Douglass Houghton Workshop, Section 2, Tue 01/20/26

Worksheet Catnip

1. Let's practice some substitution.

   (a) $\displaystyle\int z(z+3)^{1/3}\,dz$

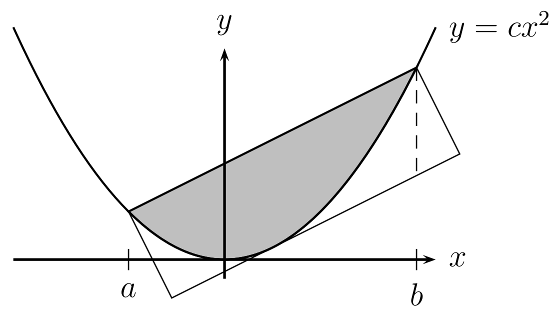
   (b) $\displaystyle\int \frac{dx}{2+2\sqrt{x}}$

   (c) $\displaystyle\int_{-3}^{0} (z+2)\sqrt{1-z}\,dz$

   (d) $\displaystyle\int_{4}^{12} \frac{3x-2}{\sqrt{2x+1}}\,dx$

2. Last time we found that:

   $$\text{SHADED AREA} = c(b-a)^3/6$$
   $$\text{SLOPE OF LINES} = c(a+b)$$
   $$\text{EQUATION OF TOP LINE}: y = c(a+b)x - abc$$



   For reasons unexplained, we want the area of the box containing the shaded area.

   (a) The bottom line is tangent to the curve. Find the point of tangency and the equation of the tangent line.

   (b) Imagine slicing a triangle off the right side of the box, along the dashed line, and gluing it onto the left side of the box. What shape do you have? Does this suggest a way to find the area of the box?

   (c) Find the area of the box.

3. Find $\displaystyle\frac{d}{dx}\int_{\cos x}^{3} e^{t^2}\,dt.$

4. Find $\displaystyle\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt.$

5. Suppose you have a function $f(x)$. You know:

   - f is a quadratic. That is, $f(x) = ax^2 + bx + c$ for some constants a, b, and c.
   - How to measure $f(-1)$, $f(0)$, and $f(1)$.

   You want to know $\displaystyle\int_{-1}^{1} f(x)\,dx.$

   (a) Let R, S, and T be the values you measure for $f(-1)$, $f(0)$, and $f(1)$. What are R, S, and T in terms of a, b, and c?

   (b) Find a formula for $f(x)$. That is, find a, b, and c in terms of R, S, and T.

   (c) Find $\displaystyle\int_{-1}^{1} f(x)\,dx$ in terms of R, S, and T.

6. A ball at an initial height h_0 is thrown straight up into the air, with an initial velocity v_0. Gravity causes the ball to accelerate downward at a constant rate, g. (This might be on another planet, so use g rather than $9.8\,\mathrm{m/sec}^2$.)

   (a) Find $v(t)$, the upward velocity of the ball at time t.

   (b) Find $h(t)$, the height of the ball at time t.

   (c) Calculate the quantity $mgh(t)+\frac{1}{2}mv(t)^2$. What do you notice about your answer?

   (d) Use part (c) to calculate the maximum height of the ball. Check using your Math 115 optimization skillz.

7. Find $\int_{-\pi}^{\pi} \cos^2(x)\sin(x)\,dx$, and then find $\int_{-\pi}^{0} \cos^2(x)\sin(x)\,dx$.

8. Suppose we want to compute $\displaystyle\int \frac{2x+5}{x^2-2x-3}\,dx$.

   (a) Factor the denominator into something like $(x-\alpha)(x-\beta)$.

   (b) Now reverse the process of finding a common denominator. That is, imagine the integrand can be written as

   $$\frac{A}{x-\alpha}+\frac{B}{x-\beta}$$

   for some constants A and B. Find what A and B have to be to make that the same as $\frac{2x+5}{x^2-2x-3}$.

   (c) Finally, rewrite the integral using the sum you found, and use substitution to solve it.