsymbol{E}}}{(vol)}$	$\frac{\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}}{(vol)}$	
		$\frac{\vec{\boldsymbol{B}}}{(vol)}$	

Diagram 130: The value $\Phi_E = \frac{Q}{\epsilon_0}$ *placed on the grid*

d/dt-ΦE

Evaluating the time derivative of Φ_E :

$$\frac{d\,\Phi_E}{dt} = \frac{d}{dt}\left(\frac{Q}{\epsilon_0}\right)$$

yields:

$$\frac{d\,\Phi_E}{dt} = \frac{1}{\epsilon_0}\left(\frac{dQ}{dt}\right) .$$

Recognizing that $\frac{dQ}{dt}$ is simply $|\vec{i}|$ leads to the expression:

$$\frac{d\,\Phi_E}{dt} = \frac{|\vec{i}|}{\epsilon_0} .$$

Place the right-hand expression in the $\frac{d\,\Phi_E}{dt}$ box on the periodic table of electromagnetic elements.

This can be interpreted according to the rules by which the table is constructed:

$\frac{d\,\Phi_E}{dt}$ $\frac{1}{\epsilon_0}\frac{dQ}{dt}=\frac{\lvert\vec{i}\rvert}{\epsilon_0}$	Φ_E $\frac{Q}{\epsilon_0}$	

Diagram 131: The relationship between $\frac{d\,\Phi_E}{dt}$ *and* Φ_E *on the grid*

	$\vec{E}(vol)$		
$\frac{d\Phi_E}{dt}$ $\frac{\lvert\vec{i}\rvert}{\epsilon_0}$	Φ_E $\frac{Q}{\epsilon_0}$	$\vec{B}(vol)$	
$\frac{dV}{dt}$	V	Φ_B	
$\frac{d\vec{E}}{dt}$	$\vec{E}$	$\vec{B}\cdot\vec{ds}$	
	$\frac{\Phi_E}{(vol)}$	$\vec{B}$	
	$\frac{V}{(vol)}$	$\frac{\Phi_B}{(vol)}$	
	$\frac{\vec{E}}{(vol)}$	$\frac{\vec{B}\cdot\vec{ds}}{(vol)}$	
		$\frac{\vec{B}}{(vol)}$	

Diagram 132: The value $\frac{d\Phi_E}{dt} = \frac{|\vec{i}|}{\epsilon_0}$ *placed on the grid*

B.ds

Examine the fourth Maxwell equation:

$$\oint \vec{B}\cdot\vec{ds} = \frac{1}{c^2}\frac{d\Phi_E}{dt} = \frac{1}{c^2}\frac{|\vec{i}|}{\epsilon_0} \ .$$

Recognize that $\frac{1}{\epsilon_0 c^2}$ is just μ_0 .

This leads to the expression:

$$\oint \vec{B}\cdot\vec{ds} = \mu_0|\vec{i}| \ .$$

Place the right-hand expression in the $\vec{B}\cdot\vec{ds}$ box on the periodic table of electromagnetic elements.

This can be interpreted according to the rules by which the table is constructed:

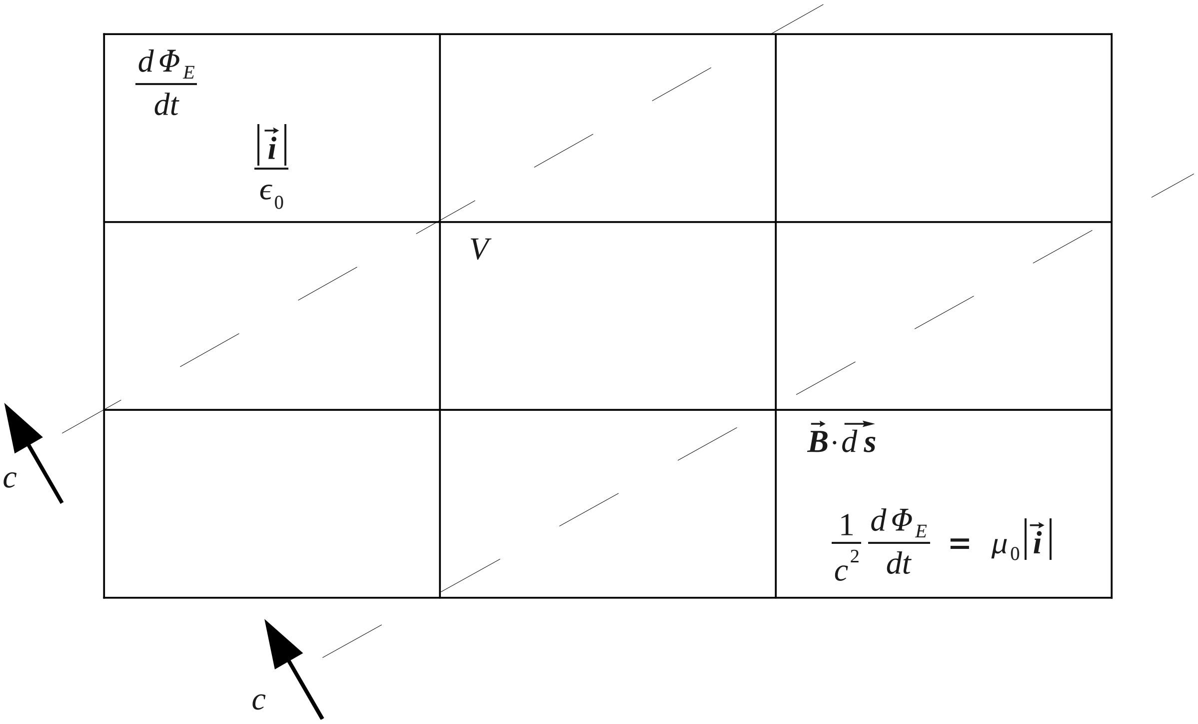


Diagram 133: The relationship between $\frac{d\Phi_E}{dt}$ *and* $\vec{B}\cdot\vec{ds}$ *on the grid*

	$\vec{\boldsymbol{E}}(vol)$		
$\frac{d\Phi_E}{dt}$ $\frac{\lvert\vec{\boldsymbol{i}}\rvert}{\epsilon_0}$	Φ_E $\frac{Q}{\epsilon_0}$	$\vec{\boldsymbol{B}}(vol)$	
$\frac{dV}{dt}$	V	Φ_B	
$\frac{d\vec{\boldsymbol{E}}}{dt}$	$\vec{\boldsymbol{E}}$	$\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}$ $\mu_0\lvert\vec{\boldsymbol{i}}\rvert$	
	$\frac{\Phi_E}{(vol)}$	$\vec{\boldsymbol{B}}$	
	$\frac{V}{(vol)}$	$\frac{\Phi_B}{(vol)}$	
	$\frac{\vec{\boldsymbol{E}}}{(vol)}$	$\frac{\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}}{(vol)}$	
		$\frac{\vec{\boldsymbol{B}}}{(vol)}$	

Diagram 134: The value $\oint \vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}} = \mu_0\lvert\vec{\boldsymbol{i}}\rvert$ *placed on the grid*

E

The $\vec{E}$ box is located to the left of the $\vec{B}\cdot\overrightarrow{ds}$ box. According to the rules by which the table is constructed, the $\vec{E}$ box differs from the $\vec{B}\cdot\overrightarrow{ds}$ box by a time derivative.

The $\vec{B}\cdot\overrightarrow{ds}$ box has the value:

$$\vec{B}\cdot\overrightarrow{ds} = \mu_0|\vec{i}| \quad .$$

So, the $\vec{E}$ box has the value:

$$\vec{E} = \frac{\partial}{\partial t}\left(\mu_0|\vec{i}|\right) = \mu_0\frac{d\vec{i}}{dt} \quad .$$

Place the right-hand expression in the $\vec{E}$ box on the periodic table of electromagnetic elements.

$\vec{E}$ $\mu_0\frac{d\vec{i}}{dt}$	$\vec{B}\cdot\overrightarrow{ds}$ $\mu_0\lvert\vec{i}\rvert$	

Diagram 135: The relationship between $\vec{E}$ *and* $\vec{B}\cdot\overrightarrow{ds}$ *on the grid*

	$\vec{\boldsymbol{E}}(vol)$		
$\frac{d\Phi_E}{dt}$ $\frac{\lvert\vec{\boldsymbol{i}}\rvert}{\epsilon_0}$	Φ_E $\frac{Q}{\epsilon_0}$	$\vec{\boldsymbol{B}}(vol)$	
$\frac{dV}{dt}$	V	Φ_B	
$\frac{d\vec{\boldsymbol{E}}}{dt}$	$\vec{\boldsymbol{E}}$ $\mu_0\frac{d\vec{\boldsymbol{i}}}{dt}$	$\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}$ $\mu_0\lvert\vec{\boldsymbol{i}}\rvert$	
	$\frac{\Phi_E}{(vol)}$	$\vec{\boldsymbol{B}}$	
	$\frac{V}{(vol)}$	$\frac{\Phi_B}{(vol)}$	
	$\frac{\vec{\boldsymbol{E}}}{(vol)}$	$\frac{\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}}{(vol)}$	
		$\frac{\vec{\boldsymbol{B}}}{(vol)}$	

Diagram 136: The value $\vec{\boldsymbol{E}} = \mu_0\frac{d\vec{\boldsymbol{i}}}{dt}$ *placed on the grid*

dE/dt

The $\frac{d\vec{E}}{dt}$ box is located to the left of the $\vec{E}$ box. According to the rules by which the table is constructed, the $\frac{d\vec{E}}{dt}$ box differs from the $\vec{E}$ box by a time derivative.

The $\vec{E}$ box has the value:

$$\vec{E} = \mu_0 \frac{d\vec{i}}{dt} \quad .$$

So, the $\frac{d\vec{E}}{dt}$ box has the value:

$$\frac{d\vec{E}}{dt} = \mu_0 \frac{d^2\vec{i}}{dt^2} \quad .$$

Place the right-hand expression in the $\frac{d\vec{E}}{dt}$ box on the periodic table of electromagnetic elements.

$\frac{d\vec{E}}{dt}$ $\mu_0 \frac{d^2\vec{i}}{dt^2}$	$\vec{E}$ $\mu_0 \frac{d\vec{i}}{dt}$	

Diagram 137: The relationship between $\frac{d\vec{E}}{dt}$ *and* $\vec{E}$ *on the grid*

	$\vec{E}(vol)$		
$\frac{d\Phi_E}{dt}$ $\frac{\lvert\vec{i}\rvert}{\epsilon_0}$	Φ_E $\frac{Q}{\epsilon_0}$	$\vec{B}(vol)$	
$\frac{dV}{dt}$	V	Φ_B	
$\frac{d\vec{E}}{dt}$ $\mu_0\frac{d^2\vec{i}}{dt^2}$	$\vec{E}$ $\mu_0\frac{d\vec{i}}{dt}$	$\vec{B}\cdot\vec{ds}$ $\mu_0\lvert\vec{i}\rvert$	
	$\frac{\Phi_E}{(vol)}$	$\vec{B}$	
	$\frac{V}{(vol)}$	$\frac{\Phi_B}{(vol)}$	
	$\frac{\vec{E}}{(vol)}$	$\frac{\vec{B}\cdot\vec{ds}}{(vol)}$	
		$\frac{\vec{B}}{(vol)}$	

Diagram 138: The value $\frac{d\vec{E}}{dt} = \mu_0\frac{d^2\vec{i}}{dt^2}$ *placed on the grid*

The box to the right of B.ds

According to the rules by which the table is constructed, the $\vec{B}\cdot\vec{ds}$ box is the time derivative of the box to the right.

The $\vec{B}\cdot\vec{ds}$ box has the value:

$$\vec{B}\cdot\vec{ds} = \mu_0|\vec{i}| \quad .$$

Recall that $|\vec{i}|$ is just the time derivative of Q :

$$|\vec{i}| = \frac{dQ}{dt} \quad .$$

So, the box to the right of $\vec{B}\cdot\vec{ds}$ has the value:

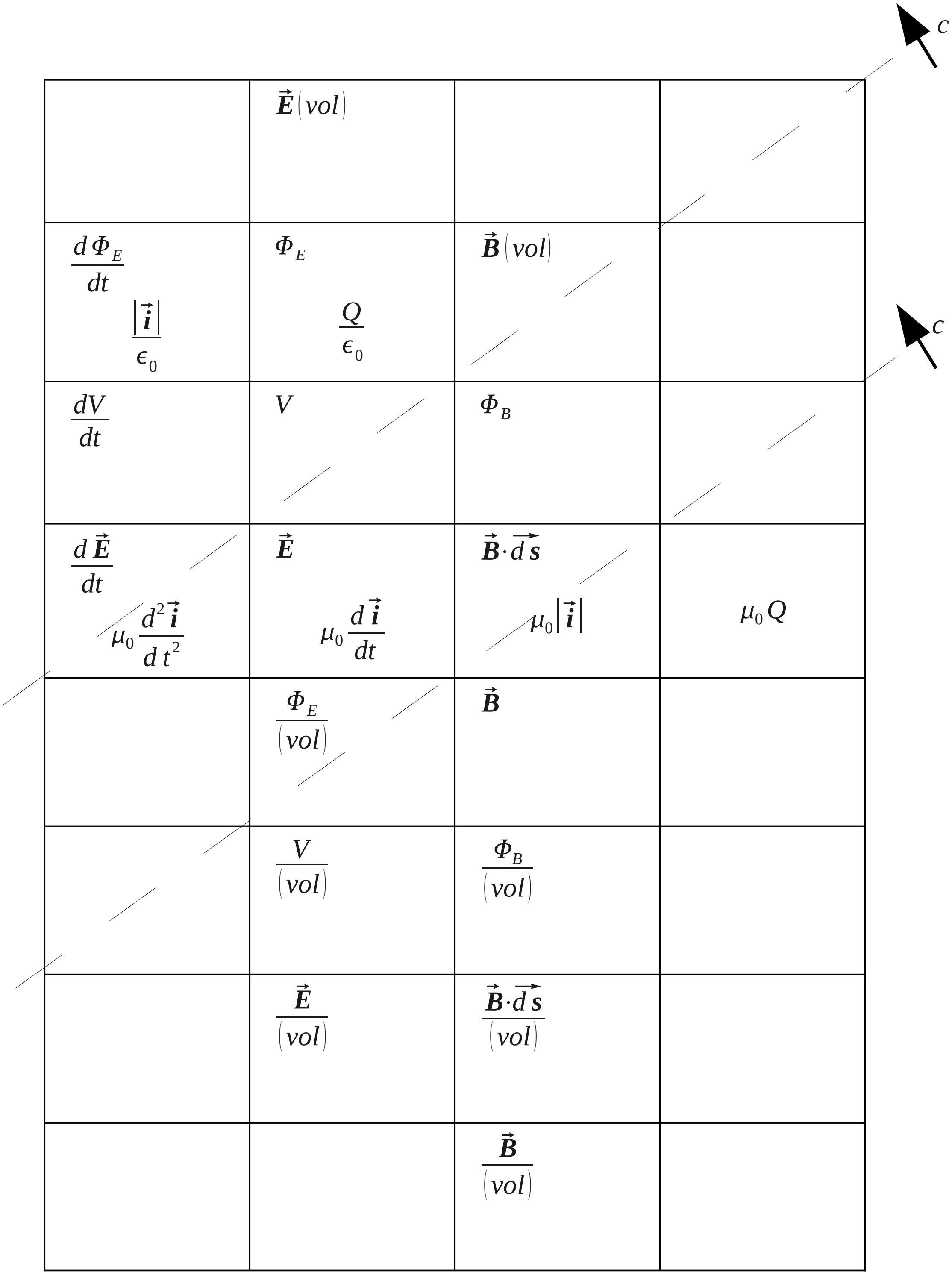
$$\mu_0 Q \quad .$$

Place this value in the box to the right of the $\vec{B}\cdot\vec{ds}$ box on the periodic table of electromagnetic elements.

