Notecard Ideas

f	f'	f''
inc	+	
dec	−	
max	+ → −	
min	− → +	
Conc up	inc	+
Conc dn	dec	−
inflection points	max	+ → −
	min	− → +

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f(h(x))h'(x) - f(g(x))g'(x)$$

For partial fractions:

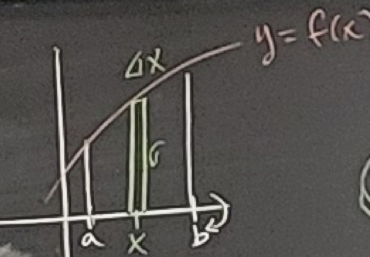
$$\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + C$$

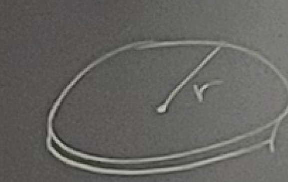
$$\int \frac{dx}{(ax+b)^2} = \frac{1}{a}\cdot\frac{-1}{ax+b} + C$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$$

$$\int \frac{x}{x^2+a^2}\,dx = \frac{1}{2}\ln|x^2+a^2| + C$$

Slicing:





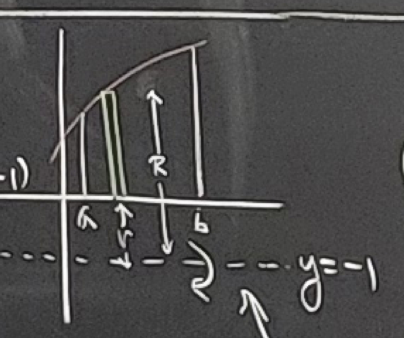
Discs:

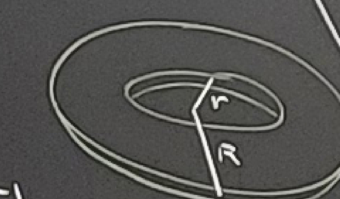
r = radius of slice = f(x)

Vol of slice = $\pi r^2 \Delta x = \pi f(x)^2 \Delta x$

Total volume = $\int_a^b \pi f(x)^2\,dx$

Washers:





r = inner radius of slice = 1

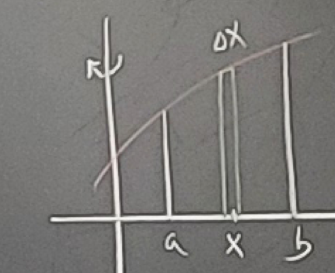
R = outer " " " = f(x) − (−1)

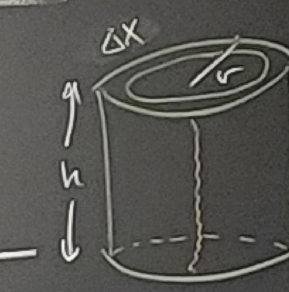
Vol of slice = $\pi(R^2 - r^2)\Delta x$

$= \pi\left((f(x)+1)^2 - 1^2\right)\Delta x$

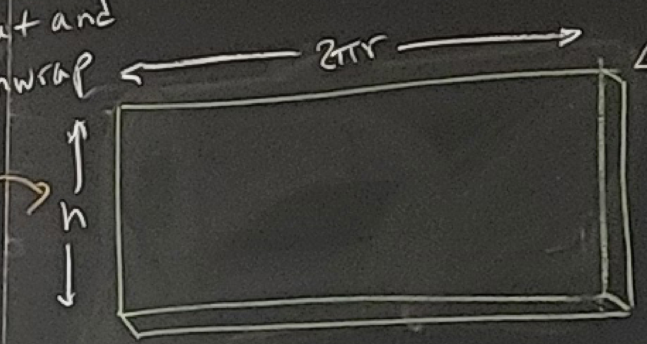
Total volume = $\int_a^b \pi\left((f(x)+1)^2 - 1^2\right)dx$

Cylindrical Shells:





Cut and unwrap



radius of shell = r

= dist from strip to axis = x

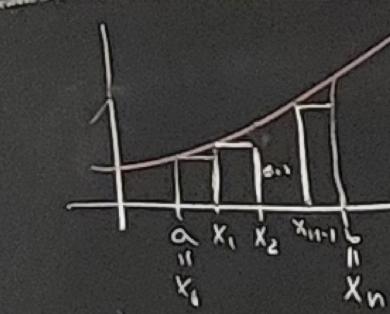
height of shell = h = f(x)

