Problem session 1

Problem 1. Describe the closed algebraic subsets of $\mathbb{A}^1$.

Problem 2. Show that every algebraically closed field is infinite.

Problem 3. Let k be an infinite field. Show that if $f_1, \ldots, f_m \in k[x_1, \ldots, x_n]$, then there is $x = (x_1, \ldots, x_n) \in k^n$ such that all $f_i(x_1, \ldots, x_n)$ are nonzero.

Problem 4. For m and $n \geq 1$, let us identify $\mathbf{A}^{mn}$ with the set of all matrices $B \in M_{m,n}(k)$. Show that the set

$$M^r_{m,n}(k) := \{B \in M_{m,n}(k) \mid \mathrm{rk}(B) \leq r\}$$

is a closed algebraic subset of $M_{m,n}(k)$.

Problem 5. Show that the following subset of $\mathbf{A}^3$

$$W_1 = \{(t, t^2, t^3) \mid t \in k\}$$

is a closed algebraic subset, and describe $I(W_1)$. Can you do the same for

$$W_2 = \{(t^2, t^3, t^4) \mid t \in k\}?$$

How about

$$W_3 = \{(t^3, t^4, t^5) \mid t \in k\}?$$

Problem 6. Show that every subspace of a Noetherian topological space is again Noetherian.