

Worksheet Love looks not with the eyes, but with the mind

And therefore is winged Cupid painted blind

With a lot of hard work, we filled the table to the right with the values of $\int_{-\pi}^{\pi} f(x)g(x) dx$, where f is the row and g is the column, and m and n are positive integers.

	1	$\sin(nx)$	$\cos(nx)$
1	2π	0	0
$\sin(mx)$	0	$\begin{cases} \pi & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$	0
$\cos(mx)$	0	0	$\begin{cases} \pi & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$

The implication was that if we know a function is of the form

$$(1) \quad f(x) = a_0 + a_1 \cos(x) + a_2 \cos(2x) + a_3 \cos(3x) + \dots + b_1 \sin(x) + b_2 \sin(2x) + b_3 \sin(3x) + \dots$$

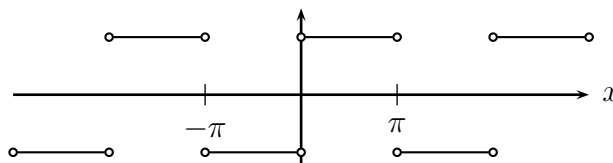
then integrating against a sine or cosine function makes almost all the terms 0, so for $n \geq 1$:

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx, \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx, \quad \text{and} \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx.$$

1. Consider the square wave:

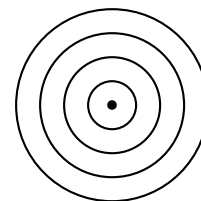
$$f(x) = \begin{cases} -1 & \text{if } -\pi < x < 0 \\ 1 & \text{if } 0 < x < \pi \end{cases}$$

and that pattern is repeated every 2π .



Suppose the square wave can be written in terms of sines and cosines, as in Equation (1) above. Find the a_n and the b_n .

2. Consider a game of “continuous darts.” The board is circular, as you expect, with radius 1. The goal is to get as close to the middle as possible. If a dart lands a distance r from the bullseye, its score is $1 - r$. (So every number between 0 and 1 is a possible score.)



A novice player throws a dart which lands randomly somewhere on the board. That means that for any region R on the board,

$$\text{Prob}(\text{dart lands in } R) = \frac{\text{area of } R}{\text{area of board}}.$$

(a) Fill in the table with the probabilities that the dart scores **below** the given value.

x	0	1/4	1/2	3/4	1
Prob(score $< x$)					

(b) Let x be any number. Find the probability that the score is less than x .

(c) Find the median score.