Volume of shell = $(2\pi r)(h)\Delta x$

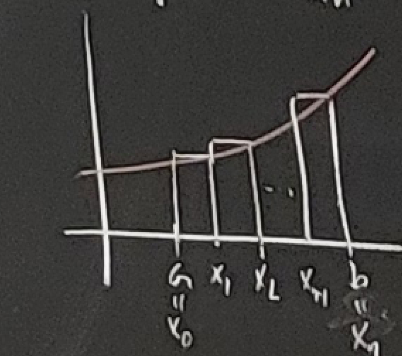
$= 2\pi x f(x)\Delta x$

Total Volume = $\int_a^b 2\pi x f(x)\,dx$

$$\Delta x = \frac{b-a}{n}$$



$$\text{LEFT}(n) = \Delta x\left[f(x_0) + f(x_1) + \dots + f(x_{n-1})\right]$$



$$\text{RIGHT}(n) = \Delta x\left[f(x_1) + f(x_2) + \dots + f(x_n)\right]$$

f inc: $\text{LEFT}(n) < \int_a^b f(x)\,dx < \text{RIGHT}(n)$

f dec: $\text{RIGHT}(n) < \int_a^b f(x)\,dx < \text{LEFT}(n)$

$$|\text{LEFT}(n) - \text{RIGHT}(n)| = |\Delta x(f(x_0) - f(x_n))|$$

$$= \frac{b-a}{n}|f(a) - f(b)|$$

So for increasing or decreasing functions, can make sure LEFT and RIGHT are within k of $\int f(x)dx$ by making sure n is big enough that $\frac{b-a}{n}|f(a)-f(b)| \le k$.

Cylindrical Shells:

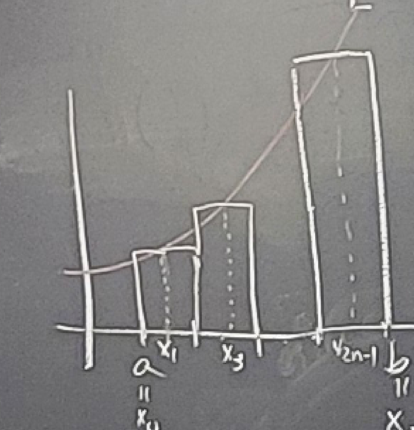
Cut and unwrap

radius of shell = r
= dist from strip to axis = x
height of shell = h = f(x)
Volume of shell = $(2\pi r)(h)\Delta x$
$= 2\pi x f(x)\Delta x$
Total Volume = $\int_a^b 2\pi x f(x)\,dx$

$\Delta x = \frac{b-a}{n}$
$\text{LEFT}(n) = \Delta x\left[f(x_0) + f(x_1) + \dots + f(x_{n-1})\right]$
$\text{RIGHT}(n) = \Delta x\left[f(x_1) + f(x_2) + \dots + f(x_n)\right]$
f inc: $\text{LEFT}(n) < \int_a^b f(x)\,dx < \text{RIGHT}(n)$
f dec: $\text{RIGHT}(n) < \int_a^b f(x)\,dx < \text{LEFT}(n)$
$|\text{LEFT}(n) - \text{RIGHT}(n)| = |\Delta x(f(x_0) - f(x_n))|$
$= \frac{b-a}{n}|f(a) - f(b)|$

$$\text{TRAP}(n) = \frac{1}{2}\left(\text{LEFT}(n) + \text{RIGHT}(n)\right)$$

$$= \Delta x\left[\frac{1}{2}f(x_0) + f(x_1) + f(x_2) + \dots + f(x_{n-1}) + \frac{1}{2}f(x_n)\right]$$



$$\text{MID}(2n) = \frac{b-a}{2n}\left[f(x_1) + f(x_3) + \dots + f(x_{2n-1})\right]$$

TRAP and MID are usually better than LEFT and RIGHT.

f conc up: $\text{MID}(n) < \int_a^b f(x)\,dx < \text{TRAP}(n)$

f conc dn: $\text{TRAP}(n) < \int_a^b f(x)\,dx < \text{MID}(n)$

$$\text{Arclen} = \int_a^b \sqrt{1 + f'(x)^2}\,dx$$