	$\vec{B}\cdot\vec{ds}$ $\mu_0\lvert\vec{i}\rvert$	$\mu_0 Q$

Diagram 139: The relationship between $\vec{B}\cdot\vec{ds}$ and the box to the right on the grid

Note that the Φ_E box is related to the box to the right of $\vec{B}\cdot\vec{ds}$ by crossing two velocity-dimension lines. Recall that $c^2\mu_0$ is just $\frac{1}{\epsilon_0}$.



	$\vec{E}(vol)$		
$\frac{d\Phi_E}{dt}$ $\frac{\lvert\vec{i}\rvert}{\epsilon_0}$	Φ_E $\frac{Q}{\epsilon_0}$	$\vec{B}(vol)$	
$\frac{dV}{dt}$	V	Φ_B	
$\frac{d\vec{E}}{dt}$ $\mu_0\frac{d^2\vec{i}}{dt^2}$	$\vec{E}$ $\mu_0\frac{d\vec{i}}{dt}$	$\vec{B}\cdot\vec{ds}$ $\mu_0\lvert\vec{i}\rvert$	$\mu_0 Q$
	$\frac{\Phi_E}{(vol)}$	$\vec{B}$	
	$\frac{V}{(vol)}$	$\frac{\Phi_B}{(vol)}$	
	$\frac{\vec{E}}{(vol)}$	$\frac{\vec{B}\cdot\vec{ds}}{(vol)}$	
		$\frac{\vec{B}}{(vol)}$	

Diagram 140: The value $\mu_0 Q$ placed on the grid

The spatial relationships between ΦE, V and E

Consider a path ($\vec{E_p}$) that is in the direction of the $\vec{E}$ field at every point along the path. If the $\vec{E}$ field has the same value at every point on the path, the third Maxwell equation can be evaluated as:

$$V = \oint \vec{E}\cdot\vec{ds} = \vec{E}\cdot\vec{E_p} \ .$$

The spatial-dimension line that separates $\vec{E}$ and V represents the electric path, $\vec{E_p}$.

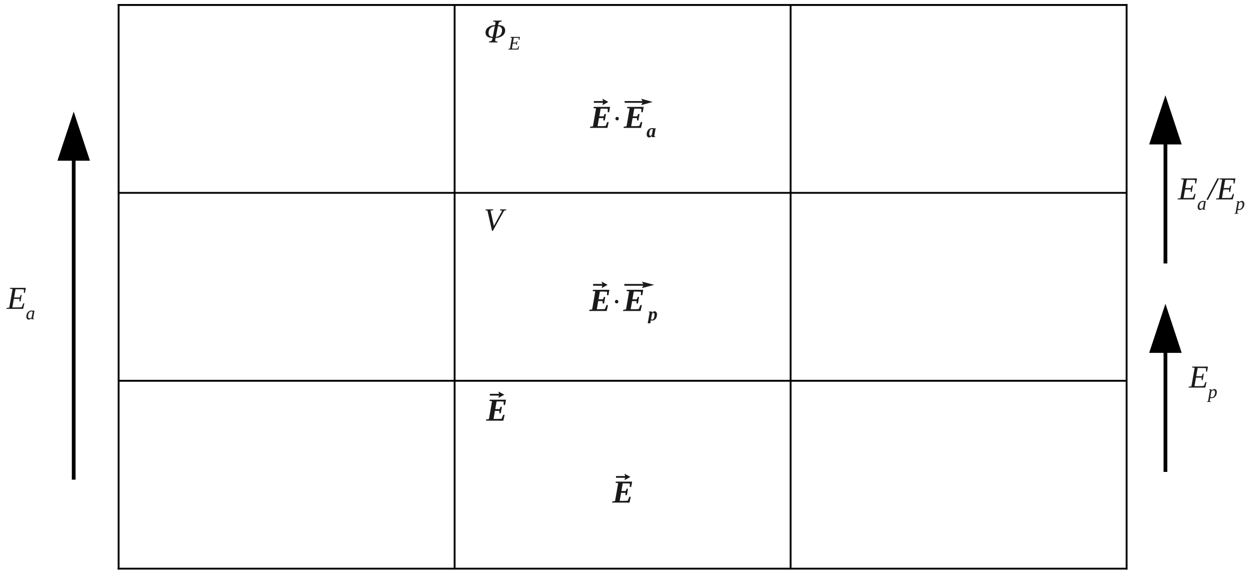
Consider an area ($\vec{E_a}$) that is in the direction of the $\vec{E}$ field at all points of the area. If the $\vec{E}$ field has the same value at every point of the area, the first Maxwell equation can be evaluated as:

$$\Phi_E = \oint \vec{E}\cdot\vec{dA} = \vec{E}\cdot\vec{E_a} \ .$$

The spatial-dimension line that separates V and Φ_E represents the ratio of electric area, $\vec{E_a}$, over electric path, $\vec{E_p}$:

$$\frac{\vec{E_a}}{\vec{E_p}} \ .$$

These two spatial-dimension lines can be interpreted on the periodic table of electromagnetic elements.



	Φ_E $\vec{E}\cdot\vec{E_a}$	
	V $\vec{E}\cdot\vec{E_p}$	
	$\vec{E}$ $\vec{E}$	

Diagram 141: Spatial relationships in the Φ_E column on the grid

The spatial relationships between ΦB, B.ds and B

Consider a path ($\overrightarrow{B_p}$) that is in the direction of the $\vec{B}$ field at every point along the path. If the $\vec{B}$ field has the same value at every point on the path, the fourth Maxwell equation can be evaluated as:

$$\oint \vec{B}\cdot\overrightarrow{ds} = \vec{B}\cdot\overrightarrow{B_p} \ .$$

The spatial-dimension line that separates $\vec{B}$ and $\vec{B}\cdot\overrightarrow{ds}$ represents the magnetic path, $\overrightarrow{B_p}$.

Consider an area ($\overrightarrow{B_a}$) that is in the direction of the $\vec{B}$ field at all points of the area. If the $\vec{B}$ field has the same value at every point of the area, the second Maxwell equation can be evaluated as:

$$\Phi_B = \oint \vec{B}\cdot\overrightarrow{dA} = \vec{B}\cdot\overrightarrow{B_a} \ .$$

The spatial-dimension line that separates $\vec{B}\cdot\overrightarrow{ds}$ and Φ_B represents the ratio of magnetic area, $\overrightarrow{B_a}$, over magnetic path, $\overrightarrow{B_p}$:

$$\frac{\overrightarrow{B_a}}{\overrightarrow{B_p}} \ .$$

These two spatial-dimension lines can be interpreted on the periodic table of electromagnetic elements.

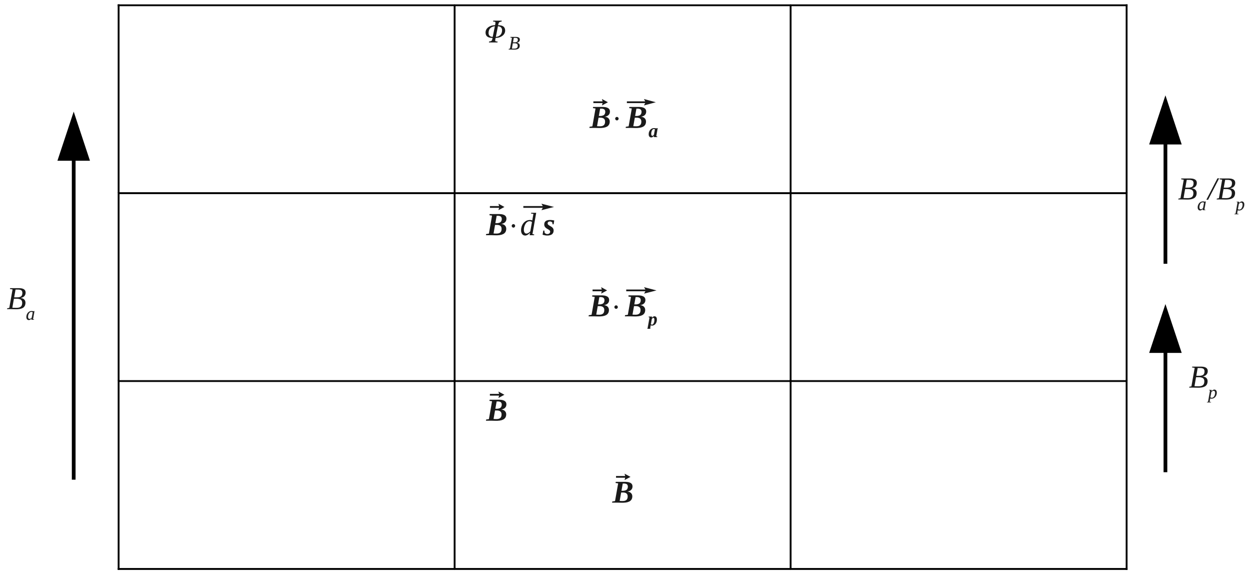


Diagram 142: Spatial relationships in the Φ_B column on the grid

Electromagnetic volume

Crossing three consecutive spatial-dimension lines on the periodic table of electromagnetic elements describes a volume. Crossing any three consecutive spatial-dimension lines describes the same volume. This is due to the fact that the sum of the volumetric parts must add up to the whole volume. For this reason the three consecutive spatial-dimension line labels repeat over and over again when proceeding up or down within a column on the periodic table of electromagnetic elements.

The volume described by three consecutive spatial-dimension lines is a quantity that has the units of volume. This volume is sometimes a readily apparent geometric volume, sometimes not. This is illustrated in the following examples.

Volume in the ΦE column

The electromagnetic volume can be interpreted in terms of the spatial dimension lines that separate the elements in the Φ_E column:

$$volume = \left(\frac{E_a}{E_p}\right)\left(E_p\right)\left(B_p\right) = \left(E_p\right)\left(B_p\right) \ .$$

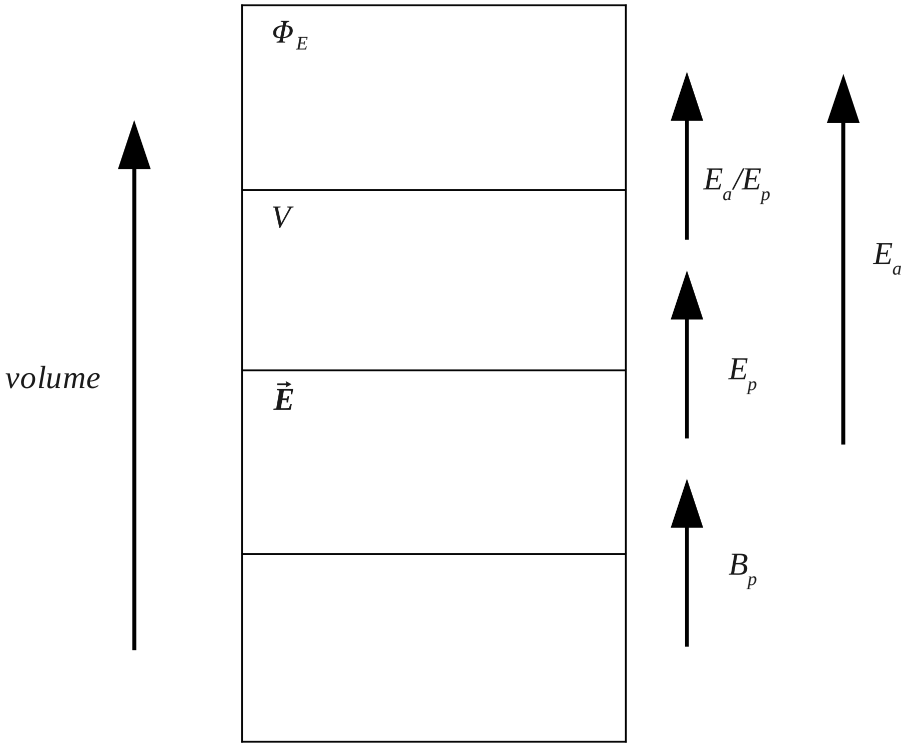


Diagram 143: The geometric relationships between Φ_E *,* V *and* $\vec{E}$ *on the grid*

The electromagnetic volume can be calculated as the product of the electric area (E_a) times the magnetic path (B_p).

Volume in the ΦB column

The electromagnetic volume can be interpreted in terms of the spatial dimension lines that separate the elements in the Φ_B column:

$$volume = \left(\frac{E_a}{E_p}\right)\left(\frac{B_a}{B_p}\right)\left(B_p\right) = \left(\frac{E_a}{E_p}\right)\left(B_a\right) .$$

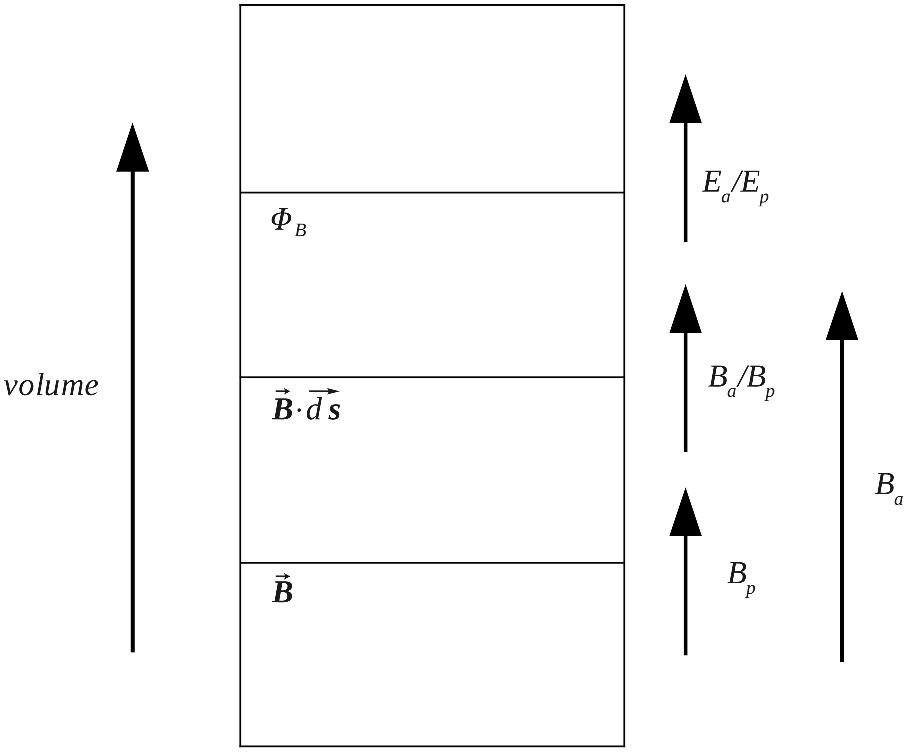


Diagram 144: The geometric relationships between Φ_B *,* $\vec{B}\cdot\vec{ds}$ *and* $\vec{B}$ *on the grid*

The electromagnetic volume can be calculated as the product of the ratio of the electric area (E_a) over the electric path (E_p) times the magnetic area (B_a).

