Problem session 6

Problem 1. Let X be a Noetherian scheme. Show that if

$$0 \to \mathcal{F}' \to \mathcal{F} \to \mathcal{F}'' \to 0$$

is an exact sequence of $\mathcal{O}_X$-modules on X and if $\mathcal{F}'$ and $\mathcal{F}''$ are (quasi)coherent, then so is $\mathcal{F}$.

Problem 2. Show that if $f\colon X \to Y$ is an affine morphism of Noetherian separated schemes, then for every quasicoherent sheaf $\mathcal{F}$ on X and every $i \geq 0$ there is a canonical isomorphism

$$H^i(Y, f_*(\mathcal{F})) \simeq H^i(X, \mathcal{F}).$$

Problem 3. Let X be a Noetherian scheme.

(1) Show that X is affine if and only if so is X_{red}.
(2) Let $X_1, \ldots, X_r$ be the irreducible components of X, with the reduced scheme structure. Show that X is affine if and only if every X_i is affine.

Problem 4. Let $X = \mathbb{A}^2_k$, where k is an algebraically closed field. If $U = X \smallsetminus \{(0,0)\}$, use a suitable cover of U by affine open subsets to show that

$$H^1(U, \mathcal{O}_U) \simeq \bigoplus_{i,j<0} k \cdot x^i y^j.$$

In particular, it is infinite dimensional.

Problem 5. Let $\mathcal{F}$ be a coherent sheaf on a Noetherian scheme X. Show that if the support of $\mathcal{F}$ has dimension r, then $H^i(X, \mathcal{F}) = 0$ for every $i > r$.