Math 420

Homework Set 2

This assignment is due on Wednesday, September 25.

Problem 1. Let V be a vector space over F and suppose that $W_1, \ldots, W_r$ are linear subspaces of V. Show that if all W_i are finite-dimensional, then $W_1 + \ldots + W_r$ is finite-dimensional and

$$\dim(W_1 + \ldots + W_r) \leq \dim(W_1) + \ldots + \dim(W_r).$$

Problem 2. Suppose that U and V are linear subspaces of F^6.

i) Show that if $\dim(U) = 3$ and $\dim(V) = 4$, then $U \cap V \neq \{0\}$.
ii) Show that if $\dim(U) = \dim(V) = 4$, then there are two vectors u and v in $U \cap V$ such that neither of them is a scalar multiple of the other.

Problem 3. Let $u_1, \ldots, u_n$ be linearly independent elements in a vector space V.

i) Show that for every $v \in V$, if
$$W = \text{span}(u_1 + v, \ldots, u_n + v),$$
then $\dim(W) \geq n - 1$.
ii) Give an example for which this is an equality.

Problem 4. Let V be a vector space and U and W linear subspaces of V such that $V = U \oplus W$. Show that if $u_1, \ldots, u_n$ is a basis of U and $w_1, \ldots, w_m$ is a basis of W, then $u_1, \ldots, u_n, w_1, \ldots, w_m$ is a basis of V.