The magnetic area (B_a) can be seen to be:

$$B_a = \left(E_p\right)\left(B_p\right) .$$

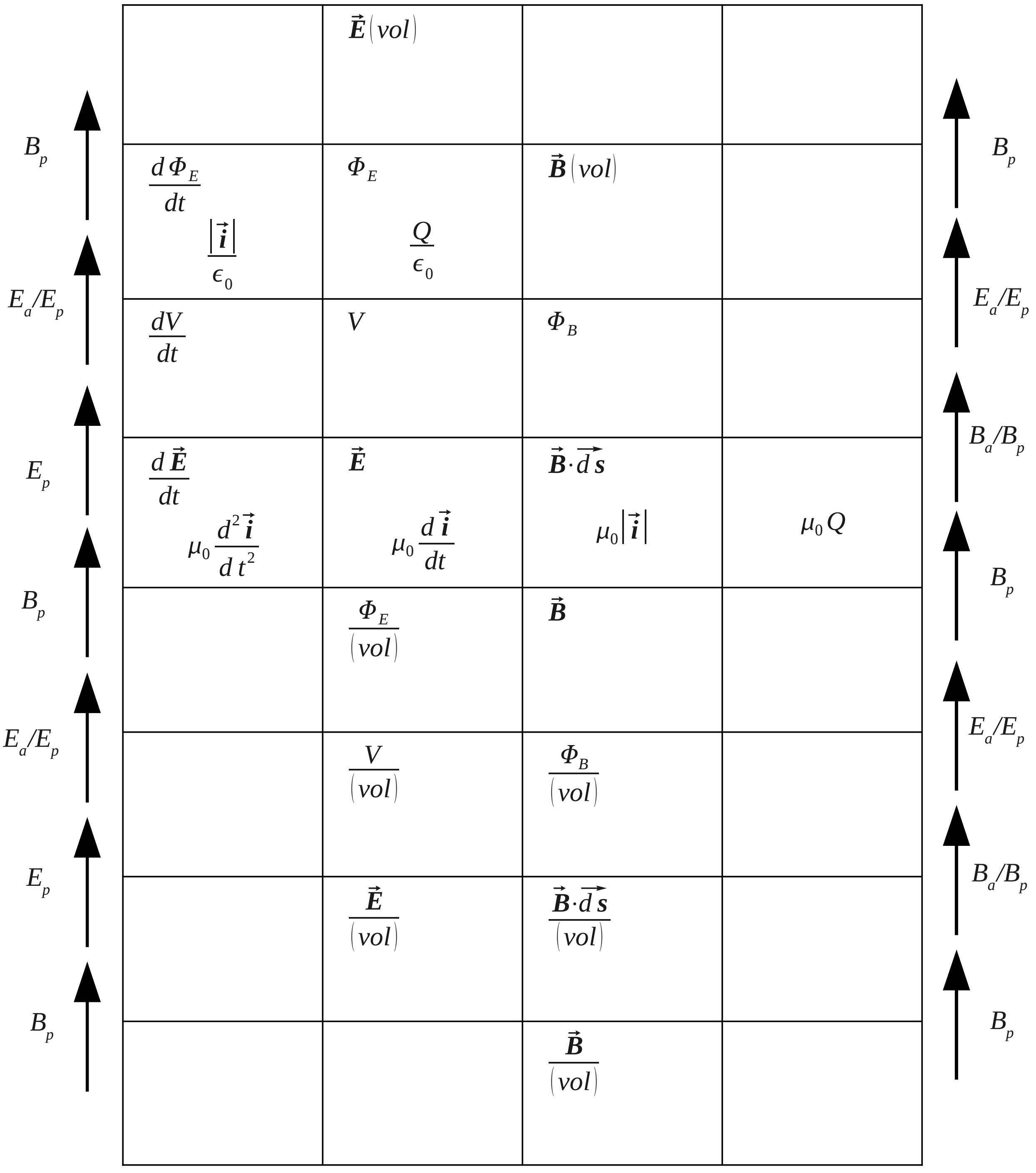
By using the fact that:

$$E_p = \frac{B_a}{B_p} ,$$

because the spatial-dimension line that separates the V box from the $\vec{E}$ box also separates the Φ_B box from the $\vec{B}\cdot\vec{ds}$ box.

Geometry of the spatial-dimension lines

Geometric labels can now be provided for the spatial-dimension lines.



	$\vec{E}(vol)$		
$\frac{d\Phi_E}{dt}$ $\frac{\lvert\vec{i}\rvert}{\epsilon_0}$	Φ_E $\frac{Q}{\epsilon_0}$	$\vec{B}(vol)$	
$\frac{dV}{dt}$	V	Φ_B	
$\frac{d\vec{E}}{dt}$ $\mu_0\frac{d^2\vec{i}}{dt^2}$	$\vec{E}$ $\mu_0\frac{d\vec{i}}{dt}$	$\vec{B}\cdot\vec{ds}$ $\mu_0\lvert\vec{i}\rvert$	$\mu_0 Q$
	$\frac{\Phi_E}{(vol)}$	$\vec{B}$	
	$\frac{V}{(vol)}$	$\frac{\Phi_B}{(vol)}$	
	$\frac{\vec{E}}{(vol)}$	$\frac{\vec{B}\cdot\vec{ds}}{(vol)}$	
		$\frac{\vec{B}}{(vol)}$	

Diagram 145: Geometric labels for the spatial-dimension lines on the grid

Filling in the ΦB column

It is now possible to fill in all of the boxes (elements) in the Φ_B column on the periodic table of electromagnetic elements by utilizing the spatial-dimension line labels to calculate the contents of each box (element). When proceeding upward from a given box, multiply the contents of the lower box by the spatial-dimension line label that separates the lower box from the upper box to determine the contents of the upper box. When proceeding downward from a given box, divide the contents of the upper box by the spatial-dimension line label that separates the upper box from the lower box to determine the contents of the lower box.

Start in the $\vec{\boldsymbol{B}} \cdot \overrightarrow{d\boldsymbol{s}}$ box which contains:

$$\mu_0 \left| \vec{\boldsymbol{i}} \right| \quad .$$

Proceed upward utilizing the spatial-dimension line label:

$$E_p$$

to yield:

$$\Phi_B = \left(\vec{\boldsymbol{B}} \cdot \overrightarrow{d\boldsymbol{s}} \right) \left(E_p \right) = \left(\mu_0 \left| \vec{\boldsymbol{i}} \right| \right) \left(E_p \right) = \mu_0 E_p \left| \vec{\boldsymbol{i}} \right| \quad .$$

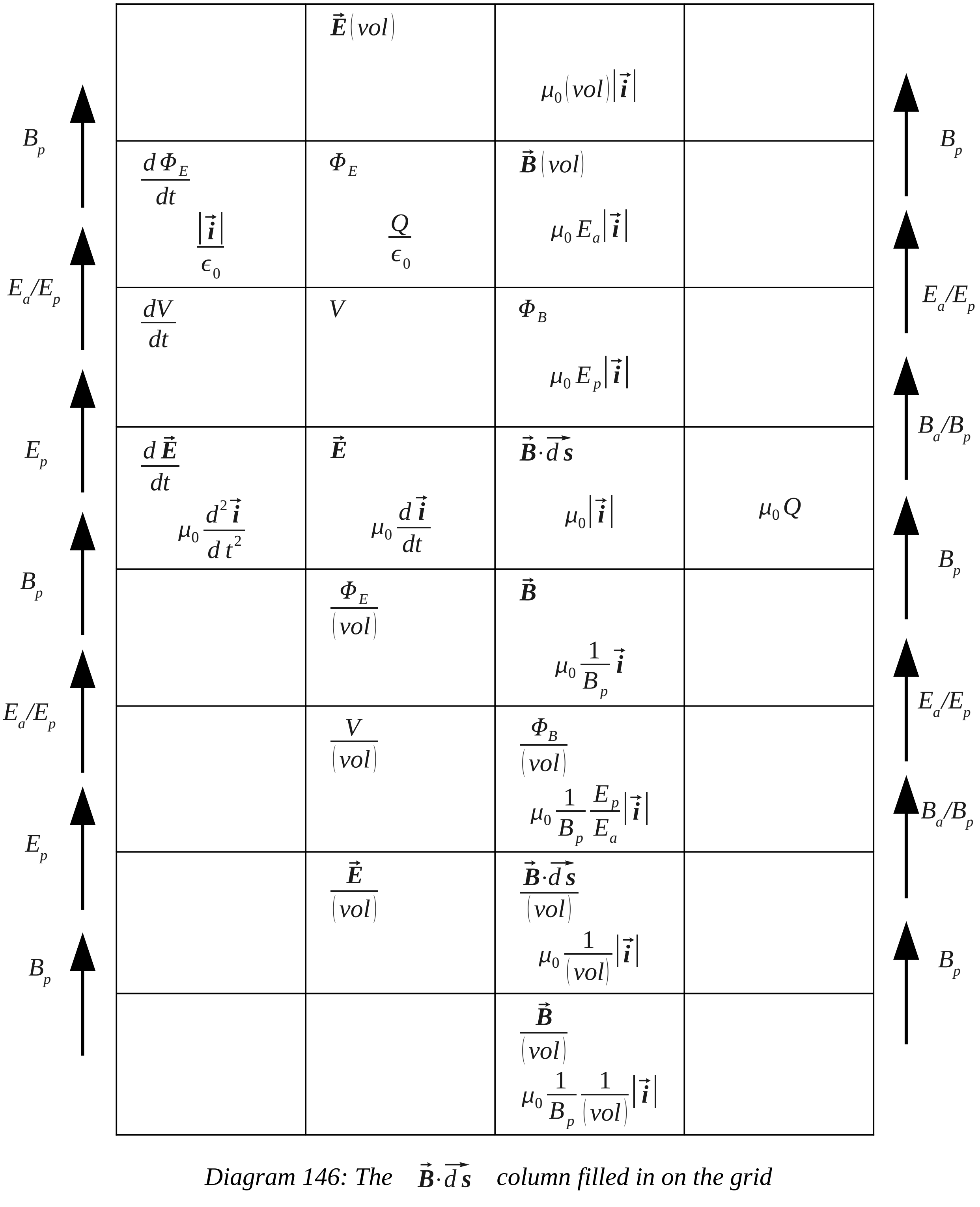
Proceed downward utilizing the spatial-dimension line label:

$$B_p$$

to yield:

$$\vec{\boldsymbol{B}} = \left(\vec{\boldsymbol{B}} \cdot \overrightarrow{d\boldsymbol{s}} \right) \left(\frac{1}{B_p} \right) = \left(\mu_0 \vec{\boldsymbol{i}} \right) \frac{1}{B_p} = \mu_0 \frac{1}{B_p} \vec{\boldsymbol{i}} \quad .$$

Apply these procedures repetitively, utilizing the appropriate spatial-dimension line labels, to fill in all of the boxes in the column.



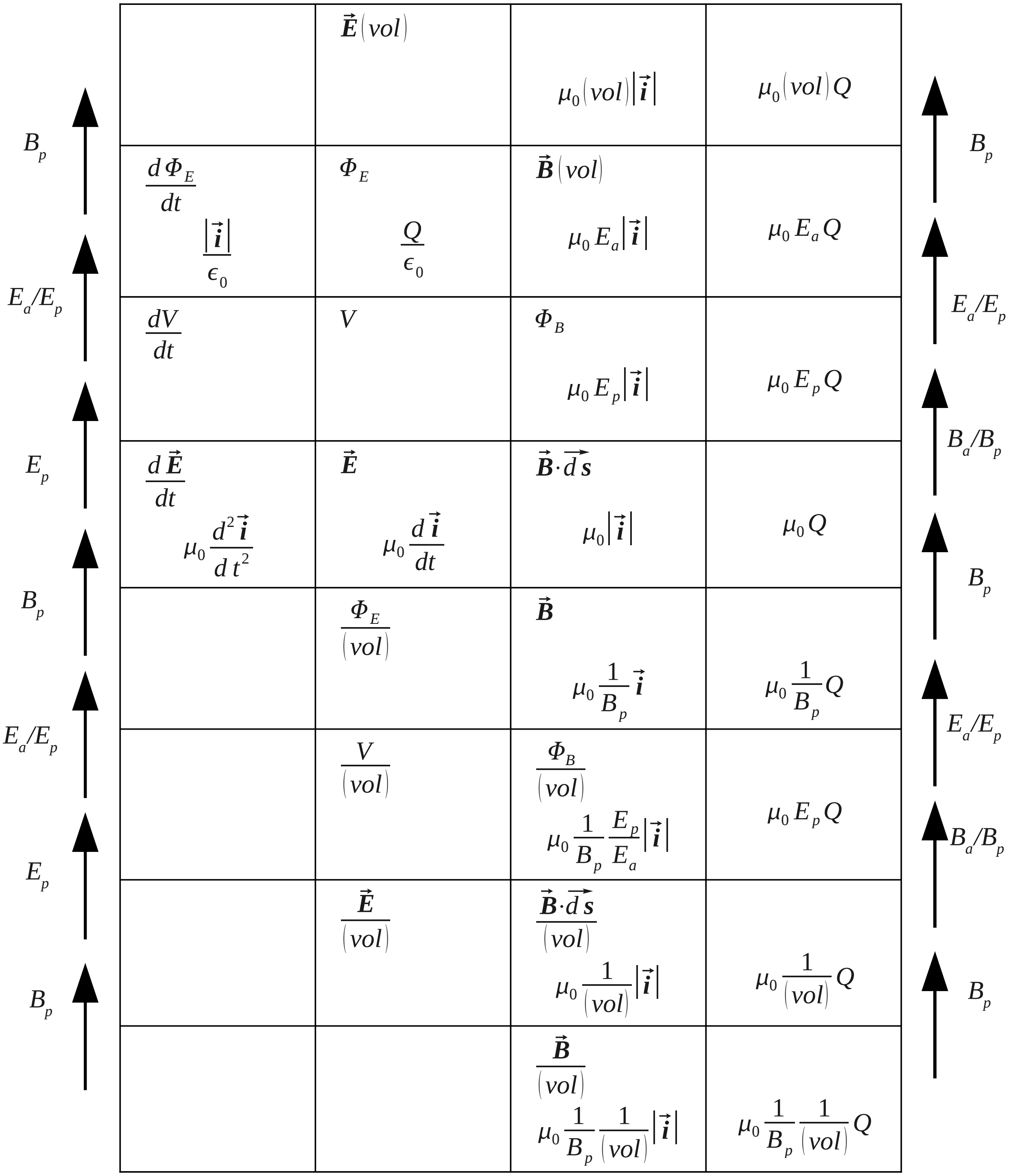
	$\vec{E}(vol)$	$\mu_0 (vol)\lvert\vec{i}\rvert$	
$\frac{d\Phi_E}{dt}$ $\frac{\lvert\vec{i}\rvert}{\epsilon_0}$	Φ_E $\frac{Q}{\epsilon_0}$	$\vec{B}(vol)$ $\mu_0 E_a \lvert\vec{i}\rvert$	
$\frac{dV}{dt}$	V	Φ_B $\mu_0 E_p \lvert\vec{i}\rvert$	
$\frac{d\vec{E}}{dt}$ $\mu_0 \frac{d^2\vec{i}}{dt^2}$	$\vec{E}$ $\mu_0 \frac{d\vec{i}}{dt}$	$\vec{B}\cdot\vec{ds}$ $\mu_0 \lvert\vec{i}\rvert$	$\mu_0 Q$
	$\frac{\Phi_E}{(vol)}$	$\vec{B}$ $\mu_0 \frac{1}{B_p}\vec{i}$	
	$\frac{V}{(vol)}$	$\frac{\Phi_B}{(vol)}$ $\mu_0 \frac{1}{B_p}\frac{E_p}{E_a}\lvert\vec{i}\rvert$	
	$\frac{\vec{E}}{(vol)}$	$\frac{\vec{B}\cdot\vec{ds}}{(vol)}$ $\mu_0 \frac{1}{(vol)}\lvert\vec{i}\rvert$	
		$\frac{\vec{B}}{(vol)}$ $\mu_0 \frac{1}{B_p}\frac{1}{(vol)}\lvert\vec{i}\rvert$	

Diagram 146: The $\vec{B}\cdot\vec{ds}$ *column filled in on the grid*

Filling in the right-hand column

It is possible to fill in all of the boxes (elements) in the right-hand column using the same procedure as was used for the Φ_B column.

