Douglass Houghton Workshop, Section 1, Mon 04/13/26

Worksheet Quo Vadimus?

Last time we began thinking about how an epidemic spreads. For some population, we defined

$$\begin{aligned} S(t) &= \text{The fraction of people who are susceptible to infection,} \\ I(t) &= \text{The fraction of people currently infected,} \\ R(t) &= \text{The fraction of people who have been removed from the pool, and} \\ p &= \text{The probability of transmission when an infected person meets a susceptible person.} \end{aligned}$$

We named one susceptible person Bob, and established that if he interacts with n people each day, then

$$\text{Probability(Bob doesn't become infected on day } t) = (1-p)^{nI(t)}$$

1. So the probability that Bob *does* become infected on day t is

   $$1 - (\text{Your answer from the previous problem}).$$

   (a) Find the first two nonzero terms of the Taylor series about $x = 0$ for

   $$f(x) = (1-x)^k$$

   where k is some constant.

   (b) Use that to approximate the probability that Bob becomes infected on day t.

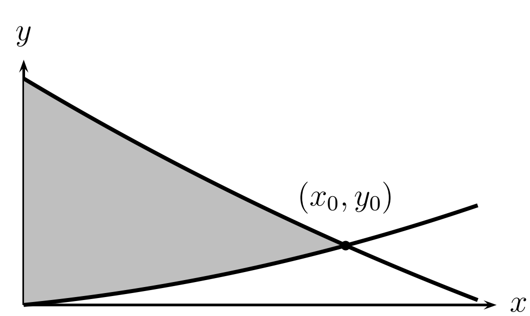
   (c) $S(t)$ is the fraction of the population who are like Bob, so multiply that last result by $S(t)$ to get the fraction of the population that becomes infected on day t.

2. So we can model the disease's spread by saying that the three types of people change accoring to these equations:

   $$\begin{aligned} S(t+1) &= S(t) - (\text{your answer to the last problem}) \\ I(t+1) &= I(t) + (\text{your answer to the last problem}) - vI(t) \\ R(t+1) &= R(t) + vI(t) \end{aligned}$$

   where v is the recovery rate. Write out those three equations—that's our model for the spread of the epidemic. We'll run it together with a spreadsheet.

3. (From the Fall, 2010 Math 116 final) The graph shows the area between the graphs of $f(x) = 6\cos(\sqrt{2x})$ and $g(x) = x^2 + x$. Let (x_0, y_0) be the intersection point between the graphs of $f(x)$ and $g(x)$.



(a) Compute $P(x)$, the function containing the first three nonzero terms of the Taylor series about $x = 0$ of $f(x) = 6\cos\left(\sqrt{2x}\right)$.

(b) Use $P(x)$ to approximate the value of x_0.

(c) Use $P(x)$ and the value of x_0 you computed in the previous question to write an integral that approximates the value of the shaded area. Find the value of this integral.

(d) Graph $f(x)$ and $g(x)$ in your calculator. Use the graphs to find an approximate value for x_0.

(e) Write a definite integral in terms of $f(x)$ and $g(x)$ that represents the value of the shaded area. Find its value using your calculator.

4. (Fall, 2014 (the robot chicken semester)) Franklin, your robot, is zipping around the kitchen making his famous "Definitely Not Poison!" soup. His coordinates in the xy-plane are given by the parametric equations

$$x = t^2 - t \qquad\qquad y = -\sin(\pi t)$$

t seconds after he starts making soup. Assume that both x and y are measured in meters.

(a) Calculate dx/dt and dy/dy.

(b) Find all times when Franklin's velocity is zero.

(c) Find Franklin's **speed** when $t = 2$ seconds.

(d) Write an integral that gives the distance traveled by Franklin during his first 5 seconds of zipping around.

5. (Adapted from a Fall, 2010 Math 116 Exam) In the picture to the right, the graphs of $r = 2$ and $r = 2 - \sin(5\theta)$ are shown.

(a) Write a definite integral that computes the shaded area.

(b) Compute the area exactly.

(c) Write an integral for the length of the boundary of the shaded area.

(d) Get an approximate answer for that length, using your calculator.