Douglass Houghton Workshop, Section 2, Thu 01/29/26

Worksheet Fate, or Folly?

1. Find $\dfrac{d}{dx}\displaystyle\int_{x^2}^{\ln(x)} e^{t^2}\,dt$.

2. A ball at an initial height h_0 is thrown straight up into the air, with an initial velocity v_0. Gravity causes the ball to accelerate downward at a constant rate, g. (This might be on another planet, so use g rather than $9.8\,\text{m/sec}^2$.)

   (a) Find $v(t)$, the upward velocity of the ball at time t.

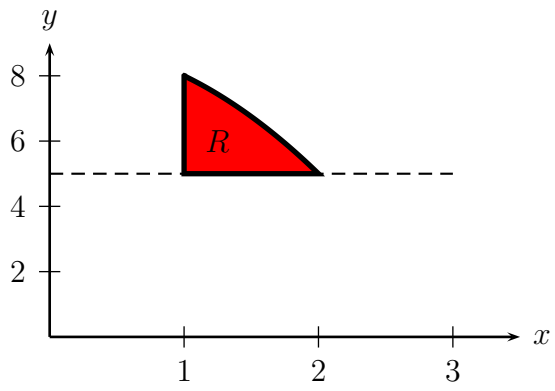
   (b) Find $h(t)$, the height of the ball at time t.

   (c) Calculate the quantity $mgh(t)+\frac{1}{2}mv(t)^2$. What do you notice about your answer?

   (d) Use part (c) to calculate the maximum height of the ball. Check using your Math 115 optimization skillz.

3. Compute the length of the ladder curve, $x^{2/3}+y^{2/3}=1$.

4. (Adapted from a Fall, 2011 Math 116 Exam) Consider the region R in the xy-plane bounded by the curves $y=9-x^2$, $x=1$, and $y=5$. This region is pictured below.



   (a) Find the area of R.

   (b) Find the volume obtained by rotating R about the y-axis. Do it with both shells and washers, and verify that the answer is the same.

   (c) Find the volume of the solid whose base is R and whose cross-sections perpendicular to the x-axis are squares.

5. (From the Fall, 2018 Math 116 Final Exam) Consider the curve $y=\sqrt{1-x^2}$. Suppose a paperweight is formed by rotating this curve around the x-axis. This paperweight has a density given by $\delta(x)=2+\cos(x)\,\text{g/cm}^3$. The units on both axes are centimeters (cm).

   (a) Write an expression that gives the approximate mass, in grams, of a slice of the paperweight taken perpendicular to the x-axis at coordinate x with thickness Δx. (Assume that Δx is small but positive.) Your expression should <u>not</u> involve any integrals.

   (b) Write an expression involving one or more integrals that gives the mass, in grams, of the paperweight.

6. Evaluate $\displaystyle\int_{-\pi}^{\pi}\sin(mx)\cos(nx)\,dx$ where m and n are positive integers. (You might want to graph a few examples.)

7. Find $\displaystyle\int_{-\pi}^{\pi}\cos(mx)\cos(nx)\,dx$, given that m and n are positive integers.

8. (Fall, 2007) For this problem, $\int_1^5 g(x)\,dx = 12$ and $f(x) = 2x - 9$. Some values of $g(x)$ are:

x	1	2	3	4	5
$g(x)$	0.1	1.5	2	5	10

(a) Find $\int_5^7 g(f(x))\,dx$. (b) Find $\int_1^5 f(x)g'(x)\,dx$.

(c) Find $\int_1^5 \dfrac{g'(x)}{g(x)(g(x)+1)}\,dx$.

9. (Winter, 2003) Below are the graphs of several functions $f(x)$, $g(x)$, $h(x)$, $i(x)$, $j(x)$, and $k(x)$. Do not assume that the y-axis scales on these graphs are equal or even comparable. We have calculated LEFT(6), RIGHT(6), TRAP(6), and MID(6) for four of these six functions. Label each column with the name of the function estimated in that column.

Function:				
LEFT(6):	64.2	.328	.255	80.0
RIGHT(6):	65.8	.444	.421	80.0
TRAP(6):	65.0	.386	.338	80.0
MID(6):	65.0	.388	.331	80.0