Start with the box labeled $\mu_0 Q$, and proceed as before.

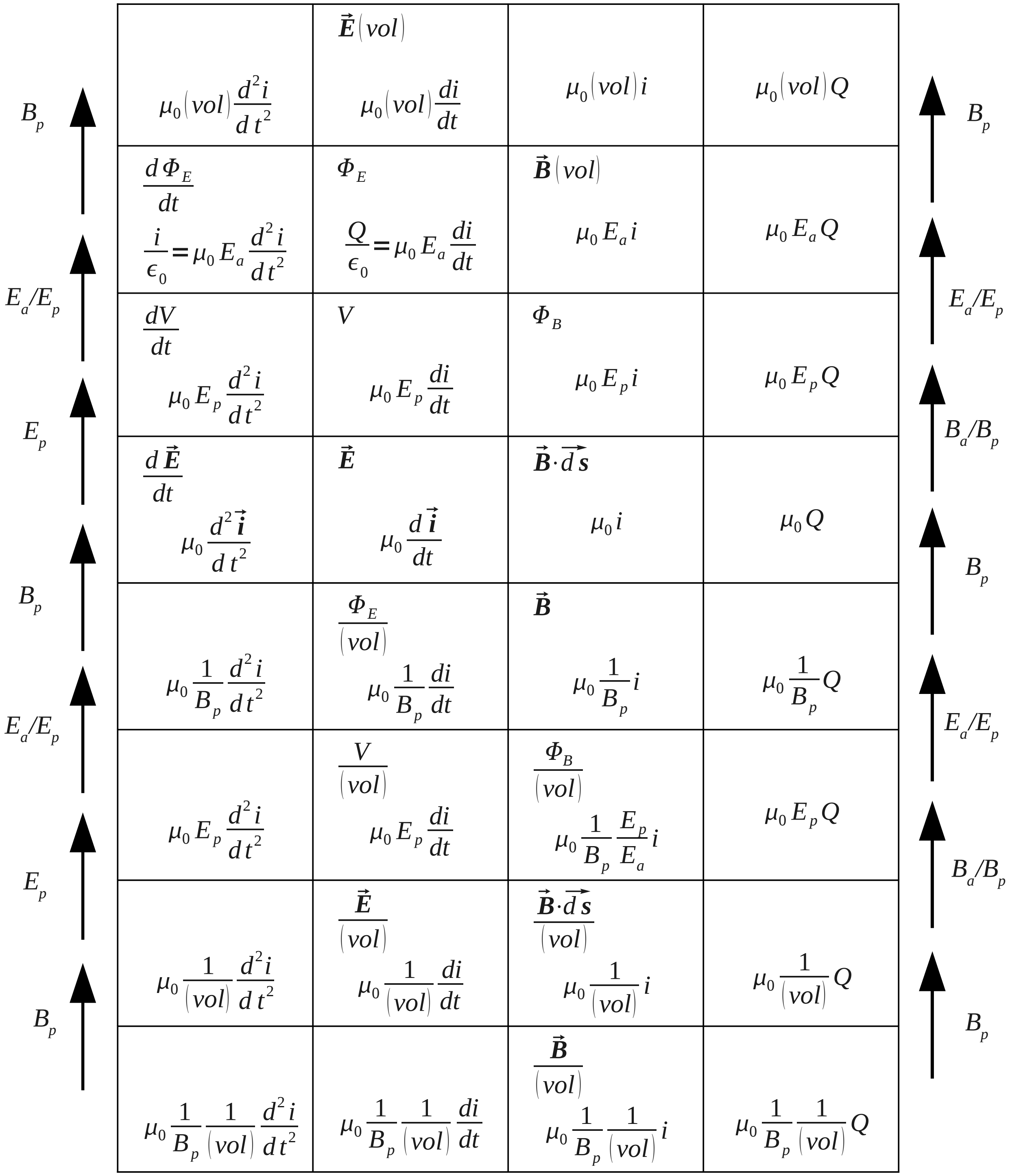


	$\vec{E}(vol)$	$\mu_0 (vol) \lvert\vec{i}\rvert$	$\mu_0 (vol) Q$
$\frac{d\Phi_E}{dt}$ $\frac{\lvert\vec{i}\rvert}{\epsilon_0}$	Φ_E $\frac{Q}{\epsilon_0}$	$\vec{B}(vol)$ $\mu_0 E_a \lvert\vec{i}\rvert$	$\mu_0 E_a Q$
$\frac{dV}{dt}$	V	Φ_B $\mu_0 E_p \lvert\vec{i}\rvert$	$\mu_0 E_p Q$
$\frac{d\vec{E}}{dt}$ $\mu_0 \frac{d^2\vec{i}}{dt^2}$	$\vec{E}$ $\mu_0 \frac{d\vec{i}}{dt}$	$\vec{B}\cdot\vec{ds}$ $\mu_0 \lvert\vec{i}\rvert$	$\mu_0 Q$
	$\frac{\Phi_E}{(vol)}$	$\vec{B}$ $\mu_0 \frac{1}{B_p} \vec{i}$	$\mu_0 \frac{1}{B_p} Q$
	$\frac{V}{(vol)}$	$\frac{\Phi_B}{(vol)}$ $\mu_0 \frac{1}{B_p} \frac{E_p}{E_a} \lvert\vec{i}\rvert$	$\mu_0 E_p Q$
	$\frac{\vec{E}}{(vol)}$	$\frac{\vec{B}\cdot\vec{ds}}{(vol)}$ $\mu_0 \frac{1}{(vol)} \lvert\vec{i}\rvert$	$\mu_0 \frac{1}{(vol)} Q$
		$\frac{\vec{B}}{(vol)}$ $\mu_0 \frac{1}{B_p} \frac{1}{(vol)} \lvert\vec{i}\rvert$	$\mu_0 \frac{1}{B_p} \frac{1}{(vol)} Q$

Diagram 147: The right-hand column filled in on the grid

Filling in the remaining columns

Inspection of the two columns filled in shows that all of the boxes in a given row contain a particular constant multiplied by a time derivative of charge (possibly the zeroth derivative, which is just charge itself). The particular time derivative of charge increases when proceeding from right to left. These two facts can be used to fill in the remaining boxes in each row.



$\mu_0 (vol) \frac{d^2 i}{d t^2}$	$\vec{\mathbf{E}}(vol)$ $\mu_0 (vol) \frac{di}{dt}$	$\mu_0 (vol) i$	$\mu_0 (vol) Q$
$\frac{d\Phi_E}{dt}$ $\frac{i}{\epsilon_0} = \mu_0 E_a \frac{d^2 i}{d t^2}$	Φ_E $\frac{Q}{\epsilon_0} = \mu_0 E_a \frac{di}{dt}$	$\vec{\mathbf{B}}(vol)$ $\mu_0 E_a i$	$\mu_0 E_a Q$
$\frac{dV}{dt}$ $\mu_0 E_p \frac{d^2 i}{d t^2}$	V $\mu_0 E_p \frac{di}{dt}$	Φ_B $\mu_0 E_p i$	$\mu_0 E_p Q$
$\frac{d\vec{\mathbf{E}}}{dt}$ $\mu_0 \frac{d^2 \vec{\mathbf{i}}}{d t^2}$	$\vec{\mathbf{E}}$ $\mu_0 \frac{d\vec{\mathbf{i}}}{dt}$	$\vec{\mathbf{B}} \cdot \vec{d\mathbf{s}}$ $\mu_0 i$	$\mu_0 Q$
$\mu_0 \frac{1}{B_p} \frac{d^2 i}{d t^2}$	$\frac{\Phi_E}{(vol)}$ $\mu_0 \frac{1}{B_p} \frac{di}{dt}$	$\vec{\mathbf{B}}$ $\mu_0 \frac{1}{B_p} i$	$\mu_0 \frac{1}{B_p} Q$
$\mu_0 E_p \frac{d^2 i}{d t^2}$	$\frac{V}{(vol)}$ $\mu_0 E_p \frac{di}{dt}$	$\frac{\Phi_B}{(vol)}$ $\mu_0 \frac{1}{B_p} \frac{E_p}{E_a} i$	$\mu_0 E_p Q$
$\mu_0 \frac{1}{(vol)} \frac{d^2 i}{d t^2}$	$\frac{\vec{\mathbf{E}}}{(vol)}$ $\mu_0 \frac{1}{(vol)} \frac{di}{dt}$	$\frac{\vec{\mathbf{B}} \cdot \vec{d\mathbf{s}}}{(vol)}$ $\mu_0 \frac{1}{(vol)} i$	$\mu_0 \frac{1}{(vol)} Q$
$\mu_0 \frac{1}{B_p} \frac{1}{(vol)} \frac{d^2 i}{d t^2}$	$\mu_0 \frac{1}{B_p} \frac{1}{(vol)} \frac{di}{dt}$	$\frac{\vec{\mathbf{B}}}{(vol)}$ $\mu_0 \frac{1}{B_p} \frac{1}{(vol)} i$	$\mu_0 \frac{1}{B_p} \frac{1}{(vol)} Q$

Diagram 148: The remaining elements filled in on the grid

Applying the template

The template has been developed using a specific value, $\mu_0 i$, for the box labeled $\vec{B}\cdot\vec{ds}$. The template can be generalized by noting that the geometric labels associated with the spatial-dimension lines apply to a wide variety of situations.

To apply the template to the analysis of a particular situation, identify the geometry ($\frac{E_a}{E_p}$, E_p and B_p) associated with the spatial-dimension lines and assign a known value to one of the boxes. Be sure the value has the correct derivative of charge for the column in which it is being placed, as shown on the template. Utilize the geometric spatial-dimension line labels and the sequence of derivatives pattern within a row to fill in the table.

For example, the classic inductor can be analyzed with the template by identifying the geometry of the spatial-dimension lines:

$$\frac{E_a}{E_p} = \frac{2\pi r H}{2\pi r N} \ ,$$

$$E_p = 2\pi r N \quad \text{and}$$

$$B_p = \frac{\pi r^2}{2\pi r} = \frac{r}{2}$$

and assigning the classic value

$$\vec{B} = \mu_0 \frac{2\pi r N}{2\pi r H} \vec{i}_0 \ .$$

This yields the results shown previously.

A current-carrying wire

The template for electromagnetic analysis can be applied to a current-carrying wire.

The geometry of the current-carrying wire

Start by examining the geometry of the current-carrying wire.

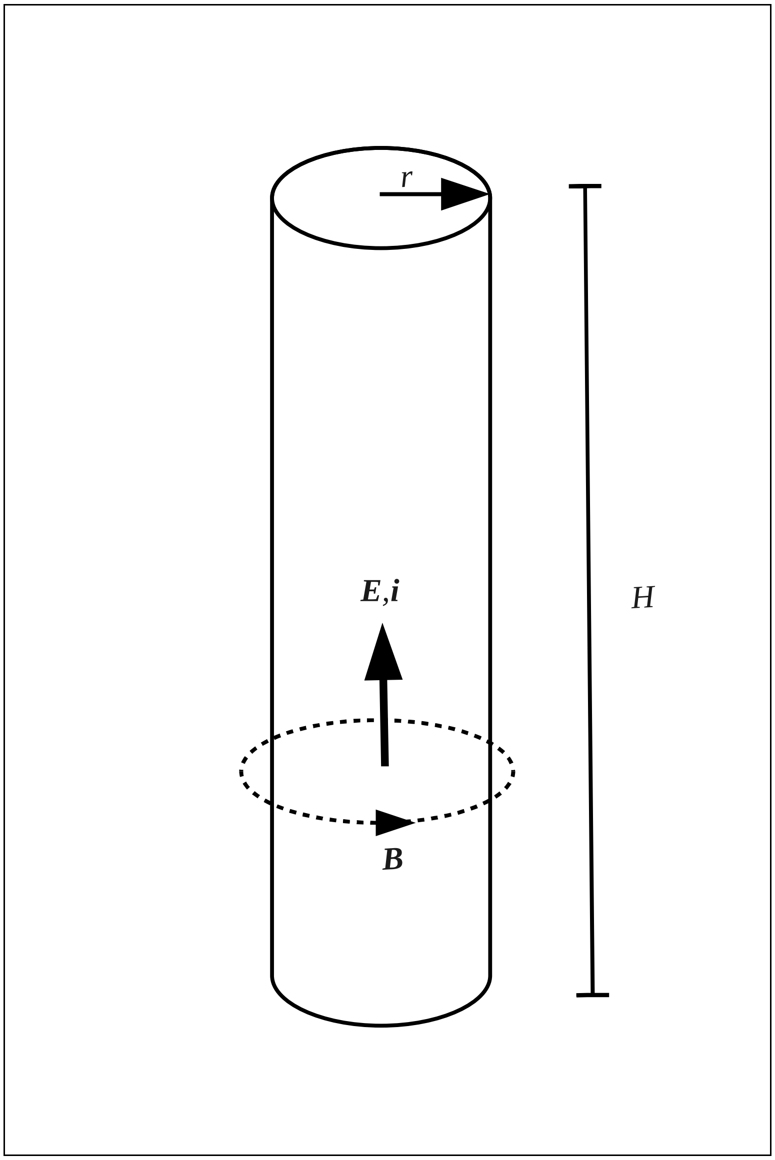


Diagram 149: The geometry of the current-carrying wire and associated fields

The wire is a cylindrical object. It has a height (H), and a radius (r). The cross section area (A) is:

$$A = \pi r^2 \quad .$$

The circumference (C) is:

$$C = 2\pi r \quad .$$

The current flows along the length of the wire. The $\vec{\boldsymbol{E}}$ field stretches the length of the wire. The length of the electric path (E_p) is the length of the wire:

$$E_p = H \quad .$$

The area (E_a) over which the electric field is distributed is the cross section of the wire:

$$E_a = \pi r^2 \quad .$$

The $\vec{\boldsymbol{B}}$ field circulates around the current. One half of the $\vec{\boldsymbol{B}}$ field lies inside the wire, and one half of the $\vec{\boldsymbol{B}}$ field lies outside the wire. The centroid of the $\vec{\boldsymbol{B}}$ field is at the surface of the wire. The length (B_p) of the path for the circulating $\vec{\boldsymbol{B}}$ field at the surface of the wire is:

