Douglass Houghton Workshop, Section 1, Wed 01/21/26

Worksheet Casino

1. Let's practice some substitution.

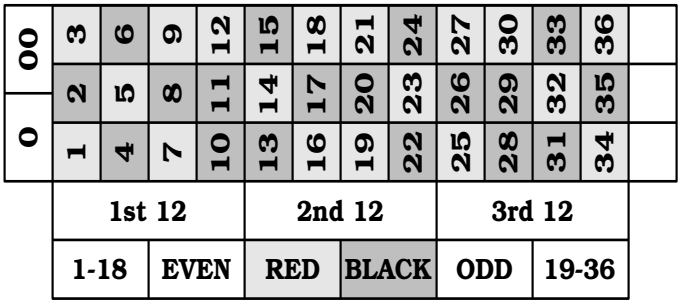
   (a) $\displaystyle\int z(z+3)^{1/3}\,dz$

   (b) $\displaystyle\int \frac{dx}{2+2\sqrt{x}}$

   (c) $\displaystyle\int_{-3}^{0}(z+2)\sqrt{1-z}\,dz$

   (d) $\displaystyle\int_{4}^{12}\frac{3x-2}{\sqrt{2x+1}}\,dx$

2. Let's calculate some probabilities for Roulette.



   (a) Suppose I put a chip on "3". That means I win if and only if the ball lands on 3. What is the probability that I win?

   (b) Fill in the table to the right with probabilities of winning the bets shown.

   (c) Suppose I keep putting a dollar chip on "red" all night long, say for 200 games. On average, how many times will I win?

   (d) The red bet pays 1:1, meaning that if I win, then I get my original dollar back, plus one more dollar. If I start the night with \$200, and play 200 times, what is the least I can have at the end? What's the most? What will I have on an average night?

Bet	**Prob**
1 or 2	
1 or 2 or 4 or 5	
odd	
red	
both odd and red	
either odd or red	

   (e) Suppose you bet on red every time the wheel spins. What's the probability you win the first two bets? How about the first 3? How about the first n?

3. Find $\displaystyle\frac{d}{dx}\int_{\cos x}^{3} e^{t^2}\,dt$.

4. Find $\displaystyle\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt$.

5. A ball at an initial height h_0 is thrown straight up into the air, with an initial velocity v_0. Gravity causes the ball to accelerate downward at a constant rate, g. (This might be on another planet, so use g rather than $9.8\,\text{m/sec}^2$.)

   (a) Find $v(t)$, the upward velocity of the ball at time t.

   (b) Find $h(t)$, the height of the ball at time t.

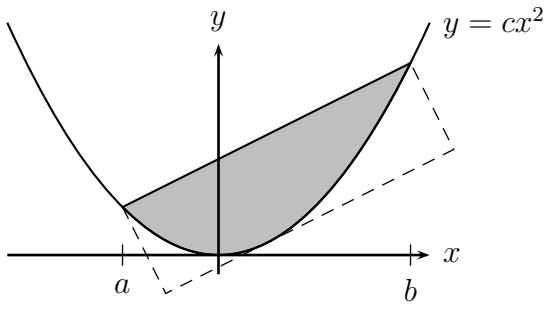
   (c) Calculate the quantity $mgh(t)+\frac{1}{2}mv(t)^2$. What do you notice about your answer?

   (d) Use part (c) to calculate the maximum height of the ball. Check using your Math 115 optimization skillz.

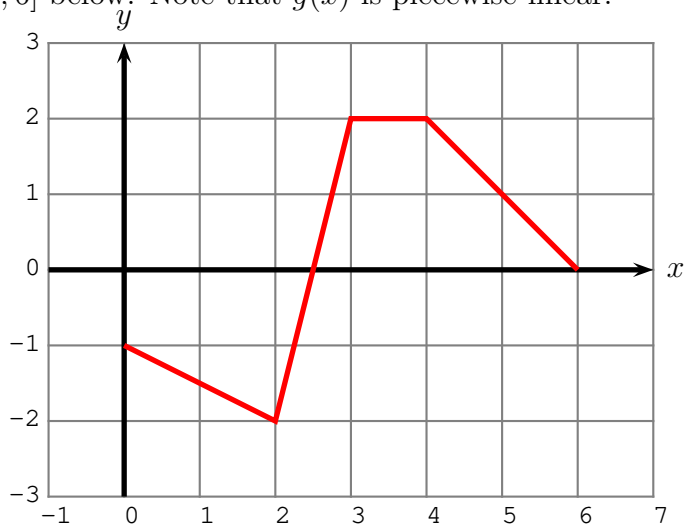
6. We're interested in the figure shown here. Last time we found that the slope of the solid line is $c(a+b)$, and the area of the trapezoid under the line is $\frac{1}{2}c(b-a)(a^2+b^2)$.

   (a) Find the area of the shaded region in the picture to the right. Make the answer as simple as possible.

   (b) Find the area of the dashed rectangle, which is tangent to the curve.



7. (Winter, 2024) An **even** function $g(x)$, which is defined for all real numbers, is graphed on the interval $[0,6]$ below. Note that $g(x)$ is piecewise linear.



   (a) The function $g(x)$ has a continuous antiderivative, $G(x)$, which satisfies $G(2) = -2$. Complete the following table of values for $G(x)$.

x	-6	-4	-2	0	2	4	6
$G(x)$					-2		

   (b) Sketch a graph of $G(x)$ on the interval $[-6,6]$. Make sure to clearly label the values at the points in your table above and also make it clear where $G(x)$ is increasing or decreasing, and where $G(x)$ is concave up, concave down, or linear.