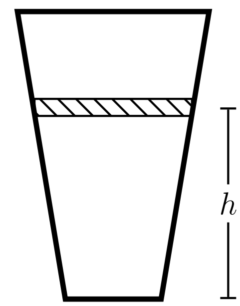
Douglass Houghton Workshop, Section 2, Tue 01/13/26

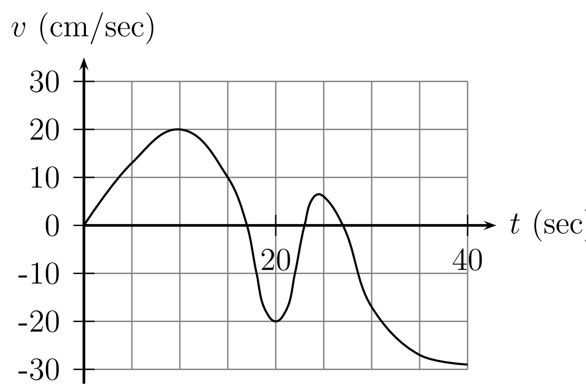
Worksheet Batrachomyomachia

1. Suppose a Solo cup has radii R_1 cm and R_2 cm and height H cm.
   (a) Consider a disk-shaped slice of the cup which is a height h above the bottom. What is its radius, in terms of h? Hint: The sides of the cup are straight, so the radius is a linear function of h.
   (b) If the thickness of the disk is Δh, what is its volume?
   (c) Write the volume of the cup as a sum of slices in sigma notation, and then turn your sum into an integral. (Don't skip the sum step.)
   (d) Evaluate your integral.
   (e) Simplify your formula in two special cases: where $R_1 = 0$ and where $R_1 = R_2$.
   (f) Compute the numerical volume when the diameters are 6 cm and 9 cm and the height is 12 cm.



2. (This is derived from Problem 47 on page 295 of your book.)

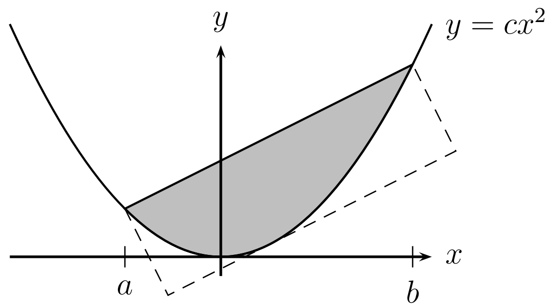
   A mouse is trapped in a psychologist's experiment. She moves back and forth in a straight tunnel. The cruel experimenter attracts the mouse with bits of cheese at one end or the other. Sometimes he also puts a frog in the tunnel to scare the mouse away, because mice are terrified of frogs. The graph of the mouse's velocity, v, is given to the right, with a positive velocity corresponding to motion toward the right end.



   Tell the story from the mouse's point of view. You might write it as a timeline, explaining what happened when. Make up explanations for all the significant features of the graph.

3. The breathing of a frog is cyclic, and when it is relaxed (because no mice are around), the time from beginning of inhalation to end of exhalation is about 5 seconds. The maximum rate of air flow into the lungs is about 50 milliliters per second.
   (a) Write a trigonometric function that models the rate of air flow into the lungs.
   (b) Use this function to find the maximum amount of inhaled air in the lungs.

4. We're interested in the figure shown here. Last time we found that the slope of the solid line is $c(a+b)$, and the area of the trapezoid under the line is $\frac{1}{2}c(b-a)(a^2+b^2)$.



   (a) Find the area of the shaded region in the picture to the right. Make the answer as simple as possible.

   (b) Find the area of the dashed rectangle, which is tangent to the curve.

5. The Michigan Lottery offers several exciting and fun ways to spend money. Let's calculate the odds of one of them.

   Daily 3 Three bins, numbered 1, 2, and 3, each contain ten ping-pong balls, numbered 0 through 9. A ball is chosen from each bin, so that the result of the drawing is a 3-digit number. Players likewise choose a 3-digit number to play.

   (a) What is the probability of getting all three digits correct?

   (b) You can also play your numbers "boxed". That means that if you match the three digits *in any order*, you win. What is the probability of winning a boxed ticket? Does it depend on what numbers you play?

6. The **expectation** of a particular bet on a particular game is the average amount you'll win if you play many times.

   (a) Suppose among a certain group of people, 54% get 1 scoop of ice cream, 32% get 2 scoops, and 14% get 3 scoops. What is the average number of scoops per person?

   (b) If you bet $1 on red in Roulette, there are 2 possbile outcomes. Write down the probabilities and payoffs for each, and find the exected payoff.

   (c) Find the expectation of The Michigan Lottery's non-boxed pick-3 game. The cost of a ticket is $1, and if your number comes up you can turn in the ticket for $500.

7. Suppose you have a function $f(x)$. You know:

   - f is a quadratic. That is, $f(x) = ax^2 + bx + c$ for some constants a, b, and c.
   - How to measure $f(-1)$, $f(0)$, and $f(1)$.

   You want to know $\int_{-1}^{1} f(x)\,dx$.

   (a) Let R, S, and T be the values you measure for $f(-1)$, $f(0)$, and $f(1)$. What are R, S, and T in terms of a, b, and c?

   (b) Find a formula for $f(x)$. That is, find a, b, and c in terms of R, S, and T.

   (c) Find $\int_{-1}^{1} f(x)\,dx$ in terms of R, S, and T.