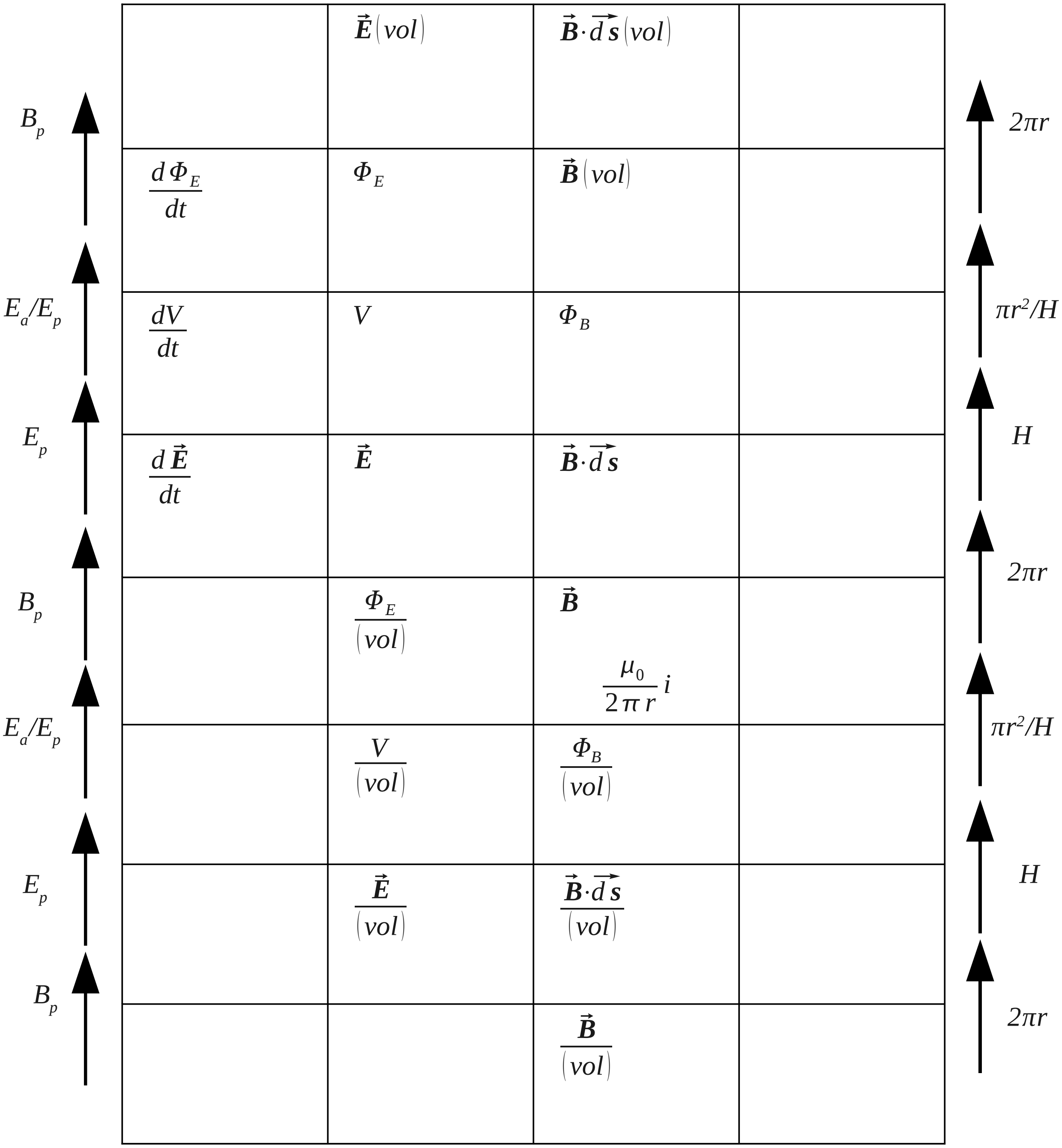
$$B_p = 2\pi r \quad .$$

The magnitude of the $\vec{\boldsymbol{B}}$ field at the surface of the wire is:

$$\left|\vec{\boldsymbol{B}}\right| = \frac{\mu_0 i}{2\pi r} \quad .$$

Labeling the template for electromagnetic analysis

The preceding information can be used to label the template for electromagnetic analysis.



	$\vec{E}(vol)$	$\vec{B}\cdot\vec{ds}(vol)$	
$\frac{d\Phi_E}{dt}$	Φ_E	$\vec{B}(vol)$	
$\frac{dV}{dt}$	V	Φ_B	
$\frac{d\vec{E}}{dt}$	$\vec{E}$	$\vec{B}\cdot\vec{ds}$	
	$\frac{\Phi_E}{(vol)}$	$\vec{B}$ $\frac{\mu_0}{2\pi r}i$	
	$\frac{V}{(vol)}$	$\frac{\Phi_B}{(vol)}$	
	$\frac{\vec{E}}{(vol)}$	$\frac{\vec{B}\cdot\vec{ds}}{(vol)}$	
		$\frac{\vec{B}}{(vol)}$	

Diagram 150: The template for electromagnetic analysis labeled with the details for the current-carrying wire

Filling in the B box

The magnitude of the $\vec{\boldsymbol{B}}$ field at the surface of the wire is:

$$\left|\vec{\boldsymbol{B}}\right| = \frac{\mu_0 i}{2\pi r} \quad .$$

This value has already been placed on the template.

Filling in the B.ds box

Utilizing the spatial-dimension line between the $\vec{\boldsymbol{B}}$ box and the $\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}$ box yields:

$$\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}} = \left(\vec{\boldsymbol{B}}\right)\left(B_p\right) \quad .$$

Substituting:

$$\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}} = \left(\frac{\mu_0 i}{2\pi r}\right)\left(2\pi r\right) \quad .$$

Resulting in:

$$\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}} = \mu_0 i \quad .$$

Place this value in the $\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}$ box on the template.

Filling in the ΦB box

Utilizing the spatial-dimension line between the $\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}$ box and the Φ_B box yields:

$$\Phi_B = \left(\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}\right)\left(E_p\right) \quad .$$

Substituting:

$$\Phi_B = \left(\mu_0 i\right)\left(H\right) \quad .$$

Resulting in:

$$\Phi_B = \mu_0 H i \quad .$$

Place this value in the Φ_B box on the template.

Filling in the B(vol) box

Utilizing the spatial-dimension line between the $\vec{B}(vol)$ box and the Φ_B box yields:

$$\vec{B}(vol) = (\Phi_B)\left(\frac{E_a}{E_p}\right) .$$

Substituting:

$$\vec{B}(vol) = (\mu_0 H i)\left(\frac{\pi r^2}{H}\right) .$$

Resulting in:

$$\vec{B}(vol) = (\mu_0 \pi r^2) i .$$

Place this value in the $\vec{B}(vol)$ box on the template.

Filling in the B.ds(vol) box

Utilizing the spatial-dimension line between the $\vec{B}\cdot\overrightarrow{ds}(vol)$ box and the $\vec{B}(vol)$ box yields:

$$\vec{B}\cdot\overrightarrow{ds}(vol) = (\vec{B}(vol))(B_p) .$$

Substituting:

$$\vec{B}\cdot\overrightarrow{ds}(vol) = (\mu_0 \pi r^2 i)(2\pi r) .$$

Resulting in:

$$\vec{B}\cdot\overrightarrow{ds}(vol) = \mu_0(\pi r^2)(2\pi r) i = \mu_0 2\pi^2 r^3 i .$$

Place this value in the $\vec{B}\cdot\overrightarrow{ds}(vol)$ box on the template.

The electromagnetic volume

Now,

$$\vec{B}\cdot\overrightarrow{ds} = \mu_0 i \quad , \quad E_a = \pi r^2 \quad \text{and} \quad B_p = 2\pi r .$$

The electromagnetic volume, (vol), is:

$$(vol) = (E_a)(B_p) = 2\pi^2 r^3 .$$

This is a quantity with the units of volume.

Filling in the ΦB/(vol) box

Utilizing the spatial-dimension line between the $\frac{\Phi_B}{(vol)}$ box and the $\vec{\boldsymbol{B}}$ box yields:

$$\frac{\Phi_B}{(vol)} = \frac{(\vec{\boldsymbol{B}})}{\left(\frac{E_a}{E_p}\right)} = (\vec{\boldsymbol{B}})\left(\frac{E_p}{E_a}\right) \quad .$$

Substituting:

$$\frac{\Phi_B}{(vol)} = \left(\frac{\mu_0}{2\pi r} i\right)\left(\frac{H}{\pi r^2}\right) \quad .$$

Resulting in:

$$\frac{\Phi_B}{(vol)} = \frac{\mu_0 H}{(2\pi r)(\pi r^2)} i = \frac{\mu_0 H}{2\pi^2 r^3} i \quad .$$

Place this value in the $\frac{\Phi_B}{(vol)}$ box on the template.

Filling in the B.ds/(vol) box

Utilizing the spatial-dimension line between the $\frac{\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}}{(vol)}$ box and the $\frac{\Phi_B}{(vol)}$ box yields:

$$\frac{\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}}{(vol)} = \frac{\left(\frac{\Phi_B}{(vol)}\right)}{(E_p)} = \frac{(\Phi_B)}{(vol)(E_p)} \quad .$$

Substituting:

$$\frac{\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}}{(vol)} = \frac{(\mu_0 H i)}{(2\pi^2 r^3)(H)} \quad .$$

Resulting in:

$$\frac{\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}}{(vol)} = \frac{(\mu_0)}{(2\pi^2 r^3)} i \quad .$$

Place this value in the $\frac{\vec{\boldsymbol{B}}\cdot\overrightarrow{d\boldsymbol{s}}}{(vol)}$ box on the template.

Filling in the B/(vol) box

Utilizing the spatial-dimension line between the $\frac{\vec{B}}{(vol)}$ box and the $\frac{\vec{B}\cdot\vec{ds}}{(vol)}$ box yields:

$$\frac{\vec{B}}{(vol)} = \frac{\left(\frac{\vec{B}\cdot\vec{ds}}{(vol)}\right)}{(B_p)} = \frac{(\vec{B}\cdot\vec{ds})}{(vol)(B_p)} \ .$$

Substituting:

$$\frac{\vec{B}}{(vol)} = \frac{(\mu_0 i)}{(2\pi^2 r^3)(2\pi r)} \ .$$

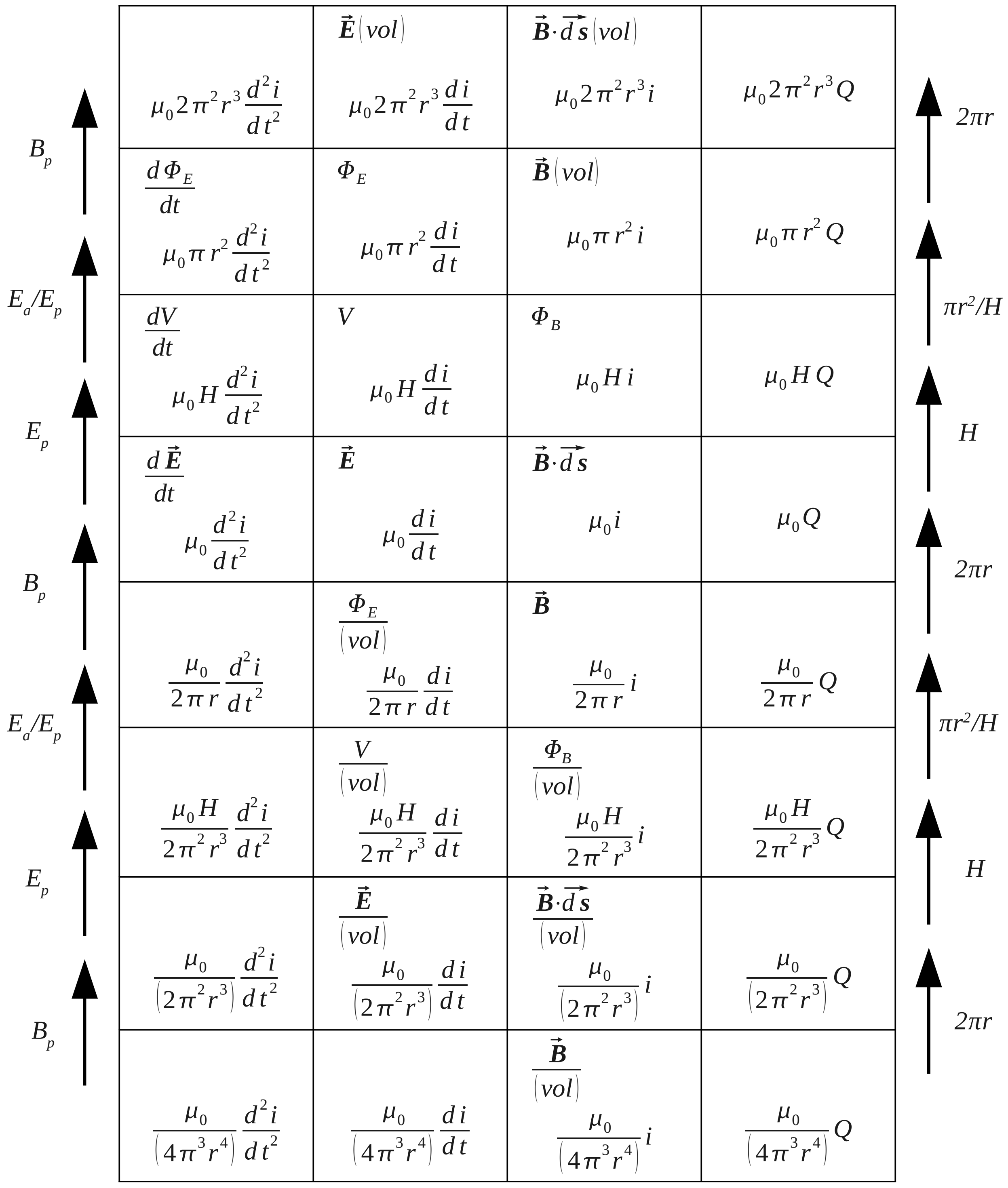
Resulting in:

$$\frac{\vec{B}}{(vol)} = \frac{(\mu_0)}{(4\pi^3 r^4)} i \ .$$

Place this value in the $\frac{\vec{B}}{(vol)}$ box on the template.

