

Note card Ideas

F	F'	F''
inc	+	
dec	-	
max	+ → -	
min	- → +	
Conc up	inc	+
Conc dn	dec	-
inflection points	max	+ → -
	min	- → +

$$\frac{d}{dx} \int_{y(x)}^{h(x)} f(t) dt = f(h(x))h'(x) - f(y(x))y'(x)$$

For partial fractions:

$$\int \frac{dx}{ax+b} = \frac{1}{a} \ln|ax+b| + C$$

$$\int \frac{dx}{(ax+b)^2} = \frac{1}{a} \cdot \frac{-1}{ax+b} + C$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$$

$$\int \frac{x}{x^2+a^2} dx = \frac{1}{2} \ln|x^2+a^2| + C$$

Slicing:

Disks:

r = radius of slice = f(x)

Vol of slice = $\pi r^2 \Delta x = \pi f(x)^2 \Delta x$

Total volume = $\int_a^b \pi f(x)^2 dx$

Washers:

r = inner radius of slice = 1

R = outer " " = f(x) - (-1)

Vol of slice = $\pi(R^2 - r^2) \Delta x$

= $\pi((f(x)+1)^2 - 1^2) \Delta x$

Total volume = $\int_a^b \pi((f(x)+1)^2 - 1^2) dx$

Cylindrical Shells:

radius of shell = r

= dist from strip to axis = x

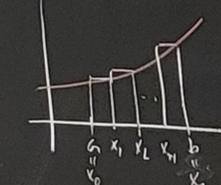
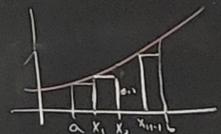
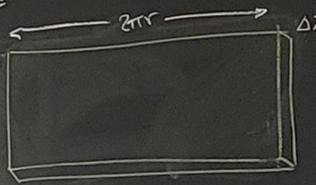
height of shell = h = f(x)

Volume of shell = $(2\pi r)(h) \Delta x$

= $2\pi x f(x) \Delta x$

Total volume = $\int_a^b 2\pi x f(x) dx$

Cut and unroll



$\Delta x = \frac{b-a}{n}$

LEFT(n) = $\Delta x [f(x_0) + f(x_1) + \dots + f(x_{n-1})]$

RIGHT(n) = $\Delta x [f(x_1) + f(x_2) + \dots + f(x_n)]$

f inc: $\text{LEFT}(n) < \int_a^b f(x) dx < \text{RIGHT}(n)$

f dec: $\text{RIGHT}(n) < \int_a^b f(x) dx < \text{LEFT}(n)$

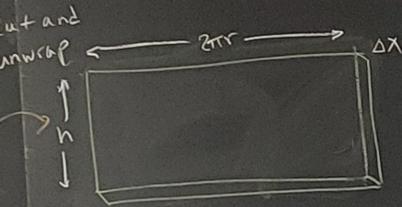
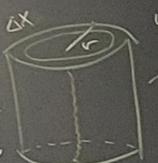
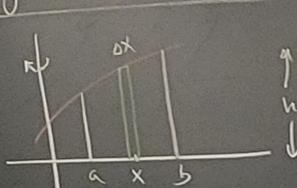
$|\text{LEFT}(n) - \text{RIGHT}(n)| = |\Delta x (f(x_0) - f(x_n))|$

= $\frac{b-a}{n} |f(a) - f(b)|$ so for increasing or decreasing functions, can make sure LEFT and RIGHT are within k of $\int_a^b f(x) dx$ by making sure n is big enough that $\frac{b-a}{n} |f(a) - f(b)| \leq k$

- Oliver: Otter
- Kareem: Mole/shrew
- Alexa: Bear
- Devayani: Raccoon
- V. nichilla
- Kyan: ...
- Alex S: Sloth
- Ilir: Ibis
- Dima: Pony
- Bobr: Beaver
- Ch: Flamingo
- Nuv: D...
- Sophia: Purrin frog
- Alissa Panda
- Mawice: Seal
- Yoram: Lemur
- Grant T: Goose
- Grant K: Gruff
- Sophia: Purrin frog
- Alex W: Elephant
- Formn: Panther (ent)
- Jenny: Mouse
- Luke M: Crane
- Fitz: Hare
- J... weasel
- Max W: Lemming
- Rohan: Thon Ak
- Aaron P: Platypus
- Kartik: Meerkat
- Vivake: Cat/fish
- Urib: Manatee
- Rhovan: ...

m-die-sees Daytabassitua...

Cylindrical Shells:



radius of shell = r

= dist from strip to axis = x

height of shell = h = f(x)

Volume of shell = $(2\pi r)(h) \Delta x$

= $2\pi x f(x) \Delta x$

Total volume = $\int_a^b 2\pi x f(x) dx$



$\Delta x = \frac{b-a}{n}$

LEFT(n) = $\Delta x [f(x_0) + f(x_1) + \dots + f(x_{n-1})]$

RIGHT(n) = $\Delta x [f(x_1) + f(x_2) + \dots + f(x_n)]$

f inc: $\text{LEFT}(n) < \int_a^b f(x) dx < \text{RIGHT}(n)$

f dec: $\text{RIGHT}(n) < \int_a^b f(x) dx < \text{LEFT}(n)$

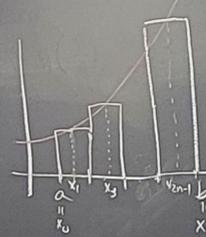
$|\text{LEFT}(n) - \text{RIGHT}(n)| = |\Delta x (f(x_0) - f(x_n))|$

= $\frac{b-a}{n} |f(a) - f(b)|$

so for increasing or decreasing functions, can make sure LEFT and RIGHT are within k of $\int_a^b f(x) dx$ by making sure n is big enough that $\frac{b-a}{n} |f(a) - f(b)| \leq k$

TRAP(n) = $\frac{1}{2} (\text{LEFT}(n) + \text{RIGHT}(n))$

= $\Delta x \left[\frac{1}{2} f(x_0) + f(x_1) + f(x_2) + \dots + f(x_{n-1}) + \frac{1}{2} f(x_n) \right]$



MID(2n) = $\frac{b-a}{2n} [f(x_1) + f(x_3) + \dots + f(x_{2n-1})]$

TRAP and MID are usually better than LEFT and RIGHT.

f conc up: $\text{MID}(n) < \int_a^b f(x) dx < \text{TRAP}(n)$

f conc dn: $\text{TRAP}(n) < \int_a^b f(x) dx < \text{MID}(n)$

Arclen = $\int_a^b \sqrt{1+f'(x)^2} dx$