Physics 390: Homework 4

For full credit, show all your working.

1. **Relativistic quantum mechanics:** We have seen that Schrödinger correctly deduced his famous wave equation by taking the relation between frequency and wavenumber $\hbar\omega = \hbar^2 k^2/2m$ and making the replacements:

$$k \to -\mathrm{i}\frac{\partial}{\partial x}, \qquad \omega \to \mathrm{i}\frac{\partial}{\partial t}.$$

   (a) Starting from the relativistic relation between energy and momentum $E^2 = p^2c^2 + m^2c^4$ find the relativistic relation between angular frequency ω and wavenumber k of de Broglie waves.

   (b) Make the replacements above and hence derive a relativistic version of the Schrödinger equation.

   (c) Substitute the wave $\Psi = \Psi_0 \mathrm{e}^{\mathrm{i}(kx-\omega t)}$ into your wave equation and show that you can recover the relation between ω and k.

2. Problem 6-16 in Tipler & Llewellyn.

3. **Moving particles:** A particle (in one dimension) is in a state with energy $E_1 = \hbar\omega_1$ having wavefunction

$$\Psi(x,t) = \psi_1(x)\,\mathrm{e}^{-\mathrm{i}\omega_1 t},$$

   where $\psi_1(x)$ is the spatial part of the wavefunction.

   (a) Calculate the probability that the particle is measured to be between x and $x + \mathrm{d}x$ at time t. Show that this probability does not depend on time t, and hence that the particle isn't moving—the probability of finding it at position x never changes.

   (b) Now suppose that the particle is in a superposition of two states with different energies $E_1 = \hbar\omega_1$ and $E_2 = \hbar\omega_2$, so that the total wavefunction is

$$\Psi(x,t) = \psi_1(x)\,\mathrm{e}^{-\mathrm{i}\omega_1 t} + \psi_2(x)\,\mathrm{e}^{-\mathrm{i}\omega_2 t}.$$

   Again find the probability that the particle is measured to be between x and $x + \mathrm{d}x$ at time t. Show that the probability now does depend on time.

   Thus in order to move, a particle has to be in a mixture of different energy states. If it is only in a single energy state then it is necessarily not moving.

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4. **Uncertainty relation:** Suppose that we measure the uncertainty in position and momentum of a particle by their standard deviations:

$$\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}, \qquad \sigma_p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}.$$

   (a) Find σ_x and σ_p for the ground state of the 1D infinite square well in terms of the length L of the well.

   (b) Find $\sigma_x \sigma_p$.