Douglass Houghton Workshop, Section 2, Thu 01/22/26

Worksheet Do the Right Thing

1. Let's practice some integration by parts.

   (a) $\int x^2 e^x \, dx$

   (b) $\int \ln x \, dx$

   (c) $\int e^x \sin x \, dx$

   (d) $\int_0^1 \tan^{-1}(x) \, dx$ Hint: $\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$

2. Last time we showed that if a quadratic function $f(x)$ goes through the points in the table to the right, then

x	$f(x)$
-1	R
0	S
1	T

$$f(x) = \left(\frac{R}{2} - S + \frac{T}{2}\right) x^2 + \left(\frac{T-R}{2}\right) x + S$$

   Find $\int_{-1}^{1} f(x) \, dx$ in terms of R, S, and T.

3. Write out the terms of each series.

   (a) $\sum_{i=0}^{6} i$

   (b) $\sum_{k=3}^{10} (-1)^k k^2$

   (c) $\sum_{\ell=1}^{5} x^{2\ell-1} \ell!$

   (d) $\sum_{m=0}^{\infty} \frac{(2m)!}{m!m!} x^m$

4. Put the series in sigma ($\sum$) notation:

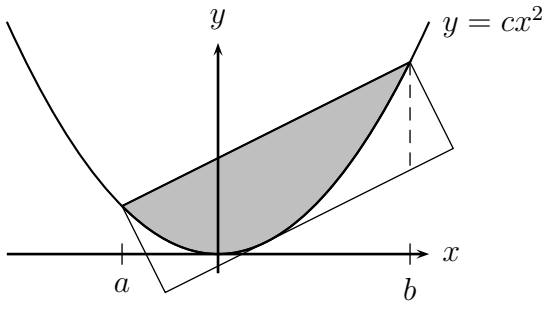
   (a) $9 + 16 + 25 + 36 + \cdots + 100$

   (b) $\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \cdots$

5. Last time we found that:

   $$\text{SHADED AREA} = c(b-a)^3/6$$
   $$\text{SLOPE OF LINES} = c(a+b)$$
   $$\text{EQUATION OF TOP LINE} : y = c(a+b)x - abc$$

   For reasons unexplained, we want the area of the box containing the shaded area.



   (a) The bottom line is tangent to the curve. Find the point of tangency and the equation of the tangent line.

   (b) Imagine slicing a triangle off the right side of the box, along the dashed line, and gluing it onto the left side of the box. What shape do you have? Does this suggest a way to find the area of the box?

   (c) Find the area of the box.

6. Consider the **gamma function**: $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1}\, dt$, for $x > 0$.

   (a) Use integration by parts to prove that $\Gamma(x+1) = x\Gamma(x)$.

   (b) Show that $\Gamma(1) = 1$. Then fill in this chart, using part (a):

x	1	2	3	4	5	6
$\Gamma(x)$						

   (c) So if x is a positive integer, what is $\Gamma(x)$?

7. Evaluate $\int \cos^2(x)\, dx$. You can use integration by parts, or try to simplify it with a trig identity. Hints: $\cos^2 x + \sin^2 x = 1$, and $\cos^2(x) - \sin^2(x) = \cos(2x)$.

8. Suppose you want to calculate

$$I = \int \frac{2x+6}{(x-3)(x-7)}\, dx.$$

   (a) Hmm, that looks hard. But what if you had something like this:

$$\int \left(\frac{A}{x-3} + \frac{B}{x-7} \right) dx$$

   where A and B are constants. You could find that, right? Do it, and get an answer in terms of A and B.

   (b) That wasn't so bad. And it's related to the initial problem. What do you get if you add the two fractions in part (a), by finding a common denominator?

   (c) Now, what do A and B have to be to make the answer to part (b) equal the integrand of I?

   (d) Now find I.

   (e) When does this trick work?

9. Currently 95% of Michigan kindergarteners have been vaccinated for measles. The measles vaccine is 93% effective, meaning that 7% of vaccinated children who are exposed to the disease will contract it, and the rest will not. That contrasts with a 10% immunity among unvaccinated children.

   (a) Suppose that all children in the community are exposed to the measles vaccine, and fill in the following table of possibilities. For instance, the upper-left corner is the probability that a randomly-chosen child is vaccinated *and* contracts measles.

		Vaccinated?	
		Yes	No
Gets measles?	Yes		
	No		

   (b) What proportion of the students who contract measles were vaccinated?

   (c) What does that mean about whether you should vaccinate your child?