Douglass Houghton Workshop, Section 1, Wed 02/25/26

Worksheet Jumbo Shrimp

With a lot of hard work, we filled the table to the right with the values of $\int_{-\pi}^{\pi} f(x)g(x)\,dx$, where f is the row and g is the column, and m and n are positive integers.

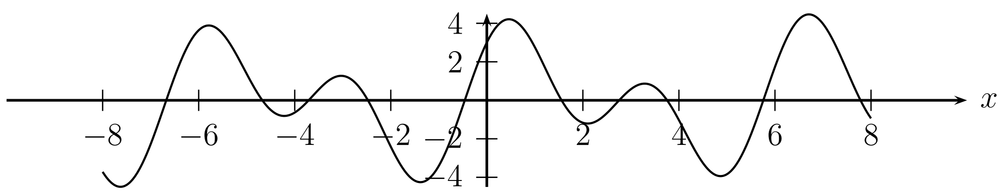
	1	$\sin(nx)$	$\cos(nx)$
1	2π	0	0
$\sin(mx)$	0	$\begin{cases} \pi & \text{if } m=n \\ 0 & \text{otherwise} \end{cases}$	0
$\cos(mx)$	0	0	$\begin{cases} \pi & \text{if } m=n \\ 0 & \text{otherwise} \end{cases}$

The implication was that if we know a function is of the form

$$\begin{aligned} f(x) = a_0 + a_1\cos(x) + a_2\cos(2x) + a_3\cos(3x) \; + \; \cdots \\ + \, b_1\sin(x) + b_2\sin(2x) + b_3\sin(3x) \; + \; \cdots \end{aligned} \tag{1}$$

then integrating against a sine or cosine function makes almost all the terms 0, so for $n \geq 1$:

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx, \quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx, \quad \text{and} \quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx.$$

1. 

Suppose $h(x)$ is some function you measure in nature, and its graph looks like the one above. You do some numerical integration and discover that

$$\int_{-\pi}^{\pi} h(x)\,dx = 0 \qquad \int_{-\pi}^{\pi} h(x)\cos(2x)\,dx = 6.28$$

$$\int_{-\pi}^{\pi} h(x)\cos(x)\,dx = 3.14 \qquad \int_{-\pi}^{\pi} h(x)\sin(2x)\,dx = 4.71$$

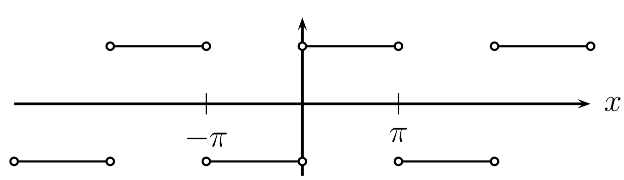
$$\int_{-\pi}^{\pi} h(x)\sin(x)\,dx = 6.28 \qquad \int_{-\pi}^{\pi} h(x)\cos(nx)\,dx = \int_{-\pi}^{\pi} h(x)\sin(nx)\,dx = 0 \text{ for } n \geq 3.$$

Can you guess a formula for $h(x)$? Use what you know, and check by graphing your formula on a calculator.

2. Consider the square wave:

$$f(x) = \begin{cases} -1 & \text{if } -\pi < x < 0 \\ 1 & \text{if } 0 < x < \pi \end{cases}$$

and that pattern is repeated every 2π.



Suppose the square wave can be written in terms of sines and cosines, as in Equation (1) above. Find the a_n and the b_n.