# Problem session 3

**Problem 1**. Let $X$ be a quasiaffine variety, and let $X_1, \ldots, X_r$ be its irreducible components. Show that there is a canonical isomorphism

$$K(X) \simeq K(X_1) \times \cdots \times K(X_r).$$

**Problem 2**. Let $f \colon X \dashrightarrow Y$ be a birational map between the quasiaffine varieties $X$ and $Y$. Show that there are open subsets $U \subseteq X$ and $V \subseteq Y$ such that $f$ induces an isomorphism $U \simeq V$.

**Problem 3**. Show that $\mathbf{A}^1$ is *not* isomorphic to any proper open subset of itself.

**Problem 4**. Let $X$ be the nodal curve

$$X = \{(u, v) \in \mathbf{A}^2 \mid u^2 = v^2(v+1)\}.$$

Show that $X$ is birational to $\mathbf{A}^1$, but that it is not isomorphic to $\mathbf{A}^1$. Hint: for the first part, consider the lines through the origin, and the their intersection with the curve; for the second part, you may consider the elements in $K(X)$ that are integral over $\mathcal{O}(X)$.

Prove the same facts for the cuspidal curve

$$Y = \{(u, v) \in \mathbf{A}^2 \mid u^2 = v^3\}.$$