Douglass Houghton Workshop, Section 1, Wed 01/28/26

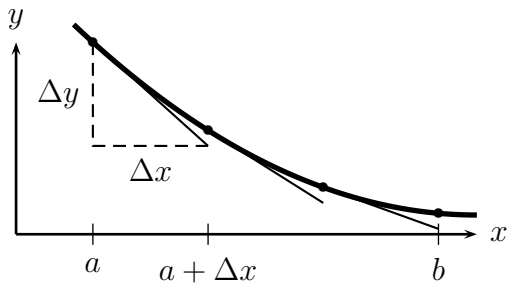
Worksheet Enter The Dragon

1. Find the area of the finite region that is bounded by the y-axis, the line $y = 1$, and the graph of $y = x^{1/4}$ in two ways:

   (a) By integrating with respect to x and

   (b) By writing x as a function of y and integrating with respect to y.

2. How can we compute the length of a curve $y = f(x)$? Consider cutting it up into small pieces, and approximating each piece with a line segment, as in the picture below.



   (a) How long is the first piece? It is tangent to the curve at a.

   (b) How long is the ith piece?

   (c) Write the left-hand Riemann sum for the length of the curve from a to b.

   (d) Now make it into an integral, which will be our formula for arc length.

3. A ball at an initial height h_0 is thrown straight up into the air, with an initial velocity v_0. Gravity causes the ball to accelerate downward at a constant rate, g. (This might be on another planet, so use g rather than $9.8\,\text{m/sec}^2$.)

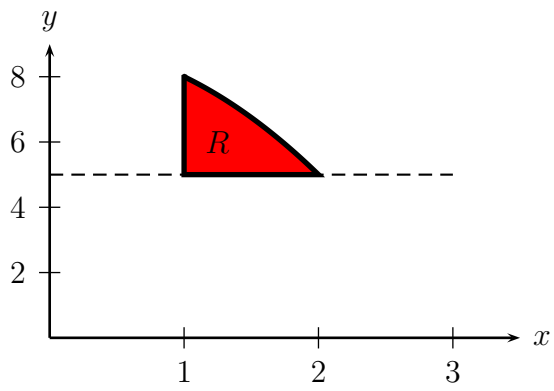
   (a) Find $v(t)$, the upward velocity of the ball at time t.

   (b) Find $h(t)$, the height of the ball at time t.

   (c) Calculate the quantity $mgh(t)+\frac{1}{2}mv(t)^2$. What do you notice about your answer?

   (d) Use part (c) to calculate the maximum height of the ball. Check using your Math 115 optimization skillz.

4. (Adapted from a Fall, 2011 Math 116 Exam) Consider the region R in the xy-plane bounded by the curves $y = 9 - x^2$, $x = 1$, and $y = 5$. This region is pictured below.



   (a) Find the area of R.

   (b) Find the volume obtained by rotating R about the y-axis. Do it with both shells and washers, and verify that the answer is the same.

   (c) Find the volume of the solid whose base is R and whose cross-sections perpendicular to the x-axis are squares.

5. Currently 95% of Michigan kindergarteners have been vaccinated for measles. The measles vaccine is 93% effective, meaning that 7% of vaccinated children who are exposed to the disease will contract it, and the rest will not. That contrasts with a 10% immunity among unvaccinated children.

   (a) Suppose that all children in the community are exposed to the measles vaccine, and fill in the following table of possibilities. For instance, the upper-left corner is the probability that a randomly-chosen child is vaccinated *and* contracts measles.

		Vaccinated?	
		Yes	No
Gets measles?	Yes		
	No		

   (b) What proportion of the students who contract measles were vaccinated?

   (c) What does that mean about whether you should vaccinate your child?

6. Evaluate $\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\,dx$ where m and n are positive integers. (You might want to graph a few examples.)

7. Find $\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\,dx$, given that m and n are positive integers.

8. (Winter, 2003) Below are the graphs of several functions $f(x)$, $g(x)$, $h(x)$, $i(x)$, $j(x)$, and $k(x)$. Do not assume that the y-axis scales on these graphs are equal or even comparable. We have calculated LEFT(6), RIGHT(6), TRAP(6), and MID(6) for four of these six functions. Label each column with the name of the function estimated in that column.

Function:				
LEFT(6):	64.2	.328	.255	80.0
RIGHT(6):	65.8	.444	.421	80.0
TRAP(6):	65.0	.386	.338	80.0
MID(6):	65.0	.388	.331	80.0