Filling in the remaining boxes

The remaining boxes in each row can now be filled in by using the value from the box in the $\vec{B}$ column for that row. The spatial (geometry) constant for each box in a given row is the same, because the boxes in a given row only differ by time derivatives. The particular time derivative of charge for each box in a given row follows a simple progression; it increases when proceeding from right to left within a row.



$\mu_0 2\pi^2 r^3 \frac{d^2 i}{d t^2}$	$\vec{E}(vol)$ $\mu_0 2\pi^2 r^3 \frac{d i}{d t}$	$\vec{B}\cdot\vec{ds}(vol)$ $\mu_0 2\pi^2 r^3 i$	$\mu_0 2\pi^2 r^3 Q$
$\frac{d\Phi_E}{dt}$ $\mu_0 \pi r^2 \frac{d^2 i}{d t^2}$	Φ_E $\mu_0 \pi r^2 \frac{d i}{d t}$	$\vec{B}(vol)$ $\mu_0 \pi r^2 i$	$\mu_0 \pi r^2 Q$
$\frac{dV}{dt}$ $\mu_0 H \frac{d^2 i}{d t^2}$	V $\mu_0 H \frac{d i}{d t}$	Φ_B $\mu_0 H i$	$\mu_0 H Q$
$\frac{d\vec{E}}{dt}$ $\mu_0 \frac{d^2 i}{d t^2}$	$\vec{E}$ $\mu_0 \frac{d i}{d t}$	$\vec{B}\cdot\vec{ds}$ $\mu_0 i$	$\mu_0 Q$
$\frac{\mu_0}{2\pi r} \frac{d^2 i}{d t^2}$	$\frac{\Phi_E}{(vol)}$ $\frac{\mu_0}{2\pi r} \frac{d i}{d t}$	$\vec{B}$ $\frac{\mu_0}{2\pi r} i$	$\frac{\mu_0}{2\pi r} Q$
$\frac{\mu_0 H}{2\pi^2 r^3} \frac{d^2 i}{d t^2}$	$\frac{V}{(vol)}$ $\frac{\mu_0 H}{2\pi^2 r^3} \frac{d i}{d t}$	$\frac{\Phi_B}{(vol)}$ $\frac{\mu_0 H}{2\pi^2 r^3} i$	$\frac{\mu_0 H}{2\pi^2 r^3} Q$
$\frac{\mu_0}{(2\pi^2 r^3)} \frac{d^2 i}{d t^2}$	$\frac{\vec{E}}{(vol)}$ $\frac{\mu_0}{(2\pi^2 r^3)} \frac{d i}{d t}$	$\frac{\vec{B}\cdot\vec{ds}}{(vol)}$ $\frac{\mu_0}{(2\pi^2 r^3)} i$	$\frac{\mu_0}{(2\pi^2 r^3)} Q$
$\frac{\mu_0}{(4\pi^3 r^4)} \frac{d^2 i}{d t^2}$	$\frac{\mu_0}{(4\pi^3 r^4)} \frac{d i}{d t}$	$\frac{\vec{B}}{(vol)}$ $\frac{\mu_0}{(4\pi^3 r^4)} i$	$\frac{\mu_0}{(4\pi^3 r^4)} Q$

Diagram 151: The completed analysis for a current-carrying wire

A parallel-plate capacitor

The template for electromagnetic analysis can be applied to a current-carrying wire.

The geometry of the parallel-plate capacitor

Start by examining the geometry of the parallel-plate capacitor.

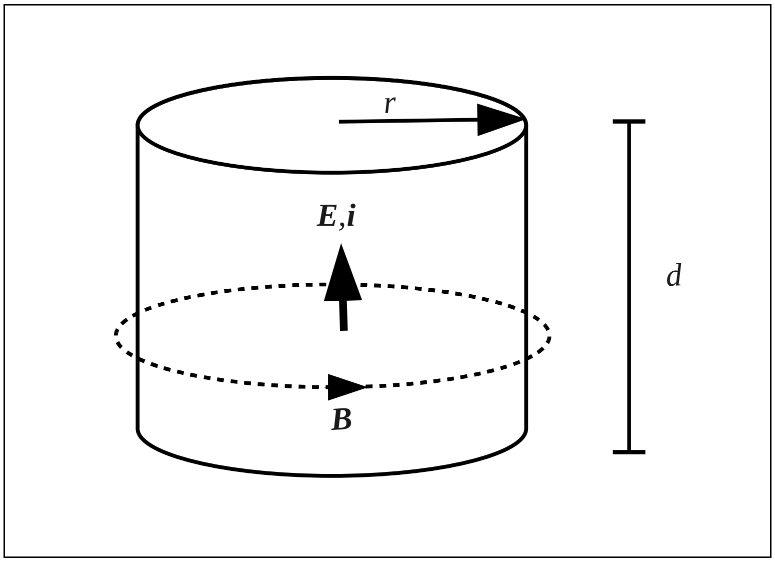


Diagram 152: The geometry of the parallel-plate capacitor and associated fields

The parallel-plate capacitor is a cylindrical object. It has a height (d) which is the distance between plates, and a radius r . The cross section area (A) is:

$$A = \pi r^2 \ .$$

The circumference (C) is:

$$C = 2\pi r \ .$$

The current flows along the length of the wire. The $\vec{E}$ field stretches between the plates. The length of the electric path (E_p) is the distance between the plates:

$$E_p = d \ .$$

The area (E_a) over which the electric field is distributed is the area between the plates:

$$E_a = \pi r^2 \ .$$

The $\vec{B}$ field circulates around the current. One half of the $\vec{B}$ field lies between the plates (inside the capacitor), and one half of the $\vec{B}$ field lies outside the capacitor. The centroid of the $\vec{B}$ field is at the edge of the plates. The length (B_p) of the path for the circulating $\vec{B}$ field at the edge of the plates is:

$$B_p = 2\pi r \ .$$

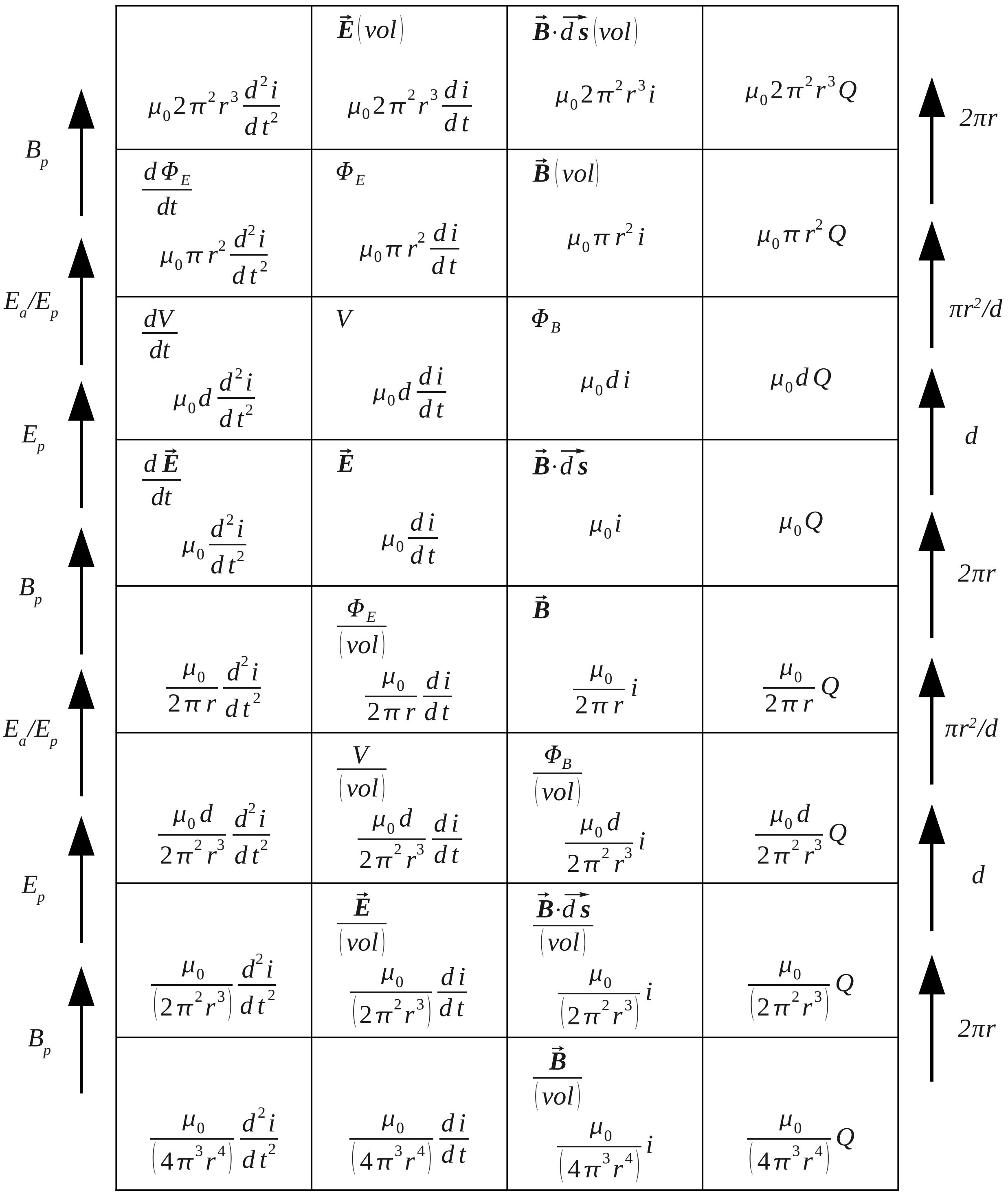
The magnitude of the $\vec{B}$ field at the edge of the plates is:

$$|\vec{B}| = \frac{\mu_0 i}{2\pi r} \ .$$

Similarity to the current-carrying wire

The geometry of the parallel-plate capacitor is remarkably similar to the geometry of the current-carrying wire. The length (H) of the wire has been replaced by the distance (d) between the plates. The current ($\vec{i}$) flows between the plates and the electric field ($\vec{E}$) stretches between the plates, while in the current-carrying wire the current flows from one end of the wire to the other and the electric field stretches the entire length of the wire. In both cases, the magnetic field ($\vec{B}$) circulates around the current. The centroid of the magnetic field lies at the edge of the plates, while in the current-carrying wire the centroid of the magnetic field lies at the surface of the wire.

The similarity allows the preceding analysis to be used in it's entirety.



$\mu_0 2\pi^2 r^3 \frac{d^2 i}{dt^2}$	$\vec{E}(vol)$ $\mu_0 2\pi^2 r^3 \frac{di}{dt}$	$\vec{B}\cdot\vec{ds}(vol)$ $\mu_0 2\pi^2 r^3 i$	$\mu_0 2\pi^2 r^3 Q$
$\frac{d\Phi_E}{dt}$ $\mu_0 \pi r^2 \frac{d^2 i}{dt^2}$	Φ_E $\mu_0 \pi r^2 \frac{di}{dt}$	$\vec{B}(vol)$ $\mu_0 \pi r^2 i$	$\mu_0 \pi r^2 Q$
$\frac{dV}{dt}$ $\mu_0 d \frac{d^2 i}{dt^2}$	V $\mu_0 d \frac{di}{dt}$	Φ_B $\mu_0 d i$	$\mu_0 d Q$
$\frac{d\vec{E}}{dt}$ $\mu_0 \frac{d^2 i}{dt^2}$	$\vec{E}$ $\mu_0 \frac{di}{dt}$	$\vec{B}\cdot\vec{ds}$ $\mu_0 i$	$\mu_0 Q$
$\frac{\mu_0}{2\pi r} \frac{d^2 i}{dt^2}$	$\frac{\Phi_E}{(vol)}$ $\frac{\mu_0}{2\pi r} \frac{di}{dt}$	$\vec{B}$ $\frac{\mu_0}{2\pi r} i$	$\frac{\mu_0}{2\pi r} Q$
$\frac{\mu_0 d}{2\pi^2 r^3} \frac{d^2 i}{dt^2}$	$\frac{V}{(vol)}$ $\frac{\mu_0 d}{2\pi^2 r^3} \frac{di}{dt}$	$\frac{\Phi_B}{(vol)}$ $\frac{\mu_0 d}{2\pi^2 r^3} i$	$\frac{\mu_0 d}{2\pi^2 r^3} Q$
$\frac{\mu_0}{(2\pi^2 r^3)} \frac{d^2 i}{dt^2}$	$\frac{\vec{E}}{(vol)}$ $\frac{\mu_0}{(2\pi^2 r^3)} \frac{di}{dt}$	$\frac{\vec{B}\cdot\vec{ds}}{(vol)}$ $\frac{\mu_0}{(2\pi^2 r^3)} i$	$\frac{\mu_0}{(2\pi^2 r^3)} Q$
$\frac{\mu_0}{(4\pi^3 r^4)} \frac{d^2 i}{dt^2}$	$\frac{\mu_0}{(4\pi^3 r^4)} \frac{di}{dt}$	$\frac{\vec{B}}{(vol)}$ $\frac{\mu_0}{(4\pi^3 r^4)} i$	$\frac{\mu_0}{(4\pi^3 r^4)} Q$

Diagram 153: The completed analysis for a parallel-plate capacitor

The inductance of a parallel-plate capacitor

The voltage (V) across an inductance (L) is given by the formula:

$$V = -L\frac{di}{dt} \quad .$$

From the completed analysis template for the parallel-plate capacitor, the voltage box contains:

$$V = \mu_0 d\frac{di}{dt} \quad .$$

Therefore the inductance is:

$$L = -\mu_0 d \quad .$$

A sphere of charge

The template for electromagnetic analysis can be used to understand the behavior of an expanding sphere of charge.

The geometry of the sphere of charge

Start by examining the geometry of the sphere of charge.

