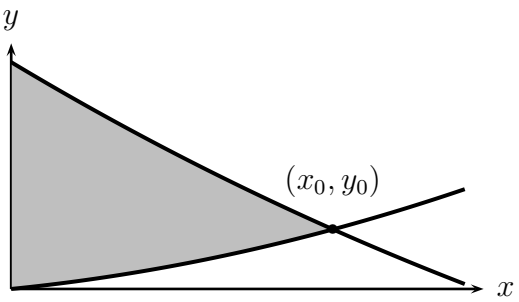
Douglass Houghton Workshop, Section 2, Tue 04/14/26

Worksheet To Infinity, and Beyond

1. (From the Fall, 2010 Math 116 final) The graph shows the area between the graphs of $f(x) = 6\cos(\sqrt{2x})$ and $g(x) = x^2 + x$. Let (x_0, y_0) be the intersection point between the graphs of $f(x)$ and $g(x)$.



   (a) Compute $P(x)$, the function containing the first three nonzero terms of the Taylor series about $x = 0$ of $f(x) = 6\cos\left(\sqrt{2x}\right)$.

   (b) Use $P(x)$ to approximate the value of x_0.

   (c) Use $P(x)$ and the value of x_0 you computed in the previous question to write an integral that approximates the value of the shaded area. Find the value of this integral.

   (d) Graph $f(x)$ and $g(x)$ in your calculator. Use the graphs to find an approximate value for x_0.

   (e) Write a definite integral in terms of $f(x)$ and $g(x)$ that represents the value of the shaded area. Find its value using your calculator.

2. (Fall, 2014 (the robot chicken semester)) Franklin, your robot, is zipping around the kitchen making his famous "Definitely Not Poison!" soup. His coordinates in the xy-plane are given by the parametric equations

$$x = t^2 - t \qquad\qquad y = -\sin(\pi t)$$

   t seconds after he starts making soup. Assume that both x and y are measured in meters.

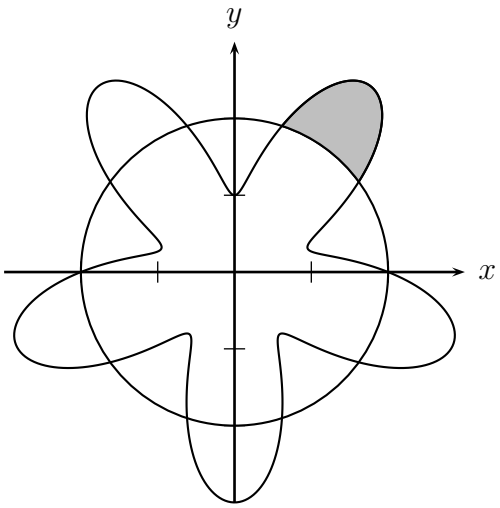
   (a) Calculate dx/dt and dy/dt.

   (b) Find all times when Franklin's velocity is zero.

   (c) Find Franklin's **speed** when $t = 2$ seconds.

   (d) Write an integral that gives the distance traveled by Franklin during his first 5 seconds of zipping around.

3. (Adapted from a Fall, 2010 Math 116 Exam) In the picture to the right, the graphs of $r = 2$ and $r = 2 - \sin(5\theta)$ are shown.



   (a) Write a definite integral that computes the shaded area.

   (b) Compute the area exactly.

   (c) Write an integral for the length of the boundary of the shaded area.

   (d) Get an approximate answer for that length, using your calculator.

4. (From a Winter, 2014 Math 116 Exam Problem)

   (a) What is the value of $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n}}{n!}$?

   (b) What is the value of $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n}(-1)^n}{(2n+1)!}$?

   (c) Suppose that $1 + x - \dfrac{1}{4}x^2 + \dfrac{1}{10}x^3$ is the third-degree Taylor polynomial for a function $f(x)$. What must the graph of f look like near $x = 0$?

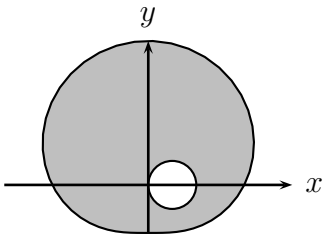
   (d) What is the Taylor series of $2xe^{x^2}$ centered at $x = 0$?

   (e) What is the interval of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(x+5)^n 5^{-n}}{n+5}$?

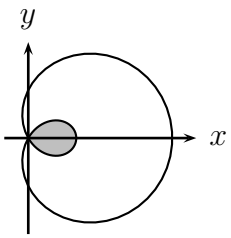
5. (This problem is from a Fall, 2014 Math 116 exam. For some reason, all the exams that term were about robots and chickens.)

   Consider the polar curves

$$r = \cos\theta \qquad \text{and} \qquad r = \sin\theta + 2.$$



   (a) Franklin's robot army occupies the shaded region between these two curves. Find the area occupied by Franklin's robot army.



   (b) Your friend, Kazilla, pours her magic potion on the ground. Suddenly, a flock of wild chickens surrounds you. The chickens occupy the shaded region enclosed within the polar curve $r = 1 + 2\cos\theta$ as shown below. Find an integral for the perimeter of the region occupied by the flock of wild chickens.