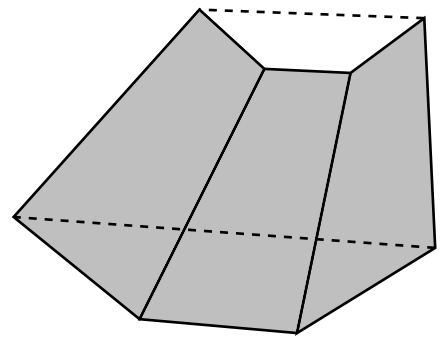
Douglass Houghton Workshop, Section 1, Mon 02/16/26

Worksheet Go Placidly Amid the Noise and Haste

1. The picture to the right shows a section of the Los Angeles river, whose sides are lined with concrete. It is currently full of water, but we need to empty it so we can film a car chase scene for a movie (as in *Terminator 2*, *Grease*, *Gone in 60 Seconds*, *Buckaroo Banzai*, etc.) It is 100 meters long, 17 meters deep, 40 meters wide at the top and 20 meters wide at the bottom. Find the work required to pump all the water up to the top of the river.



   (a) Draw a vertical cross-section of the river (i.e. a trapezoid) on the board. Label the dimensions you know.

   (b) Our slices will be slabs of water parallel to the surface. On your picture, draw a horizontal rectangle representing one end of the slice. Also, make up a variable to define where your slice is, relative to the top or the bottom (you decide). Put your variable on your picture.

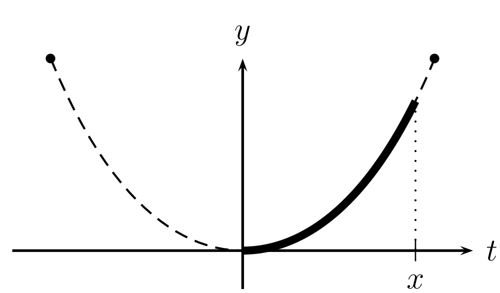
   (c) Find the width of the slice in terms of your variable. Hint: it should be a linear function, and you know two points of it.

   (d) Now find these quantities, in the order given. NO INTEGRALS UNTIL YOU FINISH THIS PART!

   - length of slice
   - volume of slice
   - mass of slice
   - weight of slice
   - dist to lift slice
   - work to lift slice

   (e) Find the total work it takes to pump out the river. NOW you can do an integral.

2. We're interested in finding an equation that describes the shape of a hanging chain. Clearly the shape is determined by the forces on the chain.



   (a) Consider the portion of the chain highlighted here. Draw it on the board, and draw arrows for all the forces that act on it.

   (b) Give the forces names. Given that the chain is not in motion, what must the forces sum to?

   (c) So how are your variables related? Write down as many equations as you can.

3. Evaluate $\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\,dx$ where m and n are positive integers. (You might want to graph a few examples.)

4. Find $\int_{-\pi}^{\pi} \sin(mx)\sin(nx)\,dx$, given that m and n are positive integers.

5. A spaceship seeks to reach a height H above the surface of the earth. The force of gravity at any time is

$$\boxed{F_g = G\frac{Mm}{r^2}}$$

$G =$ The universal gravitational constant
$M =$ The mass of the earth
$m =$ The mass of the spaceship
$r =$ The distance from the spaceship to the center of the earth.

   (a) How much work will it take to raise the spaceship from the surface of the earth to a point H meters above the surface? Use R for the radius of the earth. Don't assume gravity is constant as the ship moves upward!

   (b) How much work would it take to push the spaceship entriely beyond the reach of Earth's gravity? (Let $H \to \infty$.)

   (c) If the ship is travelling at velocity v, it will have kinetic energy $\frac{1}{2}mv^2$. That energy will be converted into work to move the ship upward. What speed must the ship be going near the surface to leave the earth's gravity well? This is the earth's *escape velocity.*

   (d) Look up the values of G, M, and R, and get a numerical answer in miles per second.

6. Sofi, working as a marine scientist, is reeling in a large shark she caught onto her boat. The edge of her boat lies 5 meters above the water as shown in the figure below. The total length of the sharking line is 30 meters. The shark weighs 500 newtons in water, and her sharking line weighs 30 newtons per meter out of water, and 10 newtons per meter in water. The figure below depicts this situation - the sharking line is the thick dark line and the boat is shaded. Write an expression which gives the work Sofi does pulling the shark's snout to the surface of the water.