Douglass Houghton Workshop, Section 1, Mon 03/09/26

Worksheet Kimono

1. Consider the "Hard Eight" bet in craps. The bet wins on double fours (⚃⚃) and loses on "soft eight" (⚁⚅ or ⚂⚄) and on 7. If something other than a 7 or 8 is rolled, the bet stays through the next roll.

   (a) Draw the addition table below on the board and fill it in.

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

   (b) Calculate these probabilities:
   - $W =$ the probability of winning on the first roll.
   - $L =$ the probability of losing on the first roll.
   - $C =$ the probability that the game continues to a second roll.

   (c) Calculate the probability of winning on the *second* roll.

   (d) Calculate the probability of winning on the kth roll.

   (e) Calculate the probability of winning on *one of* the first n rolls.

   (f) Calculate the probability of winning the hard-eight bet.

2. (From the Winter, 2011 Math 116 final exam) For $n \geq 1$, identify the properties of the four sequences below.

nth term in the sequence	Bounded?	Increasing?	Converges?
$a_n = (-1)^n + \frac{1}{n}$			
$b_n = 1 + \frac{(-1)^n}{n}$			
$c_n = \left(\frac{6}{5}\right)^n$			
$s_n = \sum_{k=1}^{n} \frac{1}{k^2}$			

3. Write out the terms of each series.

   (a) $\displaystyle\sum_{i=0}^{6} i$

   (b) $\displaystyle\sum_{k=3}^{10} (-1)^k k^2$

   (c) $\displaystyle\sum_{\ell=1}^{5} x^{2\ell-1} \ell!$

   (d) $\displaystyle\sum_{m=0}^{\infty} \frac{(2m)!}{m!m!} x^m$

4. Evaluate the following:

   (a) $\displaystyle\sum_{i=1}^{100} 2$

   (b) $\displaystyle\sum_{i=1}^{100} 1/(i+3) - 1/(i+4)$

   (c) $\displaystyle\sum_{i=1}^{\infty} 1/(i(i+1))$
   (Hint: $(1/i) - (1/(i+1)) = ?$)

   (d) $1/6 + 1/12 + 1/20 + 1/30 + 1/42 + \cdots$

5. (Fall, 2015) Determine whether the following improper integrals converge or diverge.

   (a) $\displaystyle\int_1^{\infty} \frac{\ln x}{x^{3/2}}\, dx$

   (b) $\displaystyle\int_0^{1} \frac{x}{x^2 + x^{3/2}}\, dx$

6. (This problem appeared on a Winter, 2003 Math 116 exam) Fred likes to juggle. So does Jason. The number of minutes Fred can juggle five balls without dropping one is a random variable, with probability density function $f(t) = 0.8e^{-0.8t}$. Similarly, the function $j(t) = 1.5e^{-1.5t}$ describes Jason's skill. Here t is time *in minutes*.

   (a) Find $\int_0^\infty f(t)\,dt$.

   (b) What percentage of Jason's juggling attempts are "embarrassing," meaning they last for 10 seconds or less?

   (c) How long can Fred juggle, on average?

   (d) Who is the better juggler? Give a good reason for your decision.

7. (This problem appeared on a Winter, 2009 Math 116 exam) The quantity

$$\int_1^\infty \frac{dx}{\sqrt{(a^2+x)(b^2+x)(c^2+x)}}$$

roughly models the resistance that football-shaped plankton encounter when falling through water. Note that $a = 1$, $b = 2$, and $c = 3$ are constants that describe the dimensions of the plankton. Find a value of M for which

$$\int_1^M \frac{dx}{\sqrt{(a^2+x)(b^2+x)(c^2+x)}}$$

differs from the original model of resistance by at most 0.001. Hint: make use of the integral and the comparison test.

Recall that for a function of the form

$$\begin{aligned} f(x) = a_0 + a_1\cos(x) + a_2\cos(2x) + a_3\cos(3x)\ + \cdots \\ + b_1\sin(x) + b_2\sin(2x) + b_3\sin(3x)\ + \cdots \end{aligned} \tag{1}$$

integrating against a sine or cosine function makes almost all the terms 0, so for $n \geq 1$:

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx, \quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx, \quad \text{and} \quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx.$$

Writing a function in the form of Equation (1) is called finding the **Fourier Series** of the function.

8. Let's compute the Fourier series for $f(x) = x^2$.

   (a) Compute a_0.

   (b) Fill in the table to the right.

   (c) Find the a_n and b_n for $f(x) = x^2$.

n	1	2	3	4	5	6
$\sin(n\pi)$						
$\sin(-n\pi)$						
$\cos(n\pi)$						
$\cos(-n\pi)$						