Math 420

Homework Set 1

This assignment is due on Wednesday, September 18.

Problem 1. Show that if V is a vector space over F, then for $a \in F$ and $v \in V$ we have $av = 0$ if and only if $a = 0$ or $v = 0$.

Problem 2. For each of the following subsets of F^3, determine whether it is a vector subspace. If it is, prove that this is the case; if it is not, illustrate why it is not.

i) $V_1 = \{(x_1, x_2, x_3) \in F^3 \mid x_1 + x_2 = 2x_3\}$.
ii) $V_2 = \{(x_1, x_2, x_3) \in F^3 \mid x_1 + 2x_2 + 3x_3 = 1\}$.
iii) $V_3 = \{(x_1, x_2, x_3) \in F^3 \mid x_1 = x_2 x_3\}$.

Problem 3. Prove that if W_1 and W_2 are vector subspaces of the vector space V, then $W_1 \cup W_2$ is a vector subspace of V if and only if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.

Problem 4. Prove or give a counterexample: if W_1, W_2, and W_3 are vector subspaces of the vector space V such that

$$W_1 + W_2 = W_1 + W_3,$$

then $W_2 = W_3$.

Problem 5. Let M be a nonempty set and $F(M)$ be the real vector space of all functions $f\colon M \to \mathbb{R}$, with addition and scalar multiplication of functions. Show that if $b \in M$ is any element and

$$W_1 = \{f \in F(M) \mid f(b) = 0\}$$

and

$$W_2 = \{f \in F(M) \mid f \text{ is a constant function}\},$$

then $F(M) = W_1 \oplus W_2$.