

Worksheet Just Do It

1. We've done a few integrals with sines and cosines. Fill in this table:

	1	$\sin(nx)$	$\cos(nx)$
1			
$\sin(mx)$			
$\cos(mx)$			

with the values of $\int_{-\pi}^{\pi} f(x)g(x) dx$, where f is the row and g is the column.

2. Let $h(x) = 5 + 2 \cos(x) + \sin(x) - 5 \cos(2x) + 3 \sin(2x)$.

(a) Use your calculator to compute:

$$\begin{aligned} \int_{-\pi}^{\pi} h(x) dx &= & \int_{-\pi}^{\pi} h(x) \cos(2x) dx &= \\ \int_{-\pi}^{\pi} h(x) \cos(x) dx &= & \int_{-\pi}^{\pi} h(x) \sin(2x) dx &= \\ \int_{-\pi}^{\pi} h(x) \sin(x) dx &= & &= \end{aligned}$$

(b) Explain the results using the table above.

3. Predict what the integrals in (2a) above will be if we change $h(x)$ to

$$h(x) = 2 + 3 \cos(x) - 7 \sin(x) - 4 \cos(2x) + \sin(2x).$$

4. Generalize: What will those integrals be if

$$h(x) = a_0 + a_1 \cos(x) + b_1 \sin(x) + a_2 \cos(2x) + b_2 \sin(2x).$$

5. (This problem appeared on a Winter, 2010 Math 116 Exam) Calculate the following improper integrals.

$$(a) \int_0^2 \frac{3}{x^{1/3}} dx \qquad (b) \int_0^2 \frac{e^{-1/x}}{x^2} dx$$

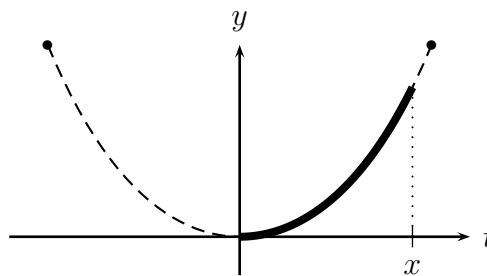
6. Prove whether the following improper integrals converge or diverge.

$$(a) \int_3^{\infty} \frac{\ln(x)}{x^2} dx \qquad (b) \int_0^{\infty} \frac{3}{4x^2 + 5\sqrt{x}} dx$$

7. We've made some progress finding the shape of a hanging chain. If the shape is given by $F(x)$, then by considering forces and arc length we've shown that

$$F''(x) = \frac{\delta g}{T_0} \sqrt{1 + F'(x)^2}$$

where T_0 is the tension at the bottom of the chain, δ is the mass density of the chain, and g is acceleration due to gravity (all constants). Where to go from here? We'd like to find a formula for $F(x)$.



- (a) Hmm. No F s, only F' s. And lots of constants. Let $y = F'(x)$, and put all the constants together into one constant. That should make it look better.
- (b) What is y when x is 0? Now you have an initial value to go with your differential equation.
- (c) Separate the variables and solve the differential equation.
8. (This problem appeared on a Winter, 2009 Math 116 exam) The quantity

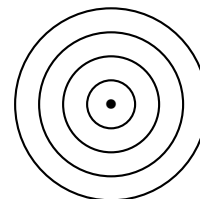
$$\int_1^{\infty} \frac{dx}{\sqrt{(a^2 + x)(b^2 + x)(c^2 + x)}}$$

roughly models the resistance that football-shaped plankton encounter when falling through water. Note that $a = 1$, $b = 2$, and $c = 3$ are constants that describe the dimensions of the plankton. Find a value of M for which

$$\int_1^M \frac{dx}{\sqrt{(a^2 + x)(b^2 + x)(c^2 + x)}}$$

differs from the original model of resistance by at most 0.001. Hint: make use of the integral and the comparison test.

9. Consider a game of "continuous darts." The board is circular, as you expect, with radius 1. The goal is to get as close to the middle as possible. If a dart lands a distance r from the bullseye, its score is $1 - r$. (So every number between 0 and 1 is a possible score.)



A novice player throws a dart which lands randomly somewhere on the board. That means that for any region R on the board,

$$\boxed{\text{Prob}(\text{dart lands in } R) = \frac{\text{area of } R}{\text{area of board}}}$$

- (a) Fill in the table with the probabilities that the dart scores **below** the given value.

x	0	1/4	1/2	3/4	1
Prob(score < x)					

- (b) Let x be any number. Find the probability that the score is less than x .
- (c) Find the median score.