MATH 731

Introduction to Hodge Theory

Contact information.

E-mail: *mmustata@umich.edu*

Office: EH 2842

Webpage: *www.math.lsa.umich.edu/˜mmustata*

- MWF 2-3 PM, East Hall 3096.
- Office hours: Tu 11-12 AM, Thu 1-2 PM, Fri 3-4 PM.

There will be no exams for this class.

The goal of the course is to give an introduction to the basic results in Hodge theory. Prerequisites: familiarity with algebraic varieties and sheaf cohomology (no familiarity with scheme theory is required) and with smooth manifolds (the tangent bundle, differential forms, integration).

Rough plan of the course.

1. The classical topology on a complex algebraic variety. Relation between algebraic properties and properties in the classical topology.
2. Holomorphic functions in several complex variables. The analytic space associated to an algebraic variety. GAGA (statements). Complex manifolds.
3. Hodge theory on Riemannian manifolds.
4. The Hodge decomposition on complex Kähler manifolds.
5. Polarizations and the Lefschetz decomposition. The Hodge Index theorem.
6. The category of (polarized) Hodge structures.
7. Local systems and vector bundles with integrable connection. Variations of Hodge structures.
8. Some homological algebra: spectral sequences, hypercohomology, rudiments of derived categories.
9. De Rham cohomology and Grothendieck's theorem (the compact case).
10. The De Rham complex with log poles and Grothendieck's theorem (the general case).
11. The mixed Hodge structure on the cohomology of a smooth variety.
12. The category of mixed Hodge structures.

Depending on the time available, we might discuss also other topics, such as:

13. Application to the proof of the Kodaira-Akizuki-Nakano vanishing theorem.
14. The construction of the mixed Hodge structure on the cohomology of singular varieties.

I will not follow very closely any book, but most of the material we will cover can be found in

Claire Voisin, Hodge theory and complex algebraic geometry. I.