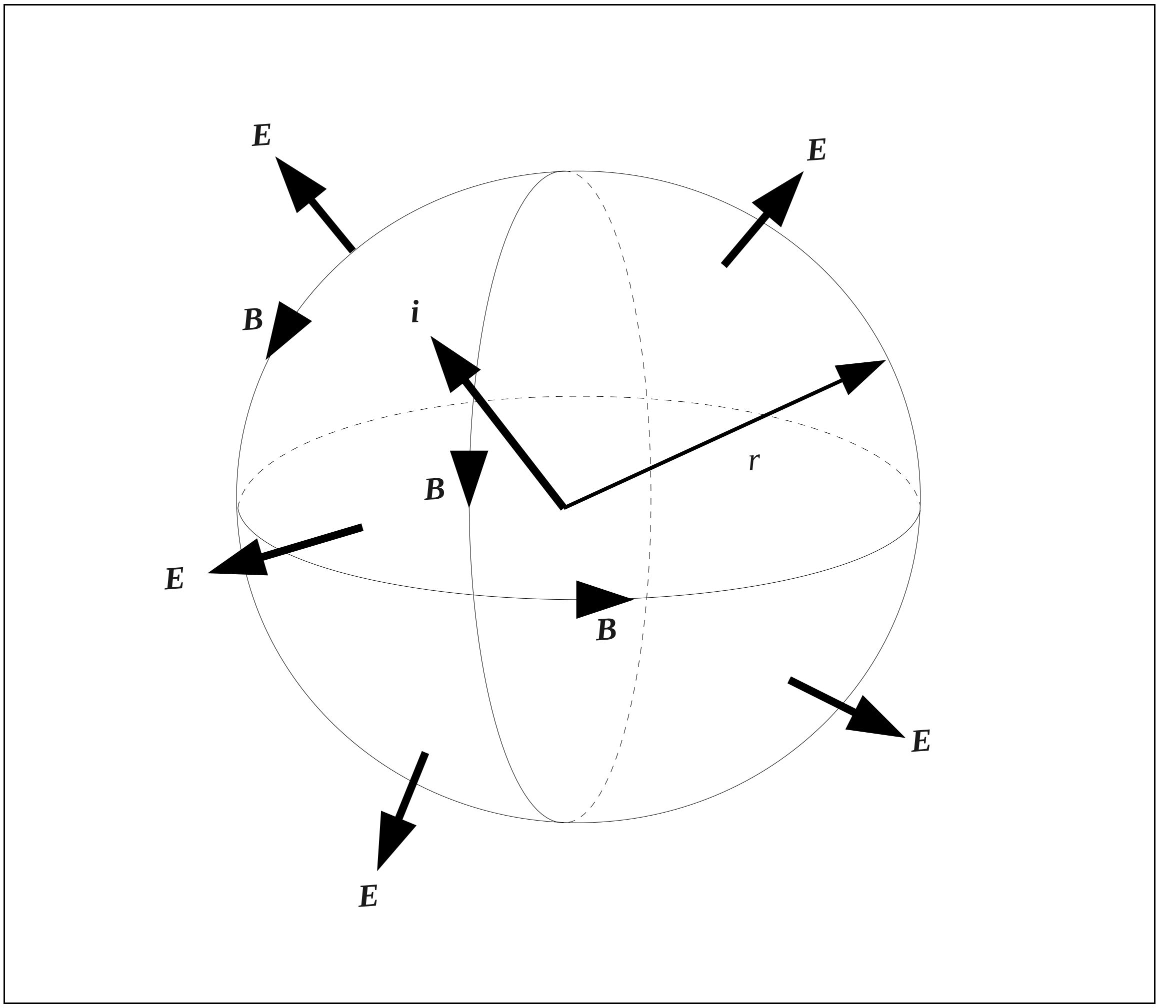


Diagram 154: The geometry of the sphere of charge and associated fields

Suppose that all of the charge (Q) starts at the center of the sphere. The charge will expand radially outward with spherical symmetry. The expanding charge represents a current ($\vec{i}$) that is flowing in a radially outward direction. Current flows in the direction of the electric field, so the electric field ($\vec{E}$) is pointing radially outward. The magnetic field ($\vec{B}$) circulates around the current in a spherically-symmetric fashion.

The length of the electric path (E_p) is equal to the radius of the sphere at any given time:

$$E_p = r \quad .$$

The charge is distributed over the surface of the sphere, so the area of the electric field (E_a) at any given time is:

$$E_a = 4\pi r^2 \quad .$$

The electromagnetic volume and the magnetic path

The template shows:

$$\frac{\Phi_E}{(vol)} = \left|\frac{d\vec{\boldsymbol{B}}}{dt}\right| \quad .$$

The template was constructed using the fact:

$$\Phi_E = |\vec{\boldsymbol{E}}| E_a \quad .$$

The template also shows:

$$\left|\frac{d\vec{\boldsymbol{B}}}{dt}\right| = \frac{|\vec{\boldsymbol{E}}|}{B_p} \quad .$$

Combining these formulas yields:

$$B_p = \frac{(vol)}{E_a} \quad .$$

Now, E_a is the surface area of the sphere:

$$E_a = 4\pi r^2 \quad .$$

If the electromagnetic volume (vol) is the volume of the sphere:

$$(vol) = \frac{4}{3}\pi r^3 \quad ,$$

then the magnetic path (B_p) is the volume over the surface area for a sphere:

$$B_p = \frac{(vol)}{E_a} = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{r}{3} \quad .$$

Filling in the B.ds box

The template was derived for the case of free-space. The $\vec{\boldsymbol{B}}\cdot\vec{d\boldsymbol{s}}$ box contains:

$$\vec{\boldsymbol{B}}\cdot\vec{d\boldsymbol{s}} = \mu_0 |\vec{\boldsymbol{i}}| \quad .$$

Filling in the template

The preceding facts can be used to fill in the template to complete the analysis.

	$\mu_0\frac{4}{3}\pi r^3\frac{d^2 i}{d t^2}$	$\vec{E}(vol)$ $\mu_0\frac{4}{3}\pi r^3\frac{d i}{d t}$	$\vec{B}\cdot\overrightarrow{ds}(vol)$ $\mu_0\frac{4}{3}\pi r^3 i$	$\mu_0\frac{4}{3}\pi r^3 Q$	
↑ B_p	$\frac{d\Phi_E}{dt}$ $\mu_0 4\pi r^2\frac{d^2 i}{d t^2}$	Φ_E $\mu_0 4\pi r^2\frac{d i}{d t}$	$\vec{B}(vol)$ $\mu_0 4\pi r^2 i$	$\mu_0 4\pi r^2 Q$	↑ $r/3$
↑ E_a/E_p	$\frac{dV}{dt}$ $\mu_0 r\frac{d^2 i}{d t^2}$	V $\mu_0 r\frac{d i}{d t}$	Φ_B $\mu_0 r i$	$\mu_0 r Q$	↑ $4\pi r$
↑ E_p	$\frac{d\vec{E}}{dt}$ $\mu_0\frac{d^2 i}{d t^2}$	$\vec{E}$ $\mu_0\frac{d i}{d t}$	$\vec{B}\cdot\overrightarrow{ds}$ $\mu_0 i$	$\mu_0 Q$	↑ r
↑ B_p	$\frac{3\mu_0}{r}\frac{d^2 i}{d t^2}$	$\frac{\Phi_E}{(vol)}$ $\frac{3\mu_0}{r}\frac{d i}{d t}$	$\vec{B}$ $\frac{3\mu_0}{r}i$	$\frac{3\mu_0}{r}Q$	↑ $r/3$
↑ E_a/E_p	$\frac{3\mu_0}{4\pi r^2}\frac{d^2 i}{d t^2}$	$\frac{V}{(vol)}$ $\frac{3\mu_0}{4\pi r^2}\frac{d i}{d t}$	$\frac{\Phi_B}{(vol)}$ $\frac{3\mu_0}{4\pi r^2}i$	$\frac{3\mu_0}{4\pi r^2}Q$	↑ $4\pi r$
↑ E_p	$\frac{3\mu_0}{(4\pi r^3)}\frac{d^2 i}{d t^2}$	$\frac{\vec{E}}{(vol)}$ $\frac{3\mu_0}{(4\pi r^3)}\frac{d i}{d t}$	$\frac{\vec{B}\cdot\overrightarrow{ds}}{(vol)}$ $\frac{3\mu_0}{(4\pi r^3)}i$	$\frac{3\mu_0}{(4\pi r^3)}Q$	↑ r
↑ B_p	$\frac{\mu_0}{(4\pi r^4)}\frac{d^2 i}{d t^2}$	$\frac{\mu_0}{(4\pi r^4)}\frac{d i}{d t}$	$\frac{\vec{B}}{(vol)}$ $\frac{\mu_0}{(4\pi r^4)}i$	$\frac{\mu_0}{(4\pi r^4)}Q$	↑ $r/3$

Diagram 155: The completed analysis for a sphere of charge

Potential and kinetic views

The classic formula:

$$\Phi_E = \frac{Q}{\epsilon_0} ,$$

and the formula from the template:

$$\Phi_E = \left(\mu_0 \frac{di}{dt}\right)(E_a)$$

are both talking about the same total electric flux. Equating the two right-hand sides and applying the identity $c^2 = \frac{1}{\mu_0 \epsilon_0}$ yields:

$$\frac{c^2}{E_a} = \frac{\frac{di}{dt}}{Q} .$$

If the charge (Q) is sinusoidal with a particular radian frequency (ω) this reduces to:

$$\frac{c^2}{E_a} = \frac{\frac{di}{dt}}{Q} = -\omega^2 .$$

Standing waves and resonant frequencies

The electromagnetic volume is the product of three spatial-dimensions:

$$(vol) = \left(\frac{E_a}{E_p}\right)(E_a)(B_p) .$$

Each of these spatial dimensions

$$\frac{E_a}{E_p} , \quad E_a \quad \text{and} \quad B_p$$

can be considered to be a path (wave) length, and has an associated frequency. Each of these frequencies can give rise to resonant behavior and standing-wave phenomena along that spatial path.

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A spherical capacitor

A spherical capacitor can be analyzed using the electromagnetic template.

The geometry of a spherical capacitor

Start by examining the geometry of the spherical capacitor.

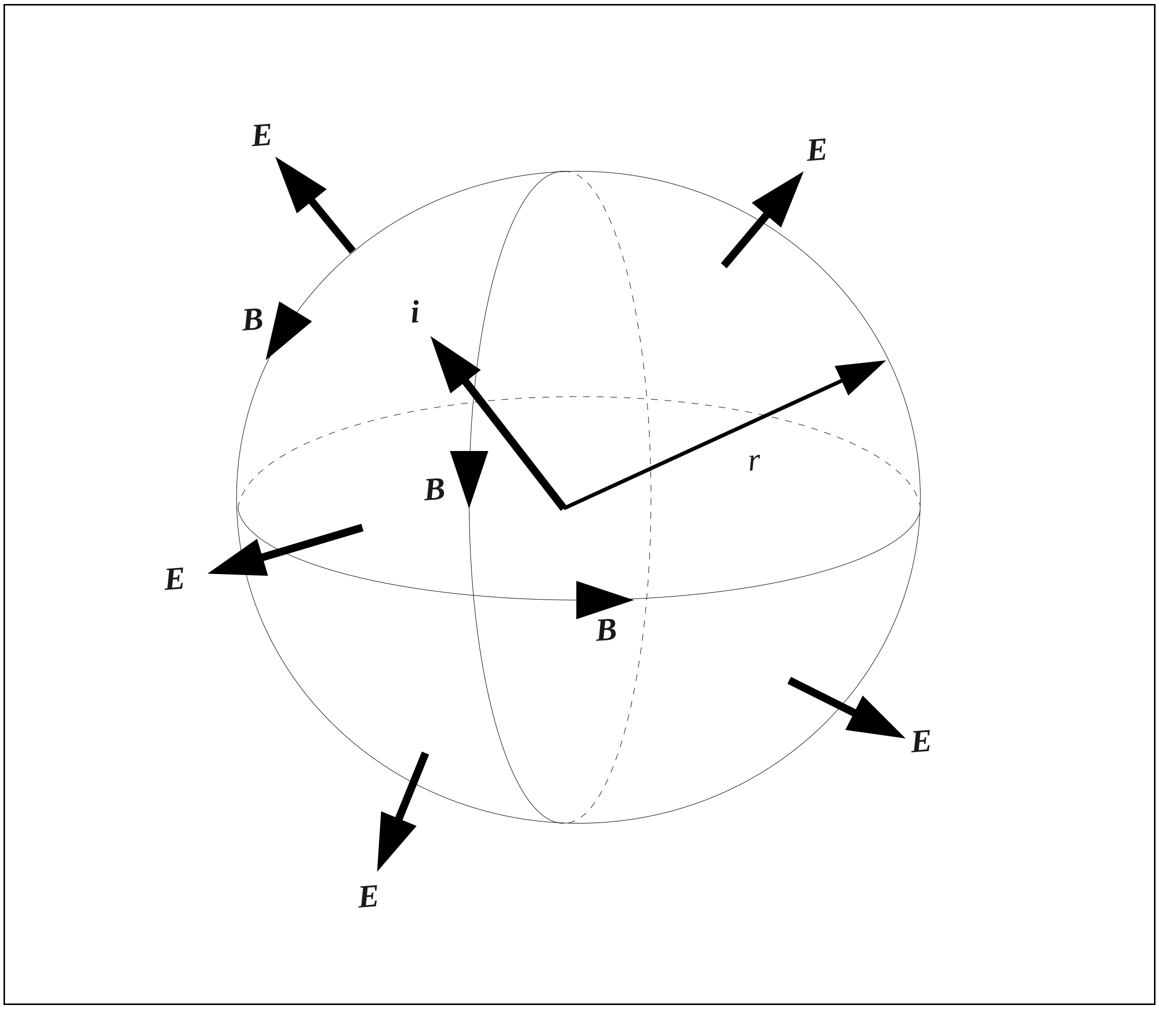

Diagram 156: The geometry of the spherical capacitor and associated fields

Similarity to the sphere of charge

The geometry of the spherical capacitor is remarkably similar to the sphere of charge. The centroid of the charge is located at the center of the sphere. In the case of the spherical capacitor the radius remains fixed, while for the sphere of charge the radius was increasing with time. The electric field points radially outward, and is distributed over the surface of the sphere.

The spherical capacitor represents a “snapshot” of the expanding sphere of charge frozen in time.

Labeling the template

The geometric labels for the spatial-dimension lines are exactly the same for the spherical capacitor and the sphere of charge.

Filling in the template

The similarity of the geometry for the spherical capacitor and the sphere of charge allows the analysis to be used in it's entirety.

