Douglass Houghton Workshop, Section 1, Mon 01/26/26

Worksheet Damn the Torpedoes

1. Let's practice some integration by parts.

   (a) $\int x^2 e^x \, dx$

   (b) $\int \ln x \, dx$

   (c) $\int e^x \sin x \, dx$

   (d) $\int_0^1 \tan^{-1}(x) \, dx$ Hint: $\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$

2. Write out the terms of each series.

   (a) $\sum_{i=0}^{6} i$

   (b) $\sum_{k=3}^{10} (-1)^k k^2$

   (c) $\sum_{\ell=1}^{5} x^{2\ell-1} \ell!$

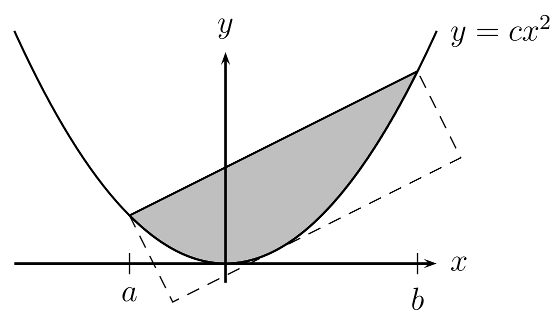
   (d) $\sum_{m=0}^{\infty} \frac{(2m)!}{m!m!} x^m$

3. Put the series in sigma ($\sum$) notation:

   (a) $9 + 16 + 25 + 36 + \cdots + 100$

   (b) $\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \cdots$

4. We're interested in the figure shown here. Last time we found that the slope of the solid line is $c(a+b)$, and the area of the trapezoid under the line is $\frac{1}{2}c(b-a)(a^2+b^2)$.



   (a) Find the area of the shaded region in the picture to the right. Make the answer as simple as possible.

   (b) Find the area of the dashed rectangle, which is tangent to the curve.

5. A ball at an initial height h_0 is thrown straight up into the air, with an initial velocity v_0. Gravity causes the ball to accelerate downward at a constant rate, g. (This might be on another planet, so use g rather than $9.8\,\text{m/sec}^2$.)

   (a) Find $v(t)$, the upward velocity of the ball at time t.

   (b) Find $h(t)$, the height of the ball at time t.

   (c) Calculate the quantity $mgh(t) + \frac{1}{2}mv(t)^2$. What do you notice about your answer?

   (d) Use part (c) to calculate the maximum height of the ball. Check using your Math 115 optimization skillz.

6. Evaluate $\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\,dx$ where m and n are positive integers. (You might want to graph a few examples.)

7. Suppose we want to compute $\int \frac{2x+5}{x^2-2x-3}\,dx$.

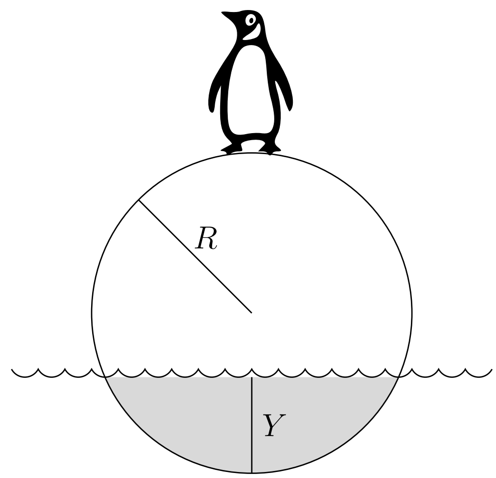
   (a) Factor the denominator into something like $(x-\alpha)(x-\beta)$.

   (b) Now reverse the process of finding a common denominator. That is, imagine the integrand can be written as
   $$\frac{A}{x-\alpha}+\frac{B}{x-\beta}$$
   for some constants A and B. Find what A and B have to be to make that the same as $\frac{2x+5}{x^2-2x-3}$.

   (c) Finally, rewrite the integral using the sum you found, and use substitution to solve it.

8. The buoyancy force on a floating object is proportional to the volume of water it displaces. Using this fact, whimsical ecologists plan to study the weights of penguins by floating beachballs on the ocean and enticing penguins to climb on top. They then measure the depth the ball sinks to, and thereby deduce the penguin's weight.

   So given a beachball of radius R that is partially submerged in the water, find a formula for the volume of the ball which is below the water line when its bottom is at depth Y. Check that your formula makes sense for the values $Y=0$, $Y=R$, and $Y=2R$.



9. (Fall, 2007) For this problem, $\int_1^5 g(x)\,dx = 12$ and $f(x)=2x-9$. Some values of $g(x)$ are:

x	1	2	3	4	5
$g(x)$	0.1	1.5	2	5	10

   (a) Find $\int_5^7 g(f(x))\,dx$. (b) Find $\int_1^5 f(x)g'(x)\,dx$.

   (c) Find $\int_1^5 \frac{g'(x)}{g(x)(g(x)+1)}\,dx$.