	$\mu_0 \frac{4}{3}\pi r^3 \frac{d^2 i}{d t^2}$	$\vec{E}(vol)$ $\mu_0 \frac{4}{3}\pi r^3 \frac{d i}{d t}$	$\vec{B}\cdot\vec{ds}(vol)$ $\mu_0 \frac{4}{3}\pi r^3 i$	$\mu_0 \frac{4}{3}\pi r^3 Q$	
↑ B_p					↑ $r/3$
	$\frac{d\Phi_E}{dt}$ $\mu_0 4\pi r^2 \frac{d^2 i}{d t^2}$	Φ_E $\mu_0 4\pi r^2 \frac{d i}{d t}$	$\vec{B}(vol)$ $\mu_0 4\pi r^2 i$	$\mu_0 4\pi r^2 Q$	
↑ E_a/E_p					↑ $4\pi r$
	$\frac{dV}{dt}$ $\mu_0 r \frac{d^2 i}{d t^2}$	V $\mu_0 r \frac{d i}{d t}$	Φ_B $\mu_0 r i$	$\mu_0 r Q$	
↑ E_p					↑ r
	$\frac{d\vec{E}}{dt}$ $\mu_0 \frac{d^2 i}{d t^2}$	$\vec{E}$ $\mu_0 \frac{d i}{d t}$	$\vec{B}\cdot\vec{ds}$ $\mu_0 i$	$\mu_0 Q$	
↑ B_p					↑ $r/3$
	$\frac{3\mu_0}{r} \frac{d^2 i}{d t^2}$	$\frac{\Phi_E}{(vol)}$ $\frac{3\mu_0}{r} \frac{d i}{d t}$	$\vec{B}$ $\frac{3\mu_0}{r} i$	$\frac{3\mu_0}{r} Q$	
↑ E_a/E_p					↑ $4\pi r$
	$\frac{3\mu_0}{4\pi r^2} \frac{d^2 i}{d t^2}$	$\frac{V}{(vol)}$ $\frac{3\mu_0}{4\pi r^2} \frac{d i}{d t}$	$\frac{\Phi_B}{(vol)}$ $\frac{3\mu_0}{4\pi r^2} i$	$\frac{3\mu_0}{4\pi r^2} Q$	
↑ E_p					↑ r
	$\frac{3\mu_0}{(4\pi r^3)} \frac{d^2 i}{d t^2}$	$\frac{\vec{E}}{(vol)}$ $\frac{3\mu_0}{(4\pi r^3)} \frac{d i}{d t}$	$\frac{\vec{B}\cdot\vec{ds}}{(vol)}$ $\frac{3\mu_0}{(4\pi r^3)} i$	$\frac{3\mu_0}{(4\pi r^3)} Q$	
↑ B_p					↑ $r/3$
	$\frac{\mu_0}{(4\pi r^4)} \frac{d^2 i}{d t^2}$	$\frac{\mu_0}{(4\pi r^4)} \frac{d i}{d t}$	$\frac{\vec{B}}{(vol)}$ $\frac{\mu_0}{(4\pi r^4)} i$	$\frac{\mu_0}{(4\pi r^4)} Q$	

Diagram 157: The completed analysis for a spherical capacitor

The capacitance

The classic formula for total electric flux is:

$$\Phi_E = \frac{Q}{\epsilon_0} \ .$$

The template shows:

$$V = \frac{\Phi_E}{\left(\frac{E_a}{E_p}\right)} = \frac{\left(\frac{Q}{\epsilon_0}\right)}{4\pi r} = \frac{Q}{\epsilon_0}\frac{1}{4\pi r} \ .$$

The definition for capacitance (C) is:

$$CV = Q \ .$$

Therefore the capacitance of a sphere is:

$$C = \frac{1}{4\pi\epsilon_0 r} \ .$$

This is in agreement with the classic derivation.

The E field box

The template also shows:

$$\vec{\boldsymbol{E}} = \frac{\Phi_E}{E_a} = \frac{\Phi_E}{4\pi r^2} = \frac{Q}{\epsilon_0}\frac{1}{4\pi\epsilon_0 r^2} \ .$$

The density of electric field lines ($\vec{\boldsymbol{E}}$) is the total electric flux (Φ_E) divided over the surface area of the sphere ($4\pi r^2$). This is in agreement with classic definitions.

The dB/dt box

Finally, the template shows:

$$\left|\frac{d\vec{\boldsymbol{B}}}{dt}\right| = \frac{Q}{\epsilon_0}\frac{1}{\frac{4}{3}\pi r^3} = \frac{\Phi_E}{(vol)} \ .$$

The right-hand side of this equation is the total electric flux divided by the volume of the sphere.

Planck units

The Planck system of units can be shown on the periodic table of mechanical elements.

Power $\frac{c^5}{G}$	Energy	Action $\hbar$	Moment of Inertia		
	Force $\frac{c^4}{G}$	Momentum	Moment		
	Spring Constant	Impedance $\frac{c^3}{G}$	Mass $m_p=\sqrt{\frac{\hbar c}{G}}$		
	Pressure		Linear Density $\frac{c^2}{G}$		
			Areal Density	$\frac{c}{G}$	
			Volumetric Density		$\frac{1}{G}$

Diagram 158: Planck's units on the periodic table of mechanical elements

Unit analysis of the gravitational constant

The force between two masses is given by the formula:

$$F = G\frac{m_1}{r}\frac{m_2}{r} \quad .$$

Convert this to a unit expression using the SI system of units:

$$\mathrm{N} = G\frac{\mathrm{kg}}{\mathrm{m}}\frac{\mathrm{kg}}{\mathrm{m}} \quad .$$

Recall that the newton should be written as:

$$\mathrm{N} = \left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 m^0 \quad .$$

Substitute and solve for G :

$$G = \frac{\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2}{\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)} \quad .$$

Take the reciprocal of both sides:

$$\frac{1}{G} = \frac{\left(\frac{\mathrm{kg}}{\mathrm{m}}\right)}{\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2} = \left(\frac{\mathrm{kg}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-2} \quad .$$

This shows that $\frac{1}{G}$ is linear density divided by the square of velocity.

Place this label on the periodic table of mechanical elements in the box two boxes down and to the right of the linear density box. Remember that the grid extends infinitely in all directions.

Power $\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{0}$	Energy $\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{1}$	Action $\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{2}$	Moment of Inertia $\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{3}$	$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-1}\mathrm{m}^{4}$	$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-2}\mathrm{m}^{5}$
$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{-1}$	Force $\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{0}$	Momentum $\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{1}$	Moment $\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{2}$	$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-1}\mathrm{m}^{3}$	$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-2}\mathrm{m}^{4}$
$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{-2}$	Spring Constant $\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{-1}$	Impedance $\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{0}$	Mass $\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{1}$	$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-1}\mathrm{m}^{2}$	$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-2}\mathrm{m}^{3}$
$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{-3}$	Pressure $\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{-2}$	$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{-1}$	Linear Density $\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{0}$	$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-1}\mathrm{m}^{1}$	$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-2}\mathrm{m}^{2}$
$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{-4}$	$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{-3}$	$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{-2}$	Areal Density $\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{-1}$	$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-1}\mathrm{m}^{0}$	$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-2}\mathrm{m}^{1}$
$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{3}\mathrm{m}^{-5}$	$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\mathrm{m}^{-4}$	$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{1}\mathrm{m}^{-3}$	Volumetric Density $\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{0}\mathrm{m}^{-2}$	$\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-1}\mathrm{m}^{-1}$	$\frac{1}{G}$ $\frac{\mathrm{kg}}{\mathrm{m}}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{-2}\mathrm{m}^{0}$

Diagram 159: $\frac{1}{G}$ *placed on the periodic table of mechanical elements*

Fill in the velocity-diagonal

Consider the velocity-diagonal to represent a speed of light (c) relationship. Label the boxes along the velocity-diagonal as:

$$\frac{c^n}{G} \ , \quad n=0,1,2,\ldots \ .$$

Power $\frac{c^5}{G}$	Energy	Action	Moment of Inertia		
	Force $\frac{c^4}{G}$	Momentum	Moment		
	Spring Constant	Impedance $\frac{c^3}{G}$	Mass		
	Pressure		Linear Density $\frac{c^2}{G}$		
			Areal Density	$\frac{c^1}{G}$	
			Volumetric Density		$\frac{c^0}{G}$

Diagram 160: $\frac{c^n}{G}$ *progression along the velocity-diagonal on the grid*

Planck's constant on the grid

Recall that Planck's constant has the units of action. Place the reduced Planck's constant ($\hbar$) in the action box on the grid.

Power $\frac{c^5}{G}$	Energy	Action $\hbar$	Moment of Inertia		
	Force $\frac{c^4}{G}$	Momentum	Moment		
	Spring Constant	Impedance $\frac{c^3}{G}$	Mass		
	Pressure		Linear Density $\frac{c^2}{G}$		
			Areal Density	$\frac{c}{G}$	
			Volumetric Density		$\frac{1}{G}$

Diagram 161: Reduced Planck's constant placed on the grid

Planck's length unit

The Planck's length unit is defined by the formula:

$$l_p = \sqrt{\frac{\hbar G}{c^3}} \ .$$

Simple rearrangement yields:

$$l_p^{\ 2} = \frac{\hbar}{\left(\frac{c^3}{G}\right)} \ .$$

This can be interpreted on the periodic table of mechanical elements as the ratio of the action box over the impedance box.

Power	Energy	Action $\hbar$	Moment of Inertia		
	Force	Momentum	Moment		
	Spring Constant	Impedance $\frac{c^3}{G}$	Mass		
	Pressure		Linear Density		
			Areal Density		
			Volumetric Density		

Diagram 162: Planck's length unit interpreted on the grid

Each of the spatial-dimension lines can be interpreted as representing a Planck's length unit spatial relationship.

Planck's time unit

The Planck's time unit is defined by the formula:

$$t_p = \sqrt{\frac{\hbar G}{c^5}} \quad .$$

Simple rearrangement yields:

$$t_p{}^2 = \frac{\hbar}{\left(\frac{c^5}{G}\right)} \quad .$$

This can be interpreted on the periodic table of mechanical elements as the ratio of the action box over the power box.

Power $\frac{c^5}{G}$	Energy	Action $\hbar$	Moment of Inertia		
	Force	Momentum	Moment		
	Spring Constant	Impedance	Mass		
	Pressure		Linear Density		
			Areal Density		
			Volumetric Density		

Diagram 163: Planck's time unit interpreted on the grid

Each of the temporal-dimension lines can be interpreted as representing a Planck's time unit temporal relationship.

Planck's mass unit

The Planck's mass unit is defined by the formula:

$$m_p = \sqrt{\frac{\hbar c}{G}} \ .$$

Simple rearrangement yields:

$$m_p^{\ 2} = \hbar\left(\frac{c}{G}\right) = \left(\hbar c\right)\left(\frac{1}{G}\right) \ .$$

This can be interpreted on the periodic table of mechanical elements as the product of the action box ($\hbar$) times the box labeled $\frac{c}{G}$. The action box is related to the mass box by a circulation relationship. The mass box is related to the box labeled $\frac{c}{G}$ by an inverse circulation relationship. Taking the product causes the circulation relationships to cancel. This leaves only the square of the mass.

This can also be interpreted on the periodic table of mechanical elements as the product of the box above the box labeled energy ($\hbar c$) times the box labeled $\frac{1}{G}$. Again the relationships cancel leaving only the square of the Planck's mass unit.

The square of the Planck's mass unit can be interpreted as the product of any two boxes that are arranged symmetrically around the mass box.

Power	Energy	Action $\hbar$	Moment of Inertia		
	Force	Momentum	Moment		
	Spring Constant	Impedance	Mass $m_p=\sqrt{\frac{\hbar c}{G}}$		
	Pressure		Linear Density		
			Areal Density	$\frac{c}{G}$	
			Volumetric Density		

Diagram 164: Planck's mass unit interpreted on the grid

Canonical Planck's units

The periodic table of mechanical elements can be labeled with canonical units written using Planck's units.

Power $\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{3} l_p^{0}$ $\frac{1}{G} c^{5} l_p^{0} t_p^{0}$	Energy $\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{2} l_p^{1}$ $\frac{1}{G} c^{4} l_p^{1} t_p^{0}$	$\hbar$ $\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{1} l_p^{2}$ $\frac{1}{G} c^{3} l_p^{2} t_p^{0}$	Moment of Inertia $\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{0} l_p^{3}$ $\frac{1}{G} c^{2} l_p^{3} t_p^{0}$	$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{-1} l_p^{4}$ $\frac{1}{G} c^{1} l_p^{4} t_p^{0}$	$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{-2} l_p^{5}$ $\frac{1}{G} c^{0} l_p^{5} t_p^{0}$
$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{3} l_p^{-1}$ $\frac{1}{G} c^{4} l_p^{0} t_p^{-1}$	Force $\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{2} l_p^{0}$ $\frac{1}{G} c^{4} l_p^{0} t_p^{0}$	Momentum $\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{1} l_p^{1}$ $\frac{1}{G} c^{3} l_p^{1} t_p^{0}$	Moment $\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{0} l_p^{2}$ $\frac{1}{G} c^{2} l_p^{2} t_p^{0}$	$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{-1} l_p^{3}$ $\frac{1}{G} c^{1} l_p^{3} t_p^{0}$	$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{-2} l_p^{4}$ $\frac{1}{G} c^{0} l_p^{4} t_p^{0}$
$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{3} l_p^{-2}$ $\frac{1}{G} c^{3} l_p^{0} t_p^{-2}$	Spring Constant $\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{2} l_p^{-1}$ $\frac{1}{G} c^{3} l_p^{0} t_p^{-1}$	Impedance $\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{1} l_p^{0}$ $\frac{1}{G} c^{3} l_p^{0} t_p^{0}$	$m_p=\sqrt{\frac{\hbar c}{G}}$ $\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{0} l_p^{1}$ $\frac{1}{G} c^{2} l_p^{1} t_p^{0}$	$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{-1} l_p^{2}$ $\frac{1}{G} c^{1} l_p^{2} t_p^{0}$	$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{-2} l_p^{3}$ $\frac{1}{G} c^{0} l_p^{3} t_p^{0}$
$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{3} l_p^{-3}$ $\frac{1}{G} c^{2} l_p^{0} t_p^{-3}$	Pressure $\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{2} l_p^{-2}$ $\frac{1}{G} c^{2} l_p^{0} t_p^{-2}$	$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{1} l_p^{-1}$ $\frac{1}{G} c^{2} l_p^{0} t_p^{-1}$	Linear Density $\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{0} l_p^{0}$ $\frac{1}{G} c^{2} l_p^{0} t_p^{0}$	$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{-1} l_p^{1}$ $\frac{1}{G} c^{1} l_p^{1} t_p^{0}$	$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{-2} l_p^{2}$ $\frac{1}{G} c^{0} l_p^{2} t_p^{0}$
$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{3} l_p^{-4}$ $\frac{1}{G} c^{1} l_p^{0} t_p^{-4}$	$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{2} l_p^{-3}$ $\frac{1}{G} c^{1} l_p^{0} t_p^{-3}$	$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{1} l_p^{-2}$ $\frac{1}{G} c^{1} l_p^{0} t_p^{-2}$	Areal Density $\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{0} l_p^{-1}$ $\frac{1}{G} c^{1} l_p^{0} t_p^{-1}$	$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{-1} l_p^{0}$ $\frac{1}{G} c^{1} l_p^{0} t_p^{0}$	$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{-2} l_p^{1}$ $\frac{1}{G} c^{0} l_p^{1} t_p^{0}$
$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{3} l_p^{-5}$ $\frac{1}{G} c^{0} l_p^{0} t_p^{-5}$	$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{2} l_p^{-4}$ $\frac{1}{G} c^{0} l_p^{0} t_p^{-4}$	$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{1} l_p^{-3}$ $\frac{1}{G} c^{0} l_p^{0} t_p^{-3}$	Volumetric Density $\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{0} l_p^{-2}$ $\frac{1}{G} c^{0} l_p^{0} t_p^{-2}$	$\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{-1} l_p^{-1}$ $\frac{1}{G} c^{0} l_p^{0} t_p^{-1}$	$\frac{1}{G}$ $\frac{m_p}{l_p}\left(\frac{l_p}{t_p}\right)^{-2} l_p^{0}$ $\frac{1}{G} c^{0} l_p^{0} t_p^{0}$

Diagram 165: Canonical Planck's units on the periodic